Transmitter Precoding for Insufficient-Cyclic-Prefix Distortion in Multicarrier Systems

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Abstract—Multicarrier modulation partitions an inter-symbol interference channel into multiple narrowband channels if the length of the cyclic prefix is longer than the channel impulse response. Otherwise, interference from the current and previous multicarrier symbols causes significant distortion at the receiver. This paper expands on the approach introduced in [1], where a transmitter based precoder suppresses insufficient-cyclic-prefix distortion. In this paper, the problem of finding an approximate optimal linear precoder to maximize the data-rate in the presence of inter-carrier interference (ICI) due to insufficient-cyclic-prefix distortion is formulated and shown to be a convex optimization problem. The paper further explores low-complexity transmitter equalization and joint transmitter-receiver processing schemes. The proposed methods result in a novel low-complexity insufficient-cyclic-prefix distortion reduction algorithms to maximize the data-rate.

I. INTRODUCTION

Multicarrier techniques have been adopted in many wireline and wireless communication systems because of their ability to divide the bandwidth into multiple independent narrowband tones while maintaining high spectral efficiency. An efficient implementation of multicarrier modulation is accomplished by encoding the symbol directly in the frequency domain using a cyclic prefix to the time-domain signal. The resulting tones are orthogonal to each other as long as the cyclic prefix is longer than the channel impulse response. Otherwise, the system suffers from insufficient-cyclic-prefix distortion, which is composed of inter-carrier-interference (ICI) and inter-symbol-interference (ISI).

Because of the severity of this distortion, the problem of alleviating insufficient-cyclic-prefix distortion has received a great deal of attention. Following the early work in [2], where the authors shorten the channel to reduce the complexity of maximum likelihood sequence estimation (MLSE), the authors in [3] propose a time-domain equalizer (TEQ) for Digital Subscriber Line (DSL) systems. The window approach introduced by Melse et al. [4] designs a filter to maximize the signal to interference ratio over a specific window of interest. More sophisticated approaches for TEQ design that attempt to directly maximize the bit-rate were proposed in [5],[6],[7]. On the other hand, unlike the time-domain approaches, equalization after the receiver FFT, where the contributions from the guard period or from the unused tones are used to aid the equalization, are proposed in [8] and [9], respectively.

More recently, transmitter based approaches have also been considered. In [1], the insufficient-cyclic-prefix distortion was eliminated by a precoder at the transmitter. An efficient implementation of the precoder in [1] was proposed in [10] for the case of transmission with no cyclic prefix. Based on the work by Troutmann and Fliege [9], Park and Im [11] used a low-complexity precoder to fully cancel the insufficient-cyclic-prefix distortion. The work in [12] extended Park and Im’s results [11] to an MMSE based precoder by optimizing the correlation between data-tones and (unused) reserved-tones. The work in [1] did not fully take into account the inherent receiver noise and the transmitter power constraint. For some channels, the resulting precoder will result in a transmit signal that could exceed the transmitter power budget and scaling down the precoder coefficients to satisfy the transmitter power constraints results in a significant data-rate loss. Moreover, the precoder in [1] essentially performs a matrix inversion and is prohibitively complex. The complexity is significantly reduced in [10] but the implementation is only applicable for systems with zero cyclic prefix and still suffers from the power increase problem. In this paper the results in [1] are extended by simultaneously optimizing the power allocation and the target distortion level to maximize the achieved throughput. Then, low-complexity precoders that still result in significant data-rate gains are explored.

II. PROBLEM FORMULATION

Following the same development as in Cheong [1], let \( h_0, h_1, \ldots, h_L \) denote the channel impulse response with \( L+1 \) taps. Also assume that the multicarrier system has \( N \) tones with a cyclic prefix of length \( \nu \), where \( \nu < L < N \). Let the \( N \times 1 \) vector \( x_k \) denote the \( k \)th transmitted multicarrier symbol so that the \( N \)-point received vector at time \( k \) (after prefix removal and taking the FFT at the receiver) is

\[
y_k = Q^H H Q x_k + Q^H G Q x_k + Q^H F Q x_{k-1} + n_k,
\]

where, without loss of generality, \( n_k \) is an independent circularly symmetric complex Gaussian vector of unit variance at time \( k \), \( Q \) denotes the \( N \times N \) unitary IFFT matrix, \( (\cdot)^H \) denotes the Hermitian transpose, \( H \) is the circular \( N \times N \) matrix channel response corresponding to the case when there
is sufficient cyclic prefix,

