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Comparing alternative models for multisite probabilistic seismic risk analysis

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ABSTRACT: The risk assessment for a building portfolio or a spatially distributed infrastructure requires multi-site probabilistic seismic hazard analysis (MSPSHA). In fact, MSPSHA accounts for the stochastic dependency between the ground motion intensity measures (*IM*s) at the sites. Multi-site hazard needs to define the correlation structure for the same *IM* at different sites (*spatial correlation*), that of different *IM*s at the same site (*cross-correlation*) and that of different *IM*s at different sites (*spatial-cross-correlation*). Literature shows that such models usually require a significant amount of regional data to be semi-empirically calibrated. An approximated yet simpler-to-model alternative option is the *conditional-hazard* approach. The latter, originally developed for single-site analyses as an alternative to vector-valued PSHA, allows computing the distribution of a *secondary IM* given the occurrence or exceedance of a value of a *primary IM*. Conditional hazard considers the spatial correlation of the primary *IM* and the cross-correlation at each site for the two *IMs*, thus, if it is adopted for MSPSHA, the spatial correlation of the secondary *IM* as well as the spatial-cross-correlation between the two *IMs* descends from these two models. In the study, the conditional hazard procedure for MSPSHA is discussed and implemented in an illustrative application. Results in terms of distribution of the total number of exceedances of selected thresholds at the sites in a given time interval are compared with the case of complete formulation of MSPSHA and the differences are quantified. It appears that conditional hazard is a solid, yet simpler alternative for MSPSHA, at least in the considered cases.

1. INTRODUCTION

Classical probabilistic seismic hazard analysis (PSHA) allows to compute the exceedance rate of arbitrary ground motion intensity measure (*IM*) thresholds at a site of interest (Cornell, 1968). The rate completely defines the homogeneous Poisson process (HPP) counting the occurrences of earthquakes at the site over time. A number of advancements of PSHA have been proposed over the years; for example, vector-valued PSHA (Bazzurro and Cornell, 2002) and *conditional hazard* (Iervolino et al., 2010). Both aim at considering multiple *IMs* in PSHA. In particular, the latter considers the distribution of a *secondary IM* conditional to a value of the *primary* for which the hazard is generally known.

Risk assessment of building portfolios or spatially distributed infrastructure requires to assess the exceedance probability, over time, of different *IMs* at different sites (e.g., Goda and Hong, 2009; Esposito et al., 2015). In these cases, PSHA may be inadequate and the so-called multi-site PSHA, or MSPSHA, has to be implemented (e.g., Eguchi, 1991). In fact, MSPSHA requires to model the correlation structure between all *IMs* at all sites.

There are several alternative strategies by which MSPSHA can be implemented for computation (Weatherill et al., 2015). In the hypothesis of joint normality of the logarithms of the *IM*s at the sites and modelling the whole correlation structure of *IM*s at the sites, a *full* MSPSHA can be performed. Alternatively, it is possible to simulate multiple *IM*s at multiple sites taking advantage of the concept of conditional hazard, yet this implies some approximations. Indeed, as discussed in the following, it only partly defines the correlation structure and let the rest descend from the defined terms.

The study presented herein is intended to quantify the effect of the approximation introduced by performing MSPSHA via conditional hazard, when the exceedance probability of a given vector of *IM* thresholds at multiple sites is of concern.

This paper is structured such that the basics of MSPSHA, along with the sources of stochastic dependence between *IM*s are introduced first. Subsequently, the conditional hazard and its implementation for MSPSHA are illustrated. Finally, an illustrative application is developed to investigate the implications of conditional hazard for multi-site seismic hazard assessment. In particular, the results from the full approach are compared with the corresponding conditional hazard counterpart, with reference to the effect of the number of sites considered and their spatial configuration (i.e., the inter-site distance).

2. MULTI-SITE PSHA

The objective of MSPSHA is to model the number of exceedances of *IM* thresholds at multiple sites. When the sites of interest are all affected by the same seismic sources, the process describing the occurrence of earthquakes causing the exceedance of the thresholds at the ensemble of the sites is not an HPP. The reason is in the stochastic dependence between the *IM*s that each single earthquake generates at the sites (e.g., Giorgio and Iervolino, 2016). Hereafter, without loss of generality, it is assumed that the *IM*s of interest are the pseudo-spectral accelerations at given spectral periods, that is $Sa(T)$.

