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A simplified fuzzy multivariable structure in a manufacturing environment

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Applications of fuzzy control to industrial processes are mainly of multivariable structure. Using the traditional Zadeh principle would require a multidimensional relation to be developed representing a fuzzy model of the system. Such a multidimensional relation would result in memory overload to most industrial computers. Moreover, it would result in a highly complex compositional rule of inference to achieve the final output(s) of the system. This paper proposes a new simplified technique that avoids such complexity as well as memory overload for multivariable structure. Section 2 describes the proposed simplified multivariable technique to avoid memory overload. Section 3 demonstrates these techniques in the form of a robotic welding example where the objective is to control the speed of a robotic arm following an irregular path of weld. The speed value is dependent on the cavity size and determined by the cavity width and cavity depth as inputs. Section 4 describes an experimental application of the technique applied to an industrial process in the manufacture of force transducers termed as the 'cornering process'. This application is composed of a two-inputs–two-outputs system.

Keywords: Fuzzy control, multivariable control, process control, industrial computers applications, human factors

1. Introduction

Fuzzy control theory can be a significant aid to developing a model that enables machine systems to imitate the intuitive skills of the human operator. Fuzzy systems have an advantage over classical control systems where such a model would be either inaccurate or non-existent. The possibility of making machine systems interpret vague statements has resulted in research workers in the field of robotics achieving the capability to control a process in a fuzzy environment. Hirota *et al.* (1985) for example proposed an intelligent robot based upon the concept of membership and vagueness where an image processor was used as a feedback to the robot in order to deal with the direction, distance and pick-and-place instruction. Lakov (1985) proposed a control strategy to control the motion of an arc welding robot online estimation and prediction. It was concluded that the fuzzy control of this arc welding robot shows good results in comparison with direct seam welding. Most of these applications use a set of assumptions as well as specific fuzzy relation equations suitable for that system. A formal description of fuzzy systems by the use of fuzzy relational equations was dealt with by Pedrycz (1989). Good work on simplified fuzzy multivariable systems was performed by Gupta *et al.* (1986). The paper commences with a description of a block diagram which is mainly composed of functional and intersectional blocks, thus allowing compatibility with the analysis of fuzzy systems and clarifying the multivariable fuzzy equations of the structure. The paper then proposes a multivariable structure of the fuzzy system that may avoid memory overload for most industrial computers. This inspired Kouatli (1990, 1993) to propose a modified version of Gupta's technique to achieve a further simplification as well as a human simulation of the fuzzy multivariable structure.

This paper describes the proposed technique in Section 2 and Section 3 demonstrates it in the form of a robotic welding example as well as comparing it with Zadeh's technique. Section 4 describes an experimental application on another industrial process in the field of transducer manufacture termed the 'cornering process' which is used to demonstrate the applicability of the technique.

2. Human simulated fuzzy multivariable structure

Starting from first principles of the traditional Zadeh fuzzy control theory, the fuzzy control of a simple open loop of a single-input–single-output system has been described by Kouatli and Jones (1990) in a robotic welding example. However, the application of the same inference strategy to a multivariable system would result in a highly complicated process of inferring the output(s) from given fuzzy input(s). This complexity arises mainly from the impractical implementation of a system that requires a substantial database and may result in memory overload. In the case of a single-input–single-output fuzzy relation, a two-dimensional fuzzy set is defined, which is the product of the dimensions of the input and the output, i.e. $\dim\{R\} = q_1 \times p_1$, where q_1 is the dimension of the fuzzy input and p_1 is the dimension of the fuzzy output. For a multivariable system of three inputs and three outputs the dimension of the fuzzy relation would be:

$$\dim\{R\} = q_1 \times q_2 \times q_3 \times p_1 \times p_2 \times p_3$$

Assuming the size of each of the $q_{(i)}$ or $p_{(i)}$ is 10 levels, each of which requires 1 byte of memory, the multi-dimensional fuzzy relation will require a memory space of 1 Mbyte. This illustrates the nature of the problem and it is clearly impractical for general use. Also, the final output is a fuzzy set of dimension $p_{(1)} \times p_{(2)} \times p_{(3)}$ and is composed of the three individual outputs which increases the complexity of using a multivariable fuzzy system. Gupta *et al.* (1986) proposed a technique for the design of a multivariable structured system which avoids such complexity. He used an analogy to the theory of linear systems where the fuzzy relation has been split into non-interactive sub-relations. Each of these describes the relationship between one of the inputs and one of the outputs where an inference of a specific output from its corresponding input can be made. The minimum operator, which is the intersection of the fuzzy sets of the input and the output, was used to infer the final output from the individual sub-relational outputs. This principle of multivariable structure led Kouatli (1990, 1993) to propose a similar technique that is simpler in design. This is based on the simulation of human behaviour which is described in the form of a robotic welding example. The proposed technique has two main characteristics as outlined below.