\[ \tilde{H} = \begin{bmatrix} h_0 & \cdots & h_L & 0 & \cdots & 0 \\ 0 & h_0 & \cdots & h_L & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_1 & \ldots & h_L & 0 & \cdots & h_0 \end{bmatrix}, \]  \quad (2)

and \( G \) and \( F \), respectively, represent the intra-symbol (ICI) distortion and inter-symbol (ISI) distortion caused by the last \( L - \nu \) channel taps and are given as,

\[ G = \begin{bmatrix} 0_{(N-L+\nu) \times \nu} & 0 & 0 \\ 0_{(L-\nu) \times \nu} & -H_t & 0_{(L-\nu) \times (N-L)} \end{bmatrix}, \quad (3) \]
\[ F = \begin{bmatrix} 0_{(N-L+\nu) \times (L-\nu)} & 0_{(N-L+\nu) \times (N-L+\nu)} \end{bmatrix}, \quad (4) \]

where,

\[ H_t = \begin{bmatrix} h_L & 0 & \cdots & 0 \\ h_{L-1} & h_L & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ h_{\nu+1} & h_{\nu+2} & \cdots & h_L \end{bmatrix}. \quad (5) \]

### III. Proposed Precoder

As in [11] and [12], it will be assumed that the distortion from the previous transmit symbol, \( Q^H F Q x_{k-1} \) in (1), is cancelled at the receiver since it is already detected by the time \( x_k \) is processed at the receiver. Or, as in [1], the distortion from the previous symbol is removed at the transmitter using modulo precoding. Note that removing the previous symbol is a relatively low-complexity operation involving an additional FFT and a matrix multiplication using the sparse matrix \( F \) defined in (4).

Therefore, the focus will be on the ICI term \( Q^H G Q x_k \). With the distortion from the previous symbol removed, the effective channel for the current multicarrier symbol is given by

\[ y_k = Q^H \tilde{H} Q x_k + Q^H G Q x_k + n_k \]
\[ = A x_k + n_k, \quad (6) \]
\[ = A x_k + n_k, \quad (7) \]

with \( A = Q^H \tilde{H} Q + Q^H G Q \). The transmit precoding is applied at the transmitter through the matrix \( W \) so that at the receiver the output is given by

\[ y_k = AW x_k + n_k. \quad (8) \]

The approach in [1] essentially chooses \( W \) to be

\[ W = A^{-1} A, \quad (9) \]

where \( A = Q^H \tilde{H} Q \) is the diagonal FFT channel gains matrix. The precoder in [1] inverts the channel and then forces the same tonal FFT channel gains as would have resulted if sufficient cyclic prefix was present.

While this approach performs well when the channel inversion does not result in a significant power increase, for some channel (see section VI) realizations the data-rate loss is significant. Since all communication systems operate with inherent noise, fully cancelling the distortion could result in a drastic power increase, and so it is not always beneficial to reduce the insufficient-cyclic-prefix distortion level below the receiver’s noise floor.

In this section, the work in [1] will first be extended by including the power constraint and the data-rate objective when designing the precoder matrix. Note first that with the precoder matrix \( W \), the effective channel is given by \( C = \text{diag}(AW) \), where \( \text{diag}(X) \) denotes the diagonal matrix with the same diagonal as \( X \). The ICI includes all the off-diagonal entries of the matrix \( AW \) because in a standard multicarrier receiver the processing is limited to a single-tap frequency-domain equalizer (FEQ). The ICI matrix is therefore given by \( V = AW - C \) and the ICI distortion power for the \( n \)th tone is given by \( [VV^H]_{nn} \). The resulting data-rate from the application of the precoder is given by

\[ b = \sum_n \log_2 \left[ \frac{1 + \frac{|C_m|^2}{1 + [VV^H]_{nn} \Gamma}}{1 + rac{|C_m|^2}{1 + [VV^H]_{nn} \Gamma}} \right], \quad (10) \]

where \( \Gamma \) denotes the gap from capacity for the used channel-code.

