An automatic direct method of interpretation of magnetic anomalies using the Hilbert transform

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AN AUTOMATIC DIRECT METHOD OF INTERPRETATION OF MAGNETIC ANOMALIES USING THE HILBERT TRANSFORM

Summary. A fast simple method to uniquely determine the physical parameters of semi-infinite dykes and horizontal cylinders is described in detail. A small desk-top calculator is necessary to carry out the interpretation.

Riassunto. Viene descritto in dettaglio un metodo semplice e veloce per determinare univocamente i parametri fisici di due strutture: il dicco semi-infinito ed il cilindro orizzontale. Per eseguire l'interpretazione è sufficiente l'utilizzo di un calcolatore da tavolo.

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1. Introduction

The need of a simple automatic direct method of interpretation is, for long, felt by most workers in the field of magnetics. The method described hereafter is intended for determining uniquely the physical parameters of two dimensional structures (the semi-infinite dyke and the horizontal cylinder). It is flexible enough to encompass total, vertical and horizontal field measurements at all latitudes. Linear regional gradients do not hamper the application of the method. Remanent magnetization does not affect the determination of the main parameters i.e. depth and location. Unlike most other methods no knowledge of origin or datum level is required. A similar scheme has been proposed by Green & Stanley (1975) for the contact anomaly.

2. Theory

The magnetic flux density $\triangle F$, in any arbitrary direction, measured along a profile normal to the strike of a semi-infinite dyke is given by the general formula (Koefoed, 1972/ unpublished notes):

$$\triangle F = (KT\triangle S \sin I \cos\beta . \varepsilon)/2\pi \sin I' \,[\cos(\xi' + i' - \alpha)\, z/(x^2 + z^2) + \\ + \sin(\xi' + i' - \alpha)\, x/(x^2 + z^2)]. \quad (1)$$

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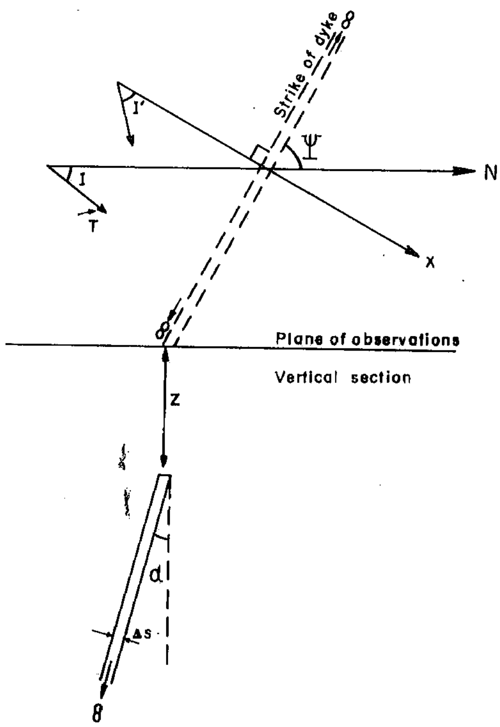

Fig. 1 — Geometrical and spatial relationships of the thin semi-infinite dyke.

Symbols not explained in Fig. 1 refer to:

K: magnetic susceptibility contrast (rationalized)
T: total geomagnetic flux density (Gammas)
i: angle between polarization and the horizontal plane
i': the projections of angle i on the x-z plane
ε: equals $[tan^2\ (I' - \alpha) + 1/(1 + K)^2]\ /\ [tan^2\ (I' - \alpha) + 1]$
β: angle between direction of measurements and its projection on the x-z plane
ξ': angle between the projected direction of measurements, on the x-z plane and the x axis (equals I', 90° or 0°)
I: The inclination of the normal geomagnetic field
I': the angle between the projection of the geomagnetic field on the x-z plane and the x-axis $= tan^{-1}\ (tanI/sin\psi)$
ψ: angle between strike of dyke and the magnetic north.

Partial differentiation of equation (1) yields:

$$(\partial/\partial\ x)\ \triangle F = (C/R^2)\ sin\ (2\theta\text{-}\gamma) \tag{2}$$

and

$$(\partial/\partial\ z)\ \triangle F = (C/R^2)\ cos\ (2\theta\text{-}y) \tag{3}$$

where $C = (KT\ sin\ I\triangle Scos\beta.\varepsilon)/2\pi\ sin\ I'$, $\gamma = \xi' + i' - \alpha$, $R = (x^2 + z^2)^{1/2}$, *and* $\theta = tan^{-1}\ (x/z)$.