2.1. *Individual sub-algorithms*

The decomposed relations as introduced by Gupta *et al.* (1986) may be further simplified by splitting the fuzzy algorithm into non-interactive sub-algorithms that describe the effect of each of the inputs individually on a certain output. This will lead to non-interactive individually decomposed fuzzy relations between each of the inputs and the outputs. This simplifies the design of the multivariable structure of the system which is easy to comprehend and to apply. Finally the inferences from all of these sub-relations can be assembled to provide the final output using a factor that corresponds to the importance level of the inputs instead of using the intersection of two inferred outputs as in the case of Gupta's technique. This factor may be termed the 'input importance factor'.

2.2. *Input importance factor*

Observations of the behaviour of a human operator indicate that he/she subjectively weighs the influence that inputs have upon the final output of the system. Thus an 'importance factor' may be introduced for each of the inputs, where the highest level of importance that might be achieved is specified as unity. Initially the system designer has to estimate the value of the importance factor based on the knowledge elicitation of the process. However, it is very important to tune the system for optimum performance by optimizing the value of this factor. This optimization can be achieved manually by repeatedly running experiments on the system, watching performance and altering the value of the factor until satisfactory performance is achieved. Where possible, the process of optimizing the importance level factor may also be automated, achieving a 'learning system' that has the ability to improve its performance based on previous history stored in a database.

A special case occurs when all of the inputs have an equal influence on the output, implying that the importance level of each of the inputs should have a value of unity divided by the number of inputs. Multiplication of each of the averaged outputs of the sub-relations with its corresponding importance level provides a term that represents the effect of that input to the output of the sub-relation. Summation of these terms leads to a final averaged value which is represented in the following equation:

$$Y_j = \sum_{i=1}^{n} \varepsilon_i \times \overline{(X_i \circ R_{ij})} \tag{1}$$

where

ε_i = importance level of input number i
$\overline{(X_i \circ R_{ij})}$ = the averaged value of the composed fuzzy variable X with the relation R_{ij}
R_{ij} = the union of the Cartesian product of all the rules of input number i and output number j
Y = the average final output number j

Figure 1 shows a schematic diagram of this technique.