As mentioned above, entirely removing the ICI is not necessarily the best approach and so the average ICI plus noise magnitude per-tone is restricted to a level of \( m_t \). The objective (data-rate) in the optimization problem (11) below makes the approximation of a uniform distortion level for all the tones, while the distortion constraint actually only requires the average per-tone distortion level to be \( m_t \). Note that with a constant ICI distortion magnitude per tone, maximizing the data rate is equivalent to maximizing the log-determinant of \( I + \frac{CC^H}{\Gamma m_t^2} \). With this average ICI approximation the problem of finding the best linear precoder \( W \) to maximize the data rate is given by:

\[ \max_W \log \left| I + \frac{CC^H}{\Gamma m_t^2} \right| \]
\[ \text{s.t.} \quad \|W\|_F \leq \sqrt{P} \]
\[ 1 + \frac{1}{N} \|V\|^2_F \leq m_t^2 \]
\[ C = \text{diag}(AW) \]
\[ V = AW - C, \]

where \( \| . \|_F \) denotes the Frobenius norm, \( P \) is the power constraint, and \( m_t \) is the approximation of background noise and ICI magnitude per data-tone. The optimization variable above is only the precoder matrix - all other variables are determined by \( W \) - since the distortion level \( m_t \) is assumed fixed. The total ICI power is taken as the sum of the powers of the off-diagonal entries of the matrix that results from the application of the precoder (\( V \) above). In the optimization problem above, no specific values are forced for the tonal channel gains (\( C \) above) but, rather, the optimization finds the best structure subject to the constraint on the ICI power. Note also that this problem is similar to the problem of finding the best linear beamformer, except that further constraints will be shortly introduced to make the precoder more practical.
Unfortunately, the optimization problem (11) above is not convex because of the maximization of the product of quadratic terms

$$\left| I + \frac{CC^H}{\Gamma m_t^2} \right| = \prod_{n=0}^{N-1} \left( 1 + \frac{|C_{nn}|^2}{\Gamma m_t^2} \right).$$

(12)

For a closely achievable lower bound, therefore, the unity term above is ignored and (11) is converted to the following convex optimization problem:

$$\max_{W} \log \left| \frac{C}{\Gamma m_t} \right|$$

s.t. \[ W \|_F \leq \sqrt{\frac{P}{u}} \] \[ 1 + \frac{1}{N} \|V\|_F^2 \leq m_t^2 \] \[ C = \text{diag}(AW) \] \[ V = AW - C. \]

(13)

Note that there is no problem with assuming that the diagonal matrix $C$ has positive entries since for any complex diagonal entries of $C$, the precoder matrix $W$ could be multiplied by the appropriate diagonal complex phasor matrix without violating any of the constraints in (13). If any of the above channel gains are small (making the ignored unity term in (13) significant) then these tones will not be used for data transmission and will serve to reduce the interference for the data-carrying tones using the tone-reservation ideas proposed in [12] and [11]. The resulting precoder, with all the approximations, still performs very well, as will be seen in section VI.

IV. OPTIMAL DISTORTION LEVEL

In the optimization problem above it is not immediately clear what should be the optimal average per-tone distortion level target $m_t$ that the optimization problem (13) is solved for. As mentioned above, making it smaller than the background noise level will not be optimal. This will be even more significant when lower-complexity precoders that cannot fully cancel the ICI distortion are explored in the next section. But even knowing that $m_t$ should not be too small, it is not obvious how to choose the optimal distortion level.

Making $m_t$ a variable in the optimization problem above would make the problem non-convex since the ICI inequality will contain strictly convex functions on both sides and the objective will be a ratio of convex functions. Fortunately, by making the substitution $u = \frac{1}{m_t^2}$, the optimal $m_t$ could also be solved for as part of the optimization problem (13). By dividing the ICI constraint by $m_t^2$ and all the other constraints (and objective) in (13) by $m_t$ the following convex optimization problem results:

$$\max_{W,u} \log \left| \frac{C}{\Gamma} \right|$$

s.t. \[ W \|_F \leq u \sqrt{\frac{P}{u}} \] \[ u^2 + \frac{1}{N} \|V\|_F^2 \leq 1 \] \[ C = \text{diag}(AW) \] \[ V = AW - C. \]

(14)

The actual precoder, ICI, and tonal gain matrices are recovered as $W = \frac{1}{u} W, V = \frac{1}{u} V, C = \frac{1}{u} C$. The optimization variables here are the precoder matrix $W$ and the interference level $u (\frac{1}{m_t})$. This optimization will result in finding the optimal average distortion level $m_t$ jointly with the optimal precoder matrix to maximize the achieved data-rate. This precoder will be termed the Optimal ICI precoder.