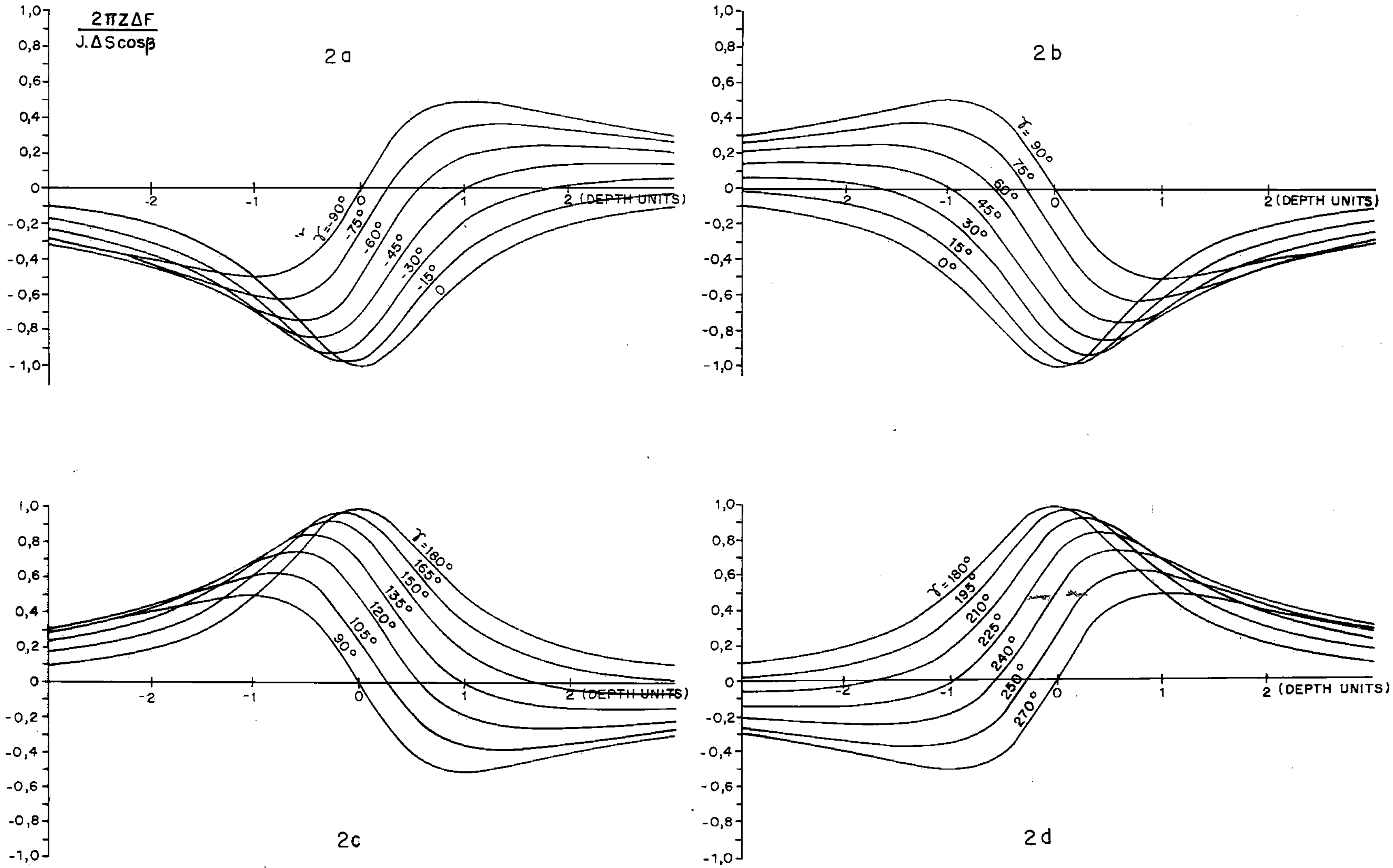

Fig. 2 — Master curves for the semi-infinite dyke: a) $(\xi' + i' - \alpha)$ from − 90 to 0 degrees,
b) $(\xi' + i' - \alpha)$ from 0 to 90 degrees,
c) $(\xi' + i' - \alpha)$ from 90 to 180 degrees,
d) $(\xi' + i' - \alpha)$ from 180 to 270 degrees.