To deepen how the stochastic dependency of pseudo-spectral accelerations has to be accounted for by the so-called ground motion prediction equations (GMPEs), let the considered sites be only two, say A and B, and T_1 and T_2 the vibration periods of interest at site A and B, respectively. The threshold of $Sa(T_1)$ at site A is identified as sa_1^* and the threshold of $Sa(T_2)$ at site B is sa_2^*. The probability that the thresholds are both exceeded given the occurrence of an earthquake (E), that is $P\left[Sa(T_1)>sa_1^* \cap Sa(T_2)>sa_2^* | E\right]$, is given in Eq. (1).

$$P\left[Sa(T_1)>sa_1^* \cap Sa(T_2)>sa_2^* | E\right] = \int_M \int_Z P\left[Sa(T_1)>sa_1^* \cap Sa(T_2)>sa_2^* | m,z\right] \times \times f_{M,Z}(m,z)\cdot dm \cdot dz \quad (1)$$

In the equation, $P\left[Sa(T_1)>sa_1^* \cap Sa(T_2)>sa_2^* | m,z\right]$ is the probability of joint exceedance conditional on the magnitude (M) and location (Z) of the earthquake; $f_{M,Z}(m,z)$ is the joint probability density function (PDF) of M and Z. The integral in the equation is over the domains of magnitude and earthquake location; $f_{M,Z}(m,z)$ depends on the characteristics of the seismic source whereas $P\left[Sa(T_1)>sa_1^* \cap Sa(T_2)>sa_2^* | m,z\right]$ is related to the probabilistic effects of a common earthquake at different sites. The latter can be modelled via (i) GMPEs and (ii) the correlation structure, which must be defined.

Under the lognormal hypothesis about one $Sa(T)$ conditional to earthquake magnitude and source-to-site distance, R (which is a deterministic function of Z), most GMPEs model the log of $Sa(T)$, at a site j due to earthquake i, according to Eq. (2).

$$log\, Sa(T)_{i,j} = E\left(log\, Sa(T) \middle| m_i, r_{i,j}, \theta\right) + \eta_i + \varepsilon_{i,j} \quad (2)$$

In the equation, $E\left(log\, Sa(T) \middle| m_i, r_{i,j}, \theta\right)$ is the mean of $log\, Sa(T)_{i,j}$ conditional on parameters such as M, R and others (θ); η_i, constant for all the sites in a given earthquake, denotes the inter-event residual, a random variable (RV) that quantifies how much the mean of $log\,Sa(T)_{i,j}$ in the *i*-th earthquake differs from $E\left(log\,Sa(T)\middle|m_i, r_{i,j}, \theta\right)$. On the other hand, $\varepsilon_{i,j}$ represents the intra-event variability at site j in earthquake i.

Typically, it is assumed that inter- and intra-event residuals are stochastically independent normal RVs with zero mean and standard deviation equal to σ_{inter} and σ_{intra}, respectively. The sum of inter- and intra-events residuals provides the *total* residual, a Gaussian RV with zero mean and standard deviation equal to $\sigma = \sqrt{\sigma_{\text{inter}}{}^2 + \sigma_{\text{intra}}{}^2}$.

Given magnitude and earthquake location, it is generally assumed that the logs of $Sa(T)$ at multiple sites form a Gaussian random field (GRF; e.g., Park et al., 2007). When the same spectral period is considered at all the sites (say s in number), e.g., T_1, the GRF has the mean vector given by the $E\left(log\,Sa(T_1)\middle|m_i, r_{i,j}, \theta\right)$ terms, one for each site, and the covariance matrix, Σ, given by Eq. (3) in which the matrices have $s \times s$ size:

$$\Sigma = \sigma_{\text{inter}}^2 \cdot \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} + \sigma_{\text{intra}}^2 \cdot \begin{bmatrix} 1 & \rho_{\text{intra}}(T_1, T_1, h_{1,2}) & \cdots & \rho_{\text{intra}}(T_1, T_1, h_{1,s}) \\ & 1 & \cdots & \vdots \\ & sym & \ddots & \vdots \\ & & & 1 \end{bmatrix} \quad (3)$$