V. REDUCED COMPLEXITY PRECODERS

A. Reduced Complexity ICI Precoder

While the Optimal ICI precoder above results in a better performance than simply inverting the channel, the complexity is still very high as the required matrix multiplication is proportional to $N^2$. To reduce the complexity further the precoder matrix $W$ could be made sparse by forcing many taps to zero. For example, inspired by the work in [13], where the authors show that the insufficient-cyclic-prefix distortion is mostly due to neighboring tones in the frequency domain, jointly processing neighboring data-tones at the transmitter will be beneficial. Therefore, to limit the complexity only a few non-zero coefficients are allowed in every row of the precoder matrix. The optimization problem for the reduced complexity Optimal ICI precoder with $2K + 1$ non-zero taps in every row of the precoder matrix is given by:

$$\max_{W,u} \log \left| \frac{C}{\Gamma} \right|$$

s.t. \[ W \|_F \leq u \sqrt{\frac{P}{u}} \] \[ u^2 + \frac{1}{N} \|V\|_F^2 \leq 1 \] \[ W_{mn} = 0 \ \forall \ |m - n| > K \] \[ C = \text{diag}(AW) \] \[ V = AW - C. \]

(15)

The low-complexity (banded) precoder matrix for the case of $N = 64$ and $K = 2$ is illustrated in Figure 3. This significantly reduces the precoder complexity, with the number of complex multiply-accumulate operations (CMACs) going from $N^2$ in the full-matrix inversion to $(2K + 1)N$ for the reduced complexity precoder.

B. Joint Transmitter-Receiver ICI Precoder

More interestingly, it is possible to easily adopt the optimization problem above to allow joint transmitter-receiver processing to achieve higher data-rates. In the receiver it is possible for a tone to reduce the interference it suffers from other detected tones by removing their contribution, similarly to the well known decision feedback equalizer (DFE) structure. This effective upper-triangular (or lower-triangular) channel structure could be forced in the optimization problem by not considering the strictly upper-triangular portion of the resulting channel as interference. However, since a low-complexity operation is desired only a small subset of the interfering tones will be removed for every tone. Again using the intuition from the results in [13], the $T$ right-adjacent tones are cancelled for
every tone. The optimization problem with limited receiver feedback processing is then:

\[
\max_{\mathbf{w}, u} \log \left| \frac{\mathbf{C}}{\mathbf{F}} \right|
\]

s.t. \[ \| \mathbf{W} \|_F \leq u \sqrt{\mathbf{P}} \]

\[
u^2 + \frac{1}{N} \sum_{j-i>T, j-i<0} |\mathbf{V}_{ij}|^2 \leq 1
\]

\[ \mathbf{W}_{mn} = 0 \quad \forall \quad |m-n| > K \]

\[ \mathbf{C} = \text{diag}(\mathbf{AW}) \]

\[ \mathbf{V} = \mathbf{AW}. \]

The resulting effective channel and ICI with limited feedback processing for \( N = 64 \) and \( T = 1 \) is illustrated in Figure 4. Note that for a decision feedback system the performance is still determined by the resulting diagonal channel entries and so the optimization objective in (16) is not changed. The total complexity at the receiver is approximately \( (T+1)N \) (including the FEQ), for a total transceiver complexity of \((2K+1+T)N\) CMACs for the ICI suppression. The feedback processing across the entire tones might result in additional latency (besides the standard FFT) and also potential error propagation. These problems could be handled by forcing a block upper-triangular structure for the channel. This will allow independent feedback processing within a block and will also limit the error propagation to within that block. Moreover, the feedback processing could be moved to the transmitter and implemented using techniques similar to the modulo precoding used in [1] to remove the insufficient-cyclic-prefix distortion caused by the previous multicarrier symbol. Intuitively, the fact that the \( T \) right-adjacent entries are not considered as ICI terms will compel the optimization to design the precoder matrix such that these feedback terms will contain much of the “ICI energy”.