Equation (3) turns out to be the Hilbert transform (Hi_T) of equation (2) i.e.

$$(\partial/\partial z)\ \triangle F = Hi_T\ [(\partial/\partial x)\ \triangle F]. \qquad (4)$$

The Hilbert transform of $f(x)$ is defined as

$$Hi_T(x) = 1/\pi \int_{\infty}^{\infty} [f(x')\ dx'] / (x' - x) \quad \text{(Bracewell, 1965);} \qquad (5)$$

by analogy with signal analysis and optics the amplitude function

$$|A(x)| = [\{(\partial/\partial z)\ \triangle F\}^2 + \{(\partial/\partial x)\ \triangle F\}^2]^{1/2} = C/R^2 \qquad (6)$$

and the phase

$$\upsilon\ (x) = tan^{-1}\ [(\partial/\partial x)\ \triangle F\ /\ (\partial/\partial z)\ \triangle F] = (2\theta\text{-}\gamma). \qquad (7)$$

Using similar notation as for the dyke, the magnetic flux density component $\triangle F$ due to a horizontal cylinder is given by:

$$\triangle F = (C/R^2)\ cos\ (\gamma\text{-}2\theta) \qquad (8)$$

where $C = -(TK\, sin\, I \triangle S\, cos\, \beta)/\pi\,(2 + K)\, sin\, I'$ and $\gamma = (\xi' + i')$, R and θ as in (3) and $\triangle S$ is the cross sectional area of the cylinder.

Now

$$Hi_T\ (\triangle F) = (C/R^2)\ sin\ (\gamma\text{-}2\theta),$$

$$|A\ (x)| = [Hi_T^2\ (\triangle F) + \triangle F^2\,]^{1/2} = C/R^2 \qquad (9)$$

and

$$\upsilon\ (x) = tan^{-1}\ [Hi_T\ (\triangle F)/\triangle F] = (2\theta\text{-}y). \qquad (10)$$

The amplitude function $|A\ (x)|$ has one maximum at $x = 0$ and half maximum at $x = z$ i.e. both the depth and location of the causitive body can be determined uniquely. The phase $\upsilon\ (x)$ has the interpretive significance that it is equal to $-\gamma$ at the origin.

3. Applications

The location and depth of the dyke are determined from $|A\ (x)|$. A good estimation

of depth is obtained from the average of the two values of $(X_{max} - X_{1/2\,max})$ and location at $|A\ (x)|$ max.

The phase angle $\upsilon\,(0) = -\gamma = -\,(\xi' + i' - \alpha)$. As the arctangent function is multi-valued, we should know beforehand, the quadrant of angle γ. This can be achieved by comparison of the field curve with the set of master curves of Fig. 2. Angle ξ' is equal to I', 90° or 0° in case of total, vertical or horizontal field measurements, respectively. Angle ψ is read from the flux density map as the angle between the magnetic north and the long axis of the anomaly. The inclination I is obtained from published magnetic charts. Entering, (Fig. 3) with the known values of I and ψ, the angle I' is determined. Now, the determination of the angle of dip depends on whether the susceptibility is known, a small value can be assumed or we have no idea about its magnitude. In the first case, the determination of angle α is straight forward since