The first matrix on the right-hand side, accounts for the *perfect* correlation of inter-event residuals. The second matrix accounts for the spatial correlation of intra-event residuals; in particular, the $\rho_{\text{intra}}(T_1, T_1, h_{k,j})$ denotes the correlation coefficient between intra-event residuals of $Sa(T_1)$ at sites k and j, (e.g., Esposito and Iervolino, 2012), which tends to decrease with the increasing of the inter-site distance, $h_{k,j}$. Note that the $Sa(T_1)$ in one earthquake are also stochastically dependent because the means of the GRF in Eq. (2) share the same event's magnitude and location (see Giorgio and Iervolino, 2016, for a discussion).

When different spectral periods are considered, the lognormal hypothesis is extended to the joint distribution of all $Sa(T)$ (i.e., accelerations for different spectral periods) at all sites. Thus, an additional correlation of residuals has to be defined. For example, in the case of two periods, T_1 and T_2, and s sites, the mean vector is made of $2s$ elements (Eq. 2), two for each site, and the covariance matrix is from Eq. (4). The matrices in the equation have size $2s \times 2s$; $\sigma_{\text{inter},1}$ and $\sigma_{\text{inter},2}$ are the standard deviations of the inter-event residuals of $Sa(T_1)$ and $Sa(T_2)$, respectively; $\sigma_{\text{intra},1}$ and $\sigma_{\text{intra},2}$ are the standard deviations of the intra-event residuals of the two $Sa(T)$; $\rho_{\text{inter}}(T_1, T_2)$ denotes the cross-correlation (or spectral correlation) coefficient between the inter-event residuals of the two $Sa(T)$ (e.g., Baker and Jayaram, 2008; Bradley, 2012); $\rho_{\text{intra}}(T_1, T_2, h_{k,j})$ is the spatial-cross-correlation of the intra-event residuals of $Sa(T_1)$ and $Sa(T_2)$ for site k and j (e.g., Loth and Baker, 2013). In fact the covariance matrix of Eq. (4) is an extension of Eq. (3) to the case of two pseudo-spectral accelerations.

$$
\Sigma = \begin{bmatrix}
\sigma^2_{\text{inter},1} & \cdots & \sigma^2_{\text{inter},1} & \rho_{\text{inter}}(T_1,T_2)\cdot\sigma_{\text{inter},1}\cdot\sigma_{\text{inter},2} & \cdots & \rho_{\text{inter}}(T_1,T_2)\cdot\sigma_{\text{inter},1}\cdot\sigma_{\text{inter},2} \\
 & \ddots & \vdots & \vdots & \ddots & \vdots \\
 & & \sigma^2_{\text{inter},1} & \rho_{\text{inter}}(T_1,T_2)\cdot\sigma_{\text{inter},1}\cdot\sigma_{\text{inter},2} & \cdots & \rho_{\text{inter}}(T_1,T_2)\cdot\sigma_{\text{inter},1}\cdot\sigma_{\text{inter},2} \\
 & & & \sigma^2_{\text{inter},2} & \cdots & \sigma^2_{\text{inter},2} \\
 & & sym & & \ddots & \vdots \\
 & & & & & \sigma^2_{\text{inter},2}
\end{bmatrix} +
$$

$$
+ \begin{bmatrix}
\sigma^2_{\text{intra},1} & \cdots & \rho_{\text{intra}}(T_1,T_1,h_{1,s})\cdot\sigma^2_{\text{intra},1} & \rho_{\text{intra}}(T_1,T_2)\cdot\sigma_{\text{intra},1}\cdot\sigma_{\text{intra},2} & \cdots & \rho_{\text{intra}}(T_1,T_2,h_{1,s})\cdot\sigma_{\text{intra},1}\cdot\sigma_{\text{intra},2} \\
 & \ddots & \vdots & \vdots & \ddots & \vdots \\
 & & \sigma^2_{\text{intra},1} & \rho_{\text{intra}}(T_1,T_2,h_{s,1})\cdot\sigma_{\text{intra},1}\cdot\sigma_{\text{intra},2} & \cdots & \rho_{\text{intra}}(T_1,T_2)\cdot\sigma_{\text{intra},1}\cdot\sigma_{\text{intra},2} \\
 & & & \sigma^2_{\text{intra},2} & \cdots & \rho_{\text{intra}}(T_2,T_2,h_{1,s})\cdot\sigma^2_{\text{intra},2} \\
 & & sym & & \ddots & \vdots \\
 & & & & & \sigma^2_{\text{intra},2}
\end{bmatrix}
\quad (4)
$$