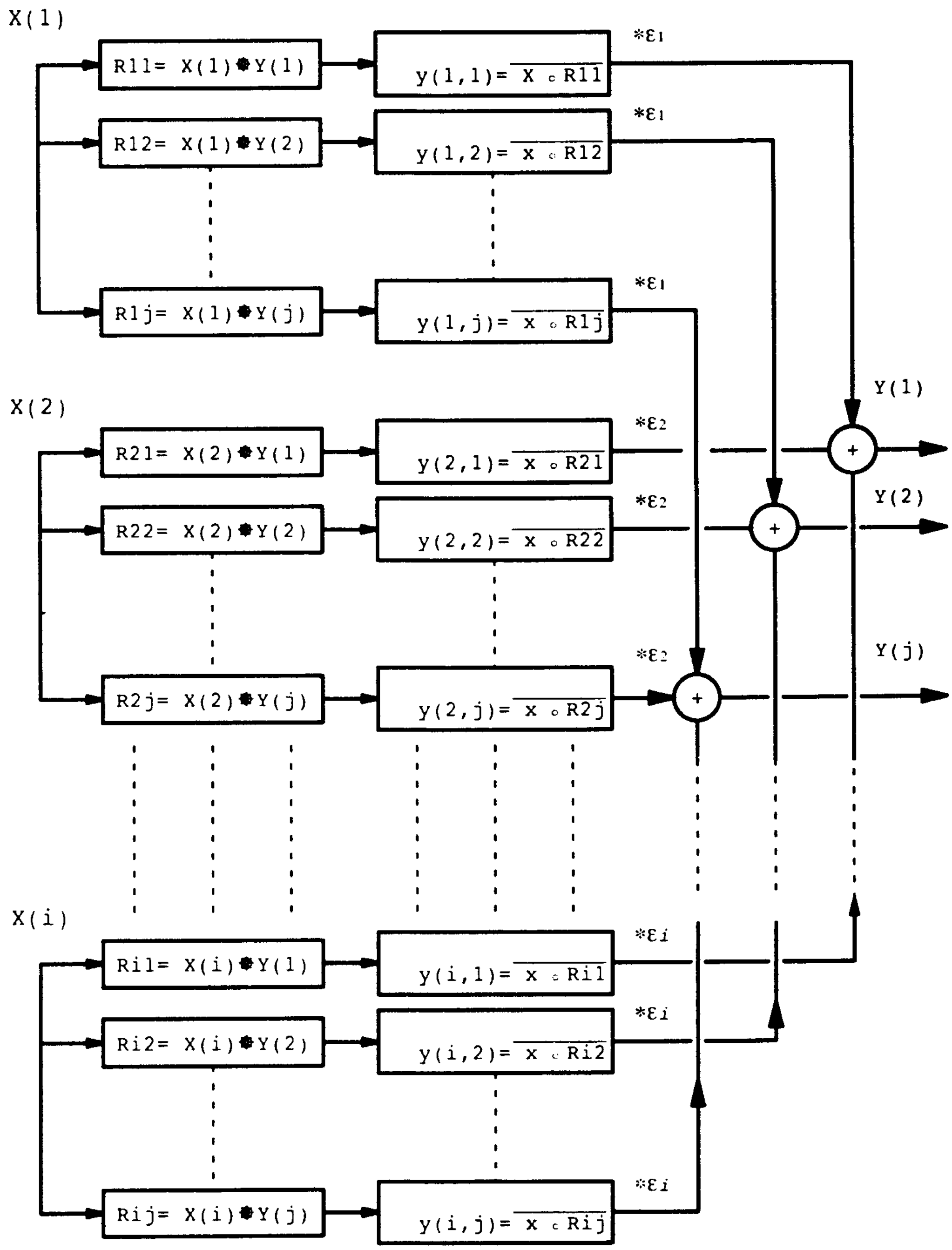

Fig. 1. Human simulated multivariable fuzzy structure.

3. A multivariable robotic welding example

In order to illustrate the technique, a simple manufacturing example has been established that imitates the human operator in performing an argon arc welding process following an irregular path of weld. Argon arc welding is one type of TIG welding (Tungsten electrode, Inert Gas shielded welding), that is usually applied to steel and a wide range of ferrous alloys without using flux, where shielding is produced by argon gas (Fig. 2). Such an application would be useful in hostile or dangerous environments such as a nuclear plant where a mobile robot has to perform the welding on an area of accidental damage. The robot should have the intelligence to identify the type of weld required, based on an assessment on the damaged material type and thickness. In welding a fractured part without preparation for example (Fig. 3), a welding expert can adjust the speed of his

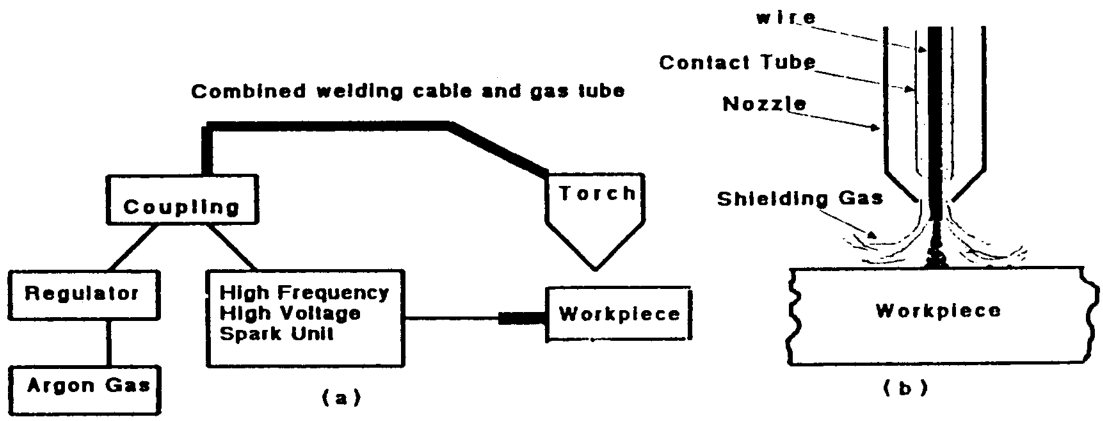

Fig. 2. (a) Setup of TIG welding; (b) torch–work contact of argon arc welding.

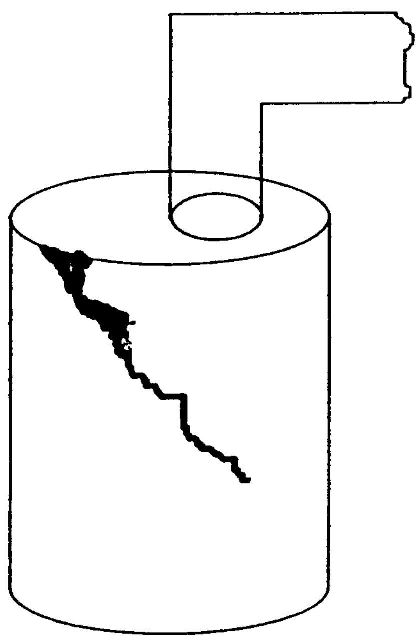

Fig. 3. Fractured part to be welded.

hand, which is dependent on the cavity width and depth at any point along the path, and hence provide optimum performance. It is assumed that a vision system is installed near to the robot in order to provide feedback to the system. Assuming that the vision system can provide adequate resolution to detect the depth of tiny cavities, the depth of a certain cavity can be detected by recognizing the degree of greyness measured. The width of the cavity can simply be found from counting the number of pixels across the path. Hence, the speed of the robotic arm is inversely proportional to either of the two inputs, *width* or *depth* of the cavity. Assuming that four linguistic variables (tiny, small, medium and large) are used to describe the level of either depth or width and four linguistic variables (minimum, slow, regular and fast) are used to describe the output speed level, then discretization of these universes may be defined.