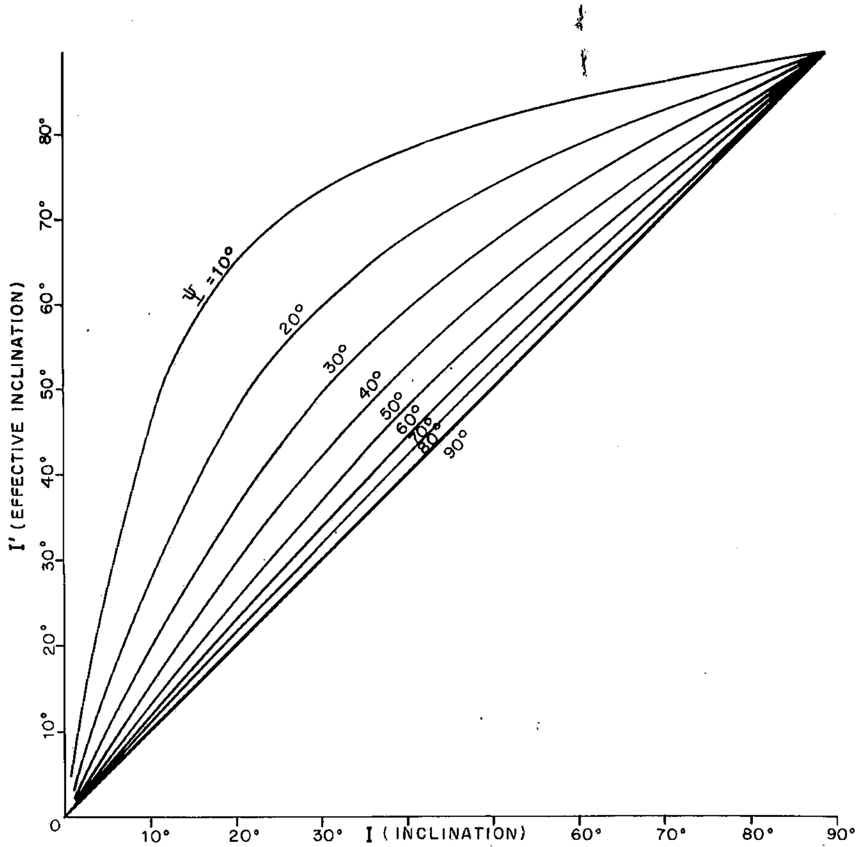

Fig. 3 — Variation of effective inclination angle (I'^{o}) with inclination (I^{o}) and strike (Ψ) as parameters.

$$i' - \alpha = tan^{-1}\,[(1 + K)\ tan\ (I' - \alpha)].$$

Furthermore from the modulus $|A\ (x)_{max}| = \dfrac{KT\ \Delta S\ \varepsilon\ cos\beta\ sinI}{2\pi z^2\ sinI'}$ the thickness of the dyke is readily determined if the susceptibility is known.

The procedure for determination of the location and depth of the horizontal cylinder from $|A\ (x)|$ is analogous to that of the infinite dyke. The phase angle $\upsilon\,(0) = -\,\gamma = -\,(\xi' + i')$. Since ξ' is always known, i' can be determined. The

maximum amplitude $\upsilon(x)$ is equal to

$$\frac{T\,K\,sinI\,\triangle S\,cos\beta}{(K+2)\,sinI'\,\pi z^2}\,.$$

In this expression, all the parameters except K and $\triangle S$ are known. If either K or $\triangle S$ can be assumed or known, the other can be determined.

4. Examples

Example 1: the semi-infinite dyke

The total magnetic flux density is calculated for a semi-infinite dyke with the following parameters: $T = 35000\gamma$, $K = 2$, $I = 30°$, $\psi = 70°$, $\triangle S = 10$ m, $z = 100$ m and $\alpha = 20°$. The anomaly of the dyke is superimposed on a linear regional gradient of 200 gamma/km. Fig. 4 and 5 show the composite anomaly curve together with the computed quantities $(\partial/\partial x)\,\triangle T$, $(\partial/\partial z)\,\triangle T$, $|A(x)|$ and $\upsilon(x)$ using cell size of 0.1 depth units.