3. CONDITIONAL HAZARD

The concept of conditional hazard was introduced by Iervolino et al. (2010) for single-site applications. It provides the distribution of a secondary $Sa(T)$ given the occurrence (or the exceedance) of a primary one. If the primary pseudo-spectral acceleration is $Sa(T_1)$ and the secondary $Sa(T_2)$, the distribution of the logarithm of $Sa(T_2)$ conditional to $sa(T_1)$, which is the realization of $Sa(T_1)$ at the same site, has conditional mean $E\left(log\,Sa(T_2)\middle|log\,sa(T_1),m_i,r_{i,j}\right)$ and conditional standard deviation $\sigma_{log\,Sa(T_2)|log\,sa(T_1)}$ as per Eq. (5).

$$
\begin{cases}
E\left(log\,Sa(T_2)\middle|log\,sa(T_1),m_i,r_{i,j}\right) = E\left(log\,Sa(T_2)\middle|m_i,r_{i,j}\right) + \\
+\sigma_2\cdot\rho(T_1,T_2)\cdot\dfrac{log\,sa(T_1)-E\left(log\,Sa(T_1)\middle|m_i,r_{i,j}\right)}{\sigma_1} \\
\sigma_{log\,Sa(T_2)|log\,sa(T_1)} = \sigma_2\cdot\sqrt{1-\rho^2(T_1,T_2)}
\end{cases}
\quad (5)
$$

In the equation, σ_2 and σ_1 are the standard deviation of total residuals of $log\,Sa(T_2)$ and $log\,Sa(T_1)$, respectively, provided by the GMPE and $\rho(T_1,T_2)$ is the cross-correlation coefficient between total residuals (Baker and Jayaram, 2008).

Under the hypothesis of bivariate lognormal distribution of the two spectral ordinates, the parameters in Eq. (5) are those of a Gaussian distribution.

When MSPSHA is of concern, and two $Sa(T)$ are considered, the concept of conditional hazard may be used. In practical terms, after simulating the primary intensity measure; i.e., $Sa(T_1)$, at the sites using the mean and the covariance matrix of Eq. (2) and Eq. (3), Eq. (5) can be applied to each site to simulate $Sa(T_2)$. This avoids the use of spatial-cross-correlation models. On the other hand, this strategy introduces an approximation with respect to the application of full MSPSHA because the spatial correlation of $Sa(T_2)$ and the spatial-cross-correlation between $Sa(T_1)$ and $Sa(T_2)$ are not explicitly modelled, yet they are consequent to the conditional hazard approach. More specifically, it is possible to demonstrate that this procedure corresponds to approximate the spatial-cross-correlation models of total residuals as shown in Eq. (6) (e.g., Goda and Hong, 2008). In other words, Eq. (4) is replaced by Eq. (7).

$$
\begin{cases}
\rho(T_1,T_2,h_{k,j}) = \rho(T_1,T_1,h_{k,j})\cdot\rho(T_1,T_2) \\
\rho(T_2,T_2,h_{k,j}) = \rho(T_1,T_1,h_{k,j})\cdot\rho^2(T_1,T_2)
\end{cases}
\quad (6)
$$