C. Complexity

A standard MMSE TEQ with \( r \) taps has a complexity of \( (N + \nu)r \) CMACs per multicarrier symbol because of the filtering operated performed at the sample rate. The ISI removal (see [1]) operation common to all the proposed precoders requires an additional FFT and an additional matrix multiplication with \((L-\nu)(L-\nu+1)/2\) non-zero entries for a total of \(N\log_2N + (L-\nu)(L-\nu+1)/2\) CMACs. The removal of ICI requires an additional \((2K + 1 + T)N\) operations, where \(K\) and \(T\) denote the number of precoder and feedback taps per tone, respectively. Therefore, the reduced-complexity precoder requires \((\log_2N + 2K + 1 + T)N + \frac{(L-\nu)(L-\nu+1)}{2}\) CMACs. Cheong’s precoder [1] requires \(N^2\) CMACs for the ICI removal and \(N\log_2N + (L-\nu)(L-\nu+1)/2\) CMACs for ISI removal. The complexity of per-tone equalization [8] and the complexity of window approach [4] are comparable to complexity of the TEQ. The complexity results are summarized in Table I.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity (in CMACs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheong’s Precoder</td>
<td>(N\log_2N + N^2 + (L-\nu)^2/2)</td>
</tr>
<tr>
<td>TEQ</td>
<td>((N + \nu)r)</td>
</tr>
<tr>
<td>Per-Tone</td>
<td>((N + \nu)r)</td>
</tr>
<tr>
<td>Optimal ICI Precoder</td>
<td>(N\log_2N + N^2 + (L-\nu)^2/2)</td>
</tr>
<tr>
<td>Reduced-Complexity Precoder</td>
<td>((2K + 1 + \log_2N)N + (L-\nu)^2/2)</td>
</tr>
<tr>
<td>Joint Tx-Rx ICI precoder</td>
<td>((2K + 1 + T + \log_2N)N + (L-\nu)^2/2)</td>
</tr>
</tbody>
</table>

VI. Simulations

To illustrate the proposed ideas, the performance of the algorithms is simulated for a standard wireless channel model. For these simulations, the system parameters were \(N = 64, \nu = 4\), and \(L = 14\). The 15 tap complex baseband channel was generated from a Rayleigh distribution with exponentially decaying parameter of \(a = .98\). The maximal tap of the channel was normalized to have a magnitude of 1. The resulting data rates were averaged over 500 random channel realizations and Figure 1 shows the performance of the proposed precoder and a few other standard channel-shortening algorithms for a range of channel SNR values. With the complex AWGN noise variance normalized to 1, the transmit power available to be used for a given SNR value was \(P = SNR \times N\).

Also plotted in Figure 1 is the Matched Filter Upper Bound (MFB) on the data-rate, which unrealistically assumes that there was sufficient cyclic prefix and all the channel energy is useful for data transmission (no ICI or ISI). For that upper bound, water-filling strategy to maximize the data-rate was employed. The figure also shows the severe degradation in data-rate that results with flat energy loading without mitigation of insufficient-cyclic-prefix distortion, and the degradation when the precoder proposed in [1] is applied to this wireless channel. Notice that the insufficient-cyclic-prefix distortion is quite severe as the channel memory beyond the cyclic prefix is almost 1/6 as long as the multicarrier symbol period.

Figure 2 illustrates the performance of the reduced-complexity variations of the Optimal ICI precoder. The low-complexity transmitter based precoder \((K=2)\) in Figure 2) shows the achievable data-rate with a 5-tap tonal precoder and standard FEQ processing at the receiver. Also illustrated is the joint transmitter-receiver \((K=2,T=1)\) in Figure 2) processing approach where once again a 5-tap tonal precoder is applied at the transmitter but a triangular structure with 1 additional feedback tap is allowed at the receiver. While the low-complexity transmitter precoding approach performs well, the low-complexity joint transmitter-receiver processing performs as well as the full-blown transmitter precoder, illustrating the usefulness of even very limited joint transmitter-receiver processing.

Notice also that with flat loading the data-rate soon saturates with higher \(SNR\) due to self-interference. The performance of the MMSE TEQ [3] and per-tone equalization [8] is also plotted in Figure 2. The MMSE TEQ performs poorly for this wireless channel because of the restrictive stationary equalizer and the difficulty of inverting an FIR channel.
The per-tone equalization performs much better as each tone has its own shortening filter. The complexity of the MMSE TEQ and the per-tone equalizer was made comparable to the reduced-complexity Optimal ICI precoders by working with a TEQ and the per-tone equalizer was made comparable to the

Fig. 2. Performance Comparison of Low-Complexity Channel-Shortening Equalizers.

VII. CONCLUSION

This work presented a new transmitter-based precoder to combat insufficient-cyclic-prefix distortion in multicarrier systems. A problem formulation that determines the optimal target distortion level to maximize the data-rate was introduced and shown to be a convex optimization problem that could be solved very efficiently. Low-complexity transmitter and joint transmitter-receiver processing were explored and shown to perform well. The approach is especially useful in situations where the average channel memory is short but occasionally longer channel responses can result. In such situations, without significantly modifying the access protocol and burdening the low-power mobile receivers, high data-rates are still achievable with limited additional complexity at the transmitter.

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