3.1. *Fuzzimetric arcs partitioning technique*

The definition of these linguistic terms is dependent on the discretization of the universe of discourse of each of the input(s) and the output. There is no standard method for partitioning the universe of discourse. However, Kouatli (1993) proposed a universe of discourse partitioning technique termed as 'fuzzimetric arcs' in which a simplified and adequate partitioning of the universe of discourse into fuzzy variable can be achieved. A brief description is mentioned here for the sake of clarity.

A fuzzimetric arc is three-quarters of a circle with a radius of unity. The universe members can be spread on this arc where the lowest member corresponds to zero radians and the highest member corresponds to $3\pi/2$ radians. For example, applying the 'fuzzimetric arcs' technique to partition the universe of discourse to the linguistic variables of tiny, small, medium, large gives (Fig. 4):

$$\text{tiny} = |\sin \pi/2 - x| \quad \forall\ 0 < x < \pi/2$$
$$= 0 \quad \text{otherwise}$$

$$\text{small} = |\sin x| \quad \forall\ 0 < x < \pi$$
$$= 0 \quad \text{otherwise}$$

$$\text{medium} = |\sin \pi/2 - x| \quad \forall\ \pi/2 < x < 3\pi/2$$
$$= 0 \quad \text{otherwise}$$

$$\text{large} = |\sin x| \quad \forall\ \pi < x < 3\pi/2$$
$$= 0 \quad \text{otherwise}$$

where

$$x = 1.5\pi X/X_{max} \text{ rad}$$

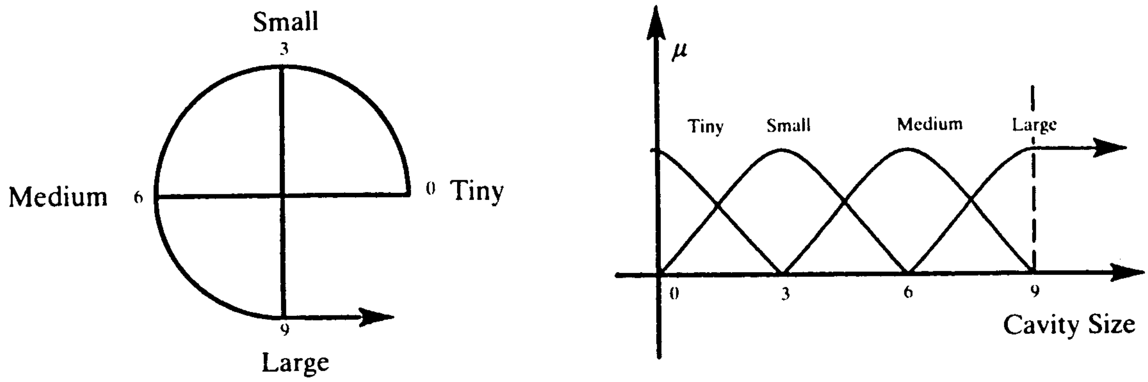

Fig. 4. Partitioning cavity size universe (width or depth).

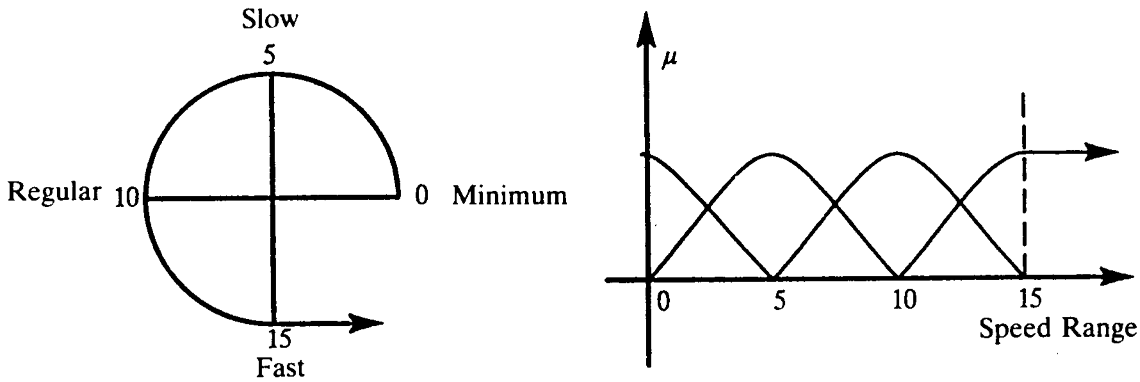

Fig. 5. Partitioning speed universe.

and

$X = 1, 2, 3 \ldots$ (maximum level in the discretized universe of discourse)