It should be noted that the conditional hazard approach to MSPSHA can be also applied when secondary $Sa(T)$ at the sites are at different periods. Similarly to the previous case, this implies the simulation of $Sa(T_1)$ at the sites and the application of Eq. (5) to each site replacing $Sa(T_2)$ with the spectral ordinate at the period of interest. This type of application is discussed in the following section.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \cdots & \rho(T_1,T_1,h_{1,s})\cdot\sigma_1^2 & \rho(T_1,T_2)\cdot\sigma_1\cdot\sigma_2 & \cdots & \rho(T_1,T_1,h_{1,s})\cdot\rho(T_1,T_2)\cdot\sigma_1\cdot\sigma_2 \\ & \ddots & \vdots & \vdots & \ddots & \vdots \\ & & \sigma_1^2 & \rho(T_1,T_1,h_{s,1})\cdot\rho(T_1,T_2)\cdot\sigma_1\cdot\sigma_2 & \cdots & \rho(T_1,T_2)\cdot\sigma_1\cdot\sigma_2 \\ & & & \sigma_2^2 & \cdots & \rho(T_1,T_1,h_{s,1})\cdot\rho^2(T_1,T_2)\cdot\sigma_2^2 \\ & & sym & & \ddots & \vdots \\ & & & & & \sigma_2^2 \end{bmatrix} \quad (7)$$

4. ILLUSTRATIVE APPLICATION

The objective of this section is quantifying, for an illustrative case, the differences on results when (i) full MSPSHA and (ii) approximated conditional hazard procedure (CH) are implemented. To this aim, a set of one hundred sites located in the district of Naples (southern Italy) are considered. The sites are distributed on a regular grid with inter-site distance equal to 1.5 km and they are assumed to represent the locations of a hypothetical heterogeneous building portfolio. It is also assumed that one intensity measure, one $Sa(T)$, is of interest for each site, but, due to buildings' heterogeneity, a different vibration period is considered for each site. The spectral ordinate of interest for each site is set among the following five: $Sa(0.6\text{s})$, $Sa(0.7\text{s})$, $Sa(0.8\text{s})$, $Sa(0.9\text{s})$ and $Sa(1\text{s})$. The one considered for each site is shown in Figure 1.

In order to define the threshold values of interest for risk assessment, a single-site (classical) PSHA is first performed at each site. Thus, the threshold values are computed as the acceleration to which classical single-site PSHA associates a return period (T_r) of 475 years. Then, the two procedures of MSPSHA are applied to compute the distribution of the number of exceedances collectively observed at the sites in a time interval (ΔT) equal to fifty years. The comparison of the two resulting distributions is discussed.

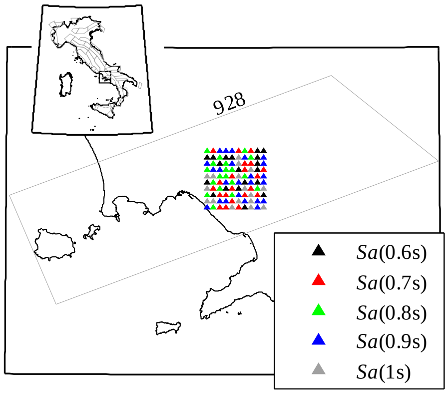

Figure 1: Pseudo-spectral acceleration of interest for each site and location of the seismic source zone 928 from Meletti et al. (2008).

In both PSHA and MSPSHA analyses, the model adopted to describe the seismic sources is that of Meletti et al. (2008), which features thirty-six seismic source zones for the whole Italy, numbered from 901 to 936. However, for the purposes of the application, only zone 928 is considered (Figure 1) for simplicity. The seismic characterization of the zone is from Barani et al. (2009), that is, a Gutenberg-Richter type magnitude distribution (Gutenberg and Richter, 1944) with minimum and maximum magnitude equal to 4.3 and 5.8, respectively, negative slope of 1.056, and annual rate of earthquakes of 0.054. The GMPE used herein is that of Akkar and Bommer (2010). It is applied within its definition ranges of magnitude (5-7.6) and Joyner and Boore (R_{JB} ; Joyner and Boore, 1981) distance (0-100 km). Epicentral distance is converted into R_{JB} according to Montaldo et al. (2005). According to Meletti et al. (2008), the normal style of faulting is considered for the source. Rock soil condition is assumed for all the sites.