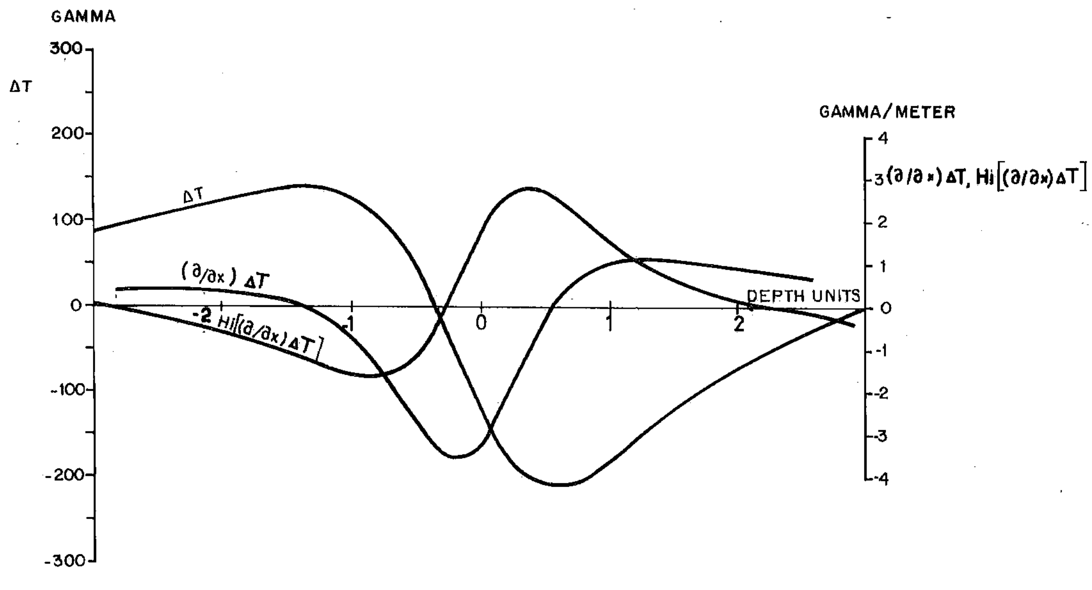

Fig. 4 — Theoretical anomaly of the dyke of Example 1, its horizontal derivative and Hilbert transform.

Linear regional gradients cause a vertical shift in $(\partial/\partial x)\,\triangle T$ but have no influence on $(\partial/\partial z)\,\triangle T$, as the Hilbert transform of a constant is zero. It follows that the position of maximum and half-maximum of the function $|A(x)|$ are not affected. Depth to dyke is found to be 103 m and location at $x = -5$ m. From $\upsilon(x)$ and Fig. 2, 3 and 4 angle $\upsilon = 66°$, $I' = 31.6°$ and $(i' - \alpha) = 34.4°$. Now if we know K to be 2, $(I' - \alpha)$ will be 12.8° and thus α is 18.8° which is a sufficiently accurate estimate of the true value (20°).

Example 2: the horizontal cylinder

The total magnetic flux density anomaly is calculated for a horizontal cylinder with the following parameters: $T = 35\,000$ gammas, $K = 2$, $I = 30°$, $\psi = 70°$, $\triangle S = 314$ m^2 and $z = 100$ m. Fig. 6 shows the flux density, its Hilbert transform, the amplitude function $A(x)$ and the phase $\upsilon(x)$.

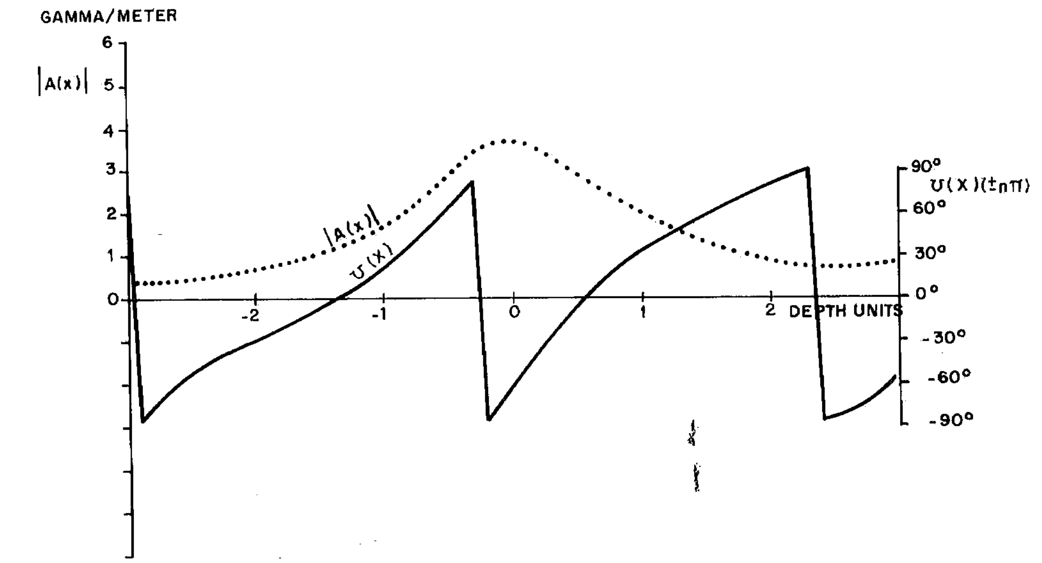

Fig. 5 — The amplitude function and the phase angle of the dyke anomaly of Example 1.