Assuming that the depth has a universe of {0,9}, i.e. has 9 grey levels where each level corresponds to 1 mm, and the width lies in a universe of {0,9}, i.e. the maximum value of the width of the cavity is 9 mm, then the fuzzy variables tiny, small, medium and large for both inputs of depth and width universes of the cavity size can be defined as shown in Table 1.

$$X_{depth} = 1, 2, 3 \ldots \text{(depth } X_{max} = 9)$$
$$X_{width} = 1, 2, 3 \ldots \text{(width } X_{max} = 9)$$

In a similar manner, the universe of the output, which is the robot's speed, should be partitioned according to the range of speed required for the TIG welding process which is between 20 and 35 cm min^{-1}. Hence, the universe should be split into 15 levels with each level representing 18° (Fig. 5). Any value above the limit of the universe is infinity and the zero level is assumed to be the minimum speed (20 cm min^{-1}) which should be added to the final output of the controller. The fuzzy variables of the speed universe will be defined as:

$$\begin{aligned}
\text{minimum} &= |\sin \pi/2 - y| && \forall\ 0 < y < \pi/2 \\
&= 0 \quad \text{otherwise} \\
\text{slow} &= |\sin y| && \forall\ 0 < y < \pi \\
&= 0 \quad \text{otherwise} \\
\text{average} &= |\sin \pi/2 - y| && \forall\ \pi/2 < y < 3\pi/2 \\
&= 0 \quad \text{otherwise} \\
\text{fast} &= |\sin y| && \forall\ \pi < y < 3\pi/2 \\
&= 0 \quad \text{otherwise}
\end{aligned}$$

where

$$y = 1.5\pi Y/Y_{max} \text{ rad}$$
$$Y = 1, 2, 3 \ldots 15$$
$$Y_{max} = 15$$

Discretized values of these definitions are shown in Table 2.

One of the features of the fuzzimetric arc is that it allows the user to use trigonometric formulae to define

Table 1. Discretized width or depth universe into linguistic variables

	Level									
Depth or width	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Tiny	1	0.866	0.5	0	0	0	0	0	0	0
Small	0	0.5	0.86	1	0.86	0.5	0	0	0	0
Medium	0	0	0	0	0.5	0.866	1	0.866	0.5	0
Large	0	0	0	0	0	0	0	0.5	0.86	1

Table 2. Discretized speed universe {0–15 cm min^{-1}} and definition of linguistic variables

	Level															
Speed	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Minimum	1	0.95	0.81	0.58	0.31	0	0	0	0	0	0	0	0	0	0	0
Slow	0	0.31	0.58	0.81	0.95	1	0.95	0.81	0.58	0.31	0	0	0	0	0	0
Regular	0	0	0	0	0	0	0.31	0.58	0.81	0.95	1	0.95	0.81	0.58	0.31	0
Fast	0	0	0	0	0	0	0	0	0	0	0	0.31	0.58	0.81	0.95	1

linguistic hedges of fuzzy sets that lie between two neighbouring sets, for example, the following trigonometric formula would be valid in the shared region of any two neighbouring sets:

$$\sin^2 x + \cos^2 x = 1$$

for example, a fuzzy set between slow and regular may be defined as:

$$\text{BSR} = \text{between slow and regular} = \text{slow}^2 + \text{regular}^2$$

where from the above definitions and Table 2 fuzzy set:

$$\text{slow} = [\sin(0), \sin(\pi/10), \sin(2\pi/10), \sin(3\pi/10), \sin(4\pi/10), \sin(\pi/2), \sin(4\pi/10), \sin(3\pi/10), \sin(2\pi/10), \sin(\pi/10), 0, 0, 0, 0, 0, 0]$$

$$\text{slow} = [0, 0.31, 0.58, 0.81, 0.95, 1, 0.95, 0.81, 0.58, 0.31, 0, 0, 0, 0, 0, 0]$$

Similarly:

$$\text{regular} = [0, 0, 0, 0, 0, 0, 0.31, 0.58, 0.81, 0.95, 1, 0.95, 0.81, 0.58, 0.31, 0]$$

The square of these fuzzy variables would be:

$$\text{slow}^2 = [0, 0.096, 0.35, 0.65, 0.904, 1, 0.904, 0.65, 0.35, 0.096, 0, 0, 0, 0, 0, 0]$$

$$\text{reg}^2 = [0, 0, 0, 0, 0, 0, 0.096, 0.35, 0.65, 0.904, 1, 0.904, 0.65, 0.34, 0.096, 0]$$

Then, the resultant of $\text{slow}^2 + \text{regular}^2$ would be:

$$= [0, 0.096, 0.35, 0.65, 0.904, 1, 1, 1, 1, 1, 1, 0.904, 0.65, 0.35, 0.096, 0]$$

Similarly the following definitions may also follow:

$$\text{BFR} = \text{between fast and regular} = \text{fast}^2 + \text{regular}^2$$

$$\text{BSM} = \text{between slow and minimum} = \text{slow}^2 + \text{minimum}^2$$