The two procedures of MSPSHA differ for the simulation of the GRF of spectral ordinates at the sites conditional to the magnitude and the location of the earthquake on the source. Such simulations, described in the following section, are repeated for each earthquake occurring on the source in the fifty years interval. Thus, at the base of the two procedures there are three-million seismic histories representing the earthquakes occurring on the source in fifty years that have been simulated via a recently developed software for regional, single-site and scenario based probabilistic seismic hazard analysis (REASSESS). The software adopts a two-step procedure to simulate the earthquake occurrence over time: the first step is addressed to simulate and collect the magnitudes and locations of the earthquakes conditional to the occurrence of a generic earthquake. Such seismic scenarios are the input of step two that consists of simulating the process of earthquakes affecting the sites in any time interval, that is the seismic history in ΔT . Further details about the simulation of the seismic histories are in Chioccarelli et al. (2018).

4.1. Simulation procedures

For each earthquake occurring on the source in the specific realization of fifty years, the first strategy (full MSPSHA) simulates a realization of the GRF made of the $Sa(T)$ at all the sites, via implementation of the full correlation structure of the type in Eq. (4). More specifically, the components $E\left(log\,Sa(T)\middle|m_i,r_{i,j},\theta\right)$ of the mean vector, one for each site and spectral period of interest, are computed according to the GMPE. Then, the vector containing the total residual for each site and period is sampled from a zero-mean multivariate normal distribution with covariance matrix as per Eq. (4). Finally, the realization of the GRF is obtained by adding the vector of the residuals' realization to the mean vector. The correlation structure of inter-event and spatial-cross-correlation of intra-event residuals, implemented in Eq. (4), are those of Baker and Jayaram (2008) and Loth and Baker (2013), respectively. The sought distribution of the total number of exceedances at the sites, in terms of probability mass function (PMF), has been carried out through the REASSESS software (Chioccarelli et al., 2018) and is pictured in Figure 2.

As alternative strategy, the GRF of realizations at the sites has been simulated through the double-step simulation described in Section 3. First, a primary spectral ordinate to be simulated at all the site has been selected. It is known that the higher is the period of the primary $Sa(T)$, the lower is the approximation introduced by the conditional hazard approach (e.g., Goda and Hong, 2008). Thus, $Sa(1\text{s})$ is chosen here as primary. Then, for each $Sa(1\text{s})$ value at each site, obtained from the GRF simulation, the realization of the secondary $Sa(T)$ of interest at each site (see Figure 1) is sampled from the normal conditional distribution of Eq. (5), in which the model of Baker and Jayaram (2008) is used for the correlation of total residuals. This procedure is equivalent to sample from a multi-variate Gaussian distribution with the correlation structure in Eq. (7). The resulting PMF of the total number of exceedances observed in fifty years is also shown in Figure 2.

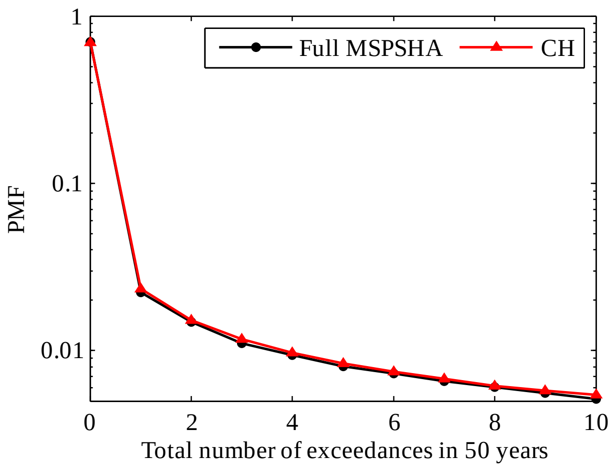

Figure 2: Distribution of the total number of exceedances observed at the considered sites in fifty years obtained through the two approaches.

4.2. Discussion

As expected, the two distributions have the same mean because the mean is not affected by the correlation. The mean is the single-site occurrence rate of the thresholds, $1/T_r$, multiplied by the time interval of interest and the number of sites, that is $0.0021 \cdot 50 \cdot 100 = 10.5$. It can be also observed that the PMFs show a very similar trend; they only slightly differ in terms of variance (VAR). In order to quantify the approximation of CH, the relative difference between variance of the full MSPSHA (VAR_{Full}) and the CH approach (VAR_{CH}), computed as $\Delta = \frac{VAR_{Full} - VAR_{CH}}{VAR_{Full}}$, is introduced. For the examined case, it results $\Delta = 1.65\%$.