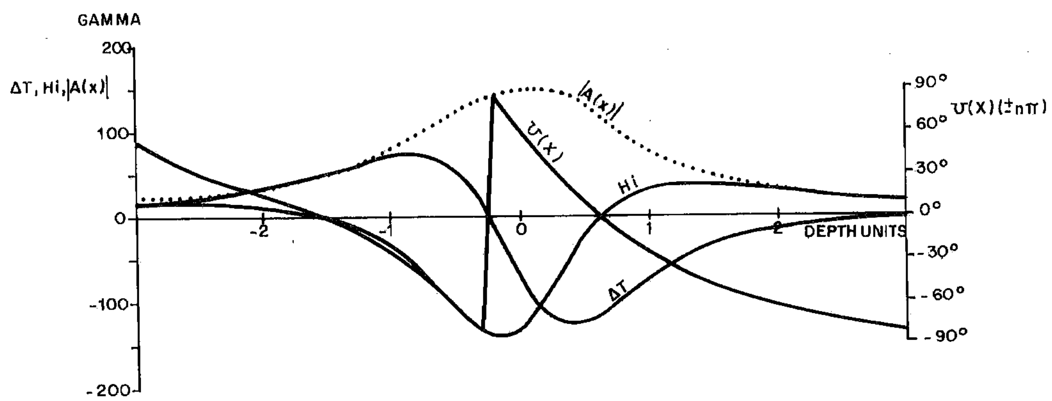

Fig. 6 — Theoretical anomaly *(ΔT)* of a horizontal cylinder, its Hilbert transform *(Hi)*, the phase angle $\upsilon(x)$ and the amplitude function.

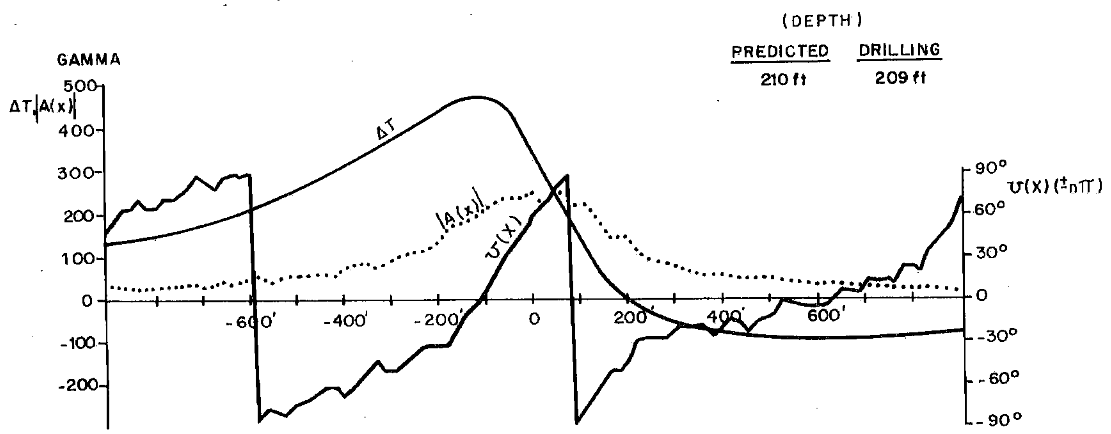

Fig. 7 — Interpretation of the Pima Copper Mine magnetic anomaly.

From $|A(x)|$ the depth is found to be 103 m and the location at $x = 5$ m. Using the plot of $\upsilon(x)$, $(\xi' + i')$ is found to be 62° and thus $i' = 30.4°$. It is seen that ξ' and i' do not differ appreciably as expected with homogeneous magnetization due to induction.

Example 3: the Pima Copper Mine anomaly

A practical example is chosen from literature (Gay, 1963). The anomaly from the Pima Copper Mine, Arizona, is shown in Fig. 7 along with the amplitude function $|A(x)|$ and the phase $\upsilon(x)$. The result of the interpretation is shown in the same figure and is almost equal to the drilling findings.

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