3.2. *Model development and inference using Zadeh's technique*

Since the speed of the robot is inversely proportional to either the width or the depth of the cavity, then the closest algorithm to such linear behaviour would be (Table 3):

Table 3. Algorithm of welding process

	Depth			
Width	*Tiny*	*Small*	*Medium*	*Large*
Tiny	Fast	BFR	Regular	BSR
Small	BFR	Regular	BSR	Slow
Medium	Regular	BSR	Slow	BSM
Large	BSR	Slow	BSM	Minimum

RULE 1 IF depth=tiny AND width=tiny THEN speed=fast
OR
RULE 2 IF depth=tiny AND width=small THEN speed=BFR
OR
RULE 3 IF depth=tiny AND width=medium THEN speed=regular
OR
RULE 4 IF depth=tiny AND width=large THEN speed=BRS
OR
RULE 5 IF depth=small AND width=tiny THEN speed=BFR
OR
RULE 6 IF depth=small AND width=small THEN speed=regular
OR
RULE 7 IF depth=small AND width=medium THEN speed=BSR
OR
RULE 8 IF depth=small AND width=large THEN speed=slow
OR
RULE 9 IF depth=medium AND width=tiny THEN speed=regular
OR
RULE 10 IF depth=medium AND width=small THEN speed=BSR
OR
RULE 11 IF depth=medium AND width=medium THEN speed=slow
OR
RULE 12 IF depth=medium AND width=large THEN speed=BSM
OR
RULE 13 IF depth=large AND width=tiny THEN speed=BSR
OR
RULE 14 IF depth=large AND width=small THEN speed=slow
OR
RULE 15 IF depth=large AND width=medium THEN speed=BSM
OR
RULE 16 IF depth=large AND width=large THEN speed=minimum

Using the algorithm and the defined linguistic variables, one can develop the final model of the system which has the dimensions of 9 × 9 × 15 using the Cartesian product of both of the inputs and the output;

$$R = U_{\text{Width}} \times U_{\text{Depth}} \times U_{\text{Speed}}$$

$$\mu_R = \underset{\text{Rule}=1}{\overset{16}{\forall}} \max[\min(\mu_{\text{Width}}, \mu_{\text{Depth}}, \mu_{\text{Speed}})]$$

Table 4 shows the model developed with all the possible values of the inputs and its inferred averaged speed as well as the final speed as an output of the model. Fuzzy singletons of the inputs assumed for a particular value and centre of gravity method were used as an averaging technique of the fuzzy output using the formula:

$$\text{average value} = \frac{\Sigma \text{ speed value} \times \mu(S)}{\Sigma \mu(S)}$$

For example, if the depth of the cavity has a grey level of 5 and the width of the cavity was 2 mm then the speed value should be:

$$\frac{0.1 \times 1 + 0.34 \times 2 + 0.66 \times (3 + 12) + 0.86 \times (4 + 5 + 6 + 7 + 8 + 9 + 10 + 11) + 0.5 \times (13 + 14 + 15)}{0.1 + 0.34 + 2 \times 0.66 + 8 \times 0.86 + 3 \times 0.5} = 8.22$$

The inferred output as well as the final speed output are also shown in Table 4. The control surface of the model relating the inputs to that of the speed output is shown in Fig. 6.

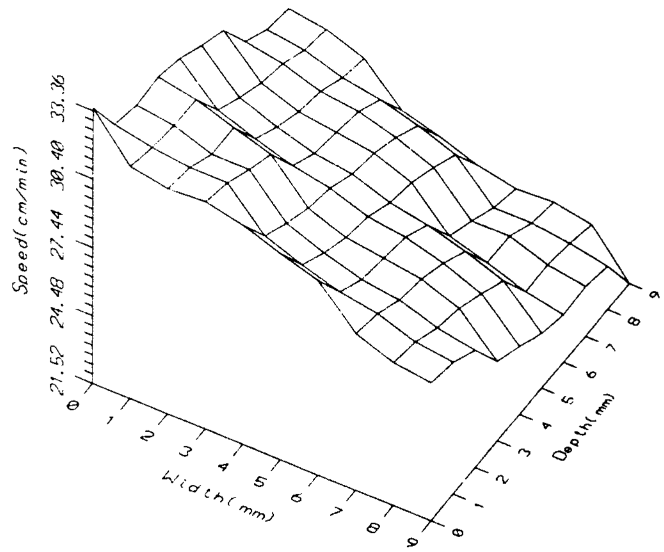

Fig. 6. Control surface using Zadeh's technique.