4.3. The effect of the number of sites and the inter-site distance

In order to study the effect on results of the inter-site distance and the number of sites, analyses have been repeated considering the same portfolio of Figure 1, but with additional inter-site distances: 0.2 km, 0.5 km and 3 km. Moreover, for each inter-site distance nine subsets of sites are considered, from $s = 4$ to $s = 100$ as shown in the upper panel of Figure 3. Thus, thirty-six analyses have been performed. For each combination of inter-site distance, h, and number of sites, s, the investigated PMF has been computed through full MSPSHA and CH and the differences of variance of distributions are reported in terms of Δ in Figure 3(b).

Each curve in the figure provides, for a given inter-site distance, the trend of Δ as a function of the number of sites. It can be noted first that the inter-site distance has a minor effect. Then, although a slight increase of Δ is shown for the increasing number of sites up to $s = 25$, the general trend of Δ is non-monotonic and the values are always within the range of -0.5% and 2%.

5. CONCLUSIONS

When the hazard assessment for multiple sites is of interest, that is MSPSHA, the key issue is to account for the stochastic dependence of the site-specific counting processes, which derives from the correlations of pseudo-spectral accelerations at the sites. In fact, it was shown that modelling all the sources of dependency in the $Sa(T)$ simulations at multiple sites requires models of spatial-cross-correlation that have to be fitted on a relevant amount of data, which can also be region-dependent.

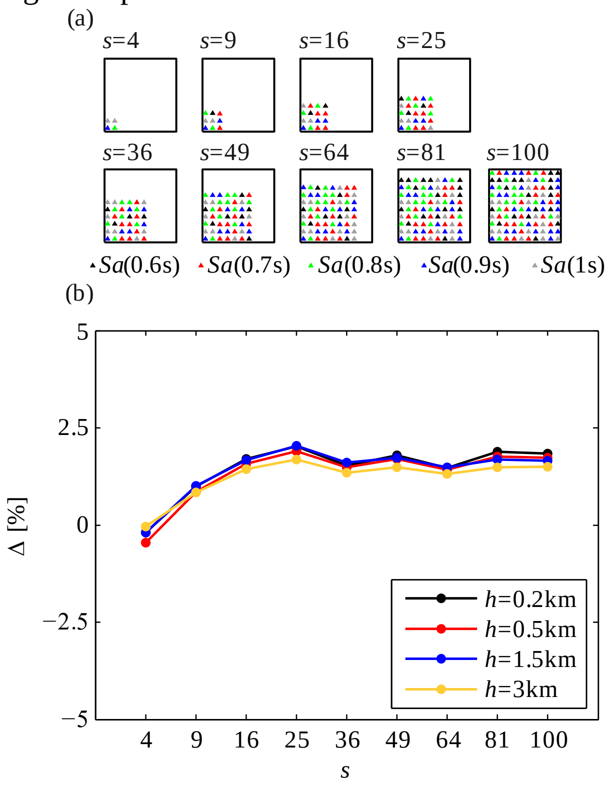

Figure 3. (a) Configuration of sites for each subset; (b) Approximation of conditional hazard for different inter-site distances and number of sites.

On the other hand, conditional hazard allows to obtain the distribution of a secondary $Sa(T)$, given a primary one, at a site of interest. Thus, CH can be applied to generate random fields of a secondary $Sa(T)$ at the sites, conditional to the spatially correlated realizations of the primary $Sa(T)$. This procedure can be adopted for MSPSHA and does not require the covariance structure of different pseudo-spectral accelerations at different sites to be modelled. Nevertheless, it introduces some approximations, which were quantified in an illustrative application. To this aim, a portfolio of one-hundred equally spaced sites located in the southern Italy is considered. The distribution of the total number of exceedances observed at the ensemble of the sites in fifty years was computed through the full and CH approach. The approximation of CH was evaluated in terms of relative difference between the variances of the two distributions. In the case the inter-site distance is 1.5 km, it was found that the relative difference is equal to 1.65%. In order to investigate the effect of the inter-site distance and the number of sites, different spatial configurations for the portfolio were also considered. Still with reference to the variance of the distributions of the total number of exceedances observed at the sites in fifty years, it was shown that the relative difference of CH with respect to the full approach is negligible at least in the investigated cases.

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