3.3. *Model development and inference using the proposed technique*

The algorithm may further be simplified using the individual relation between each of the inputs (depth and width) and the output (speed value). Since the depth and the width have the same effect on the output (inversely proportional), the same strategy can be used for both sub-algorithms to relate them to speed. In other words, the sub-algorithm that relates the depth to speed should be:

RULE 1 IF depth=tiny THEN speed=fast
OR
RULE 2 IF depth=small THEN speed=regular
OR
RULE 3 IF depth=medium THEN speed=slow
OR
RULE 4 IF depth=large THEN speed=minimum

and the sub-algorithm that relates the width of the cavity to the speed of the arm of the robot is represented by the following algorithm:

RULE 1 IF width=tiny THEN speed=fast
OR
RULE 2 IF width=small THEN speed=regular
OR
RULE 3 IF width=medium THEN speed=slow
OR
RULE 4 IF width=large THEN speed=minimum

The relations representing the two sub-algorithms R_{11} and R_{21} can now be developed. Relation R_{11} is defined as the Cartesian product of all rules in the sub-algorithm

Table 4. Fuzzy relation *R* (width × depth × speed) using Zadeh's technique and the inferred and final speed

Inputs level	*Speed level*																	
	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*Average value*	*Final speed*
Width = 0, Depth = 0→	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.95	1.00	13.48	33.48
Width = 0, Depth = 1→	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	11.61	31.61
Width = 0, Depth = 2→	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.31	31.31
Width = 0, Depth = 3→	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.34	0.66	0.90	1.00	1.00	0.99	0.99	1.00	1.00	11.42	31.42
Width = 0, Depth = 4→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.07	31.07
Width = 0, Depth = 5→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	10.49	30.49
Width = 0, Depth = 6→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	10.00	30.00
Width = 0, Depth = 7→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.36	28.36
Width = 0, Depth = 8→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.74	27.74
Width = 0, Depth = 9→	0.00	0.10	0.34	0.66	0.90	1.00	1.00	0.99	0.99	1.00	1.00	0.90	0.66	0.34	0.10	0.00	7.50	27.50
Width = 1, Depth = 0→	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	11.61	31.61
Width = 1, Depth = 1→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	11.28	31.28
Width = 1, Depth = 2→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.07	31.07
Width = 1, Depth = 3→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.07	31.07
Width = 1, Depth = 4→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	9.45	29.45
Width = 1, Depth = 5→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	8.86	28.86
Width = 1, Depth = 6→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.36	28.36
Width = 1, Depth = 7→	0.00	0.31	0.50	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.05	28.05
Width = 1, Depth = 8→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.50	27.50
Width = 1, Depth = 9→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	7.26	27.26
Width = 2, Depth = 0→	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.31	31.31
Width = 2, Depth = 1→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.07	31.07
Width = 2, Depth = 2→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	10.49	30.49
Width = 2, Depth = 3→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	10.49	30.49
Width = 2, Depth = 4→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	8.86	28.86
Width = 2, Depth = 5→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.50	0.50	8.22	28.22
Width = 2, Depth = 6→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.74	27.74
Width = 2, Depth = 7→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.50	27.50
Width = 2, Depth = 8→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.50	0.31	0.00	6.95	26.95
Width = 2, Depth = 9→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.64	26.64
Width = 3, Depth = 0→	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.34	0.66	0.90	1.00	1.00	0.99	0.99	1.00	1.00	11.42	31.42
Width = 3, Depth = 1→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.07	31.07
Width = 3, Depth = 2→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	10.49	30.49
Width = 3, Depth = 3→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	10.00	30.00
Width = 3, Depth = 4→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.36	28.36
Width = 3, Depth = 5→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.74	27.74
Width = 3, Depth = 6→	0.00	0.10	0.34	0.66	0.90	1.00	1.00	0.99	0.99	1.00	1.00	0.90	0.66	0.34	0.10	0.00	7.50	27.50
Width = 3, Depth = 7→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	7.26	27.26
Width = 3, Depth = 8→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.64	26.64
Width = 3, Depth = 9→	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	5.00	25.00
Width = 4, Depth = 0→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	11.07	31.07
Width = 4, Depth = 1→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	9.45	29.45
Width = 4, Depth = 2→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	8.86	28.86
Width = 4, Depth = 3→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.36	28.36
Width = 4, Depth = 4→	0.00	0.31	0.50	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.05	28.05
Width = 4, Depth = 5→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.50	27.50
Width = 4, Depth = 6→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	7.26	27.26
Width = 4, Depth = 7→	0.50	0.50	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	6.78	26.78
Width = 4, Depth = 8→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.14	26.14
Width = 4, Depth = 9→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	4.51	24.51
Width = 5, Depth = 0→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	10.49	30.49
Width = 5, Depth = 1→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	8.86	28.86
Width = 5, Depth = 2→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.50	0.50	8.22	28.22

Table 4. (contd.) Fuzzy relation *R* (width × depth × speed) using Zadeh's technique and the inferred and final speed

Inputs level	*Speed level* 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	*Average value*	*Final speed*
Width = 5, Depth = 3→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.74	27.74
Width = 5, Depth = 4→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.50	27.50
Width = 5, Depth = 5→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.50	0.31	0.00	6.95	26.95
Width = 5, Depth = 6→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.64	26.64
Width = 5, Depth = 7→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.14	26.14
Width = 5, Depth = 8→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.50	0.50	0.50	0.50	0.34	0.10	0.00	5.55	25.55
Width = 5, Depth = 9→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.93	23.93
Width = 6, Depth = 0→	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	10.00	30.00
Width = 6, Depth = 1→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.36	28.36
Width = 6, Depth = 2→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.74	27.74
Width = 6, Depth = 3→	0.00	0.10	0.34	0.66	0.90	1.00	1.00	0.99	0.99	1.00	1.00	0.90	0.66	0.34	0.10	0.00	7.50	27.50
Width = 6, Depth = 4→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	7.26	27.26
Width = 6, Depth = 5→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.64	26.64
Width = 6, Depth = 6→	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	5.00	25.00
Width = 6, Depth = 7→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	4.51	24.51
Width = 6, Depth = 8→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.93	23.93
Width = 6, Depth = 9→	1.00	1.00	0.99	0.99	1.00	1.00	0.90	0.66	0.34	0.10	0.00	0.00	0.00	0.00	0.00	0.00	3.58	23.58
Width = 7, Depth = 0→	0.00	0.10	0.34	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.36	28.36
Width = 7, Depth = 1→	0.00	0.31	0.50	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.05	28.05
Width = 7, Depth = 2→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.50	27.50
Width = 7, Depth = 3→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	7.26	27.26
Width = 7, Depth = 4→	0.50	0.50	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	6.78	26.78
Width = 7, Depth = 5→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.14	26.14
Width = 7, Depth = 6→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	4.51	24.51
Width = 7, Depth = 7→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	4.51	24.51
Width = 7, Depth = 8→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.93	23.93
Width = 7, Depth = 9→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	0.00	0.00	0.00	0.00	0.00	3.69	23.69
Width = 8, Depth = 0→	0.00	0.10	0.34	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.74	27.74
Width = 8, Depth = 1→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	7.50	27.50
Width = 8, Depth = 2→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.50	0.31	0.00	6.95	26.95
Width = 8, Depth = 3→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.64	26.64
Width = 8, Depth = 4→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.14	26.14
Width = 8, Depth = 5→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.50	0.50	0.50	0.50	0.34	0.10	0.00	5.55	25.55
Width = 8, Depth = 6→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.93	23.93
Width = 8, Depth = 7→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.93	23.93
Width = 8, Depth = 8→	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.72	23.72
Width = 8, Depth = 9→	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	0.00	0.00	0.00	0.00	0.00	3.39	23.39
Width = 9, Depth = 0→	0.00	0.10	0.34	0.66	0.90	1.00	1.00	0.99	0.99	1.00	1.00	0.90	0.66	0.34	0.10	0.00	7.50	27.50
Width = 9, Depth = 1→	0.00	0.31	0.50	0.66	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	7.26	27.26
Width = 9, Depth = 2→	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	6.64	26.64
Width = 9, Depth = 3→	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	5.00	25.00
Width = 9, Depth = 4→	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	4.51	24.51
Width = 9, Depth = 5→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.93	23.93
Width = 9, Depth = 6→	1.00	1.00	0.99	0.99	1.00	1.00	0.90	0.66	0.34	0.10	0.00	0.00	0.00	0.00	0.00	0.00	3.58	23.58
Width = 9, Depth = 7→	0.86	0.86	0.86	0.86	0.86	0.86	0.86	0.66	0.34	0.10	0.00	0.00	0.00	0.00	0.00	0.00	3.69	23.69
Width = 9, Depth = 8→	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.34	0.10	0.00	0.00	0.00	0.00	0.00	0.00	3.39	23.39
Width = 9, Depth = 9→	1.00	0.95	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.52	21.52

describing the relation of the first input (depth) with the output (speed):

$$R_{11} = (\text{depth})_{\substack{\text{tiny}\\ \text{small}\\ \text{medium}\\ \text{large}}} \times (\text{speed value})_{\substack{\text{fast}\\ \text{regular}\\ \text{slow}\\ \text{minimum}}}$$

i.e. relation R_{11} should have the following form:

Depth universe	*Speed universe* 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	*Average*
0	0	0	0	0	0	0	0	0	0	0	0	0.31	0.58	0.81	0.95	1	13.48
1	0	0	0	0	0	0	0.31	0.5	0.5	0.5	0.5	0.5	0.58	0.81	0.86	0.86	11.2
2	0	0	0	0	0	0	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.5	0.5	10.4
3	0	0	0	0	0	0	0.31	0.58	0.81	0.95	1	0.95	0.81	0.58	0.31	0	10
4	0	0.31	0.5	0.5	0.5	0.5	0.5	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0	8.05
5	0	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.5	0.5	0.5	0.5	0.5	0.31	0	6.95
6	0	0.31	0.58	0.81	0.95	1	0.95	0.81	0.58	0.31	0	0	0	0	0	0	5
7	0.5	0.5	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0	0	0	0	0	0	4.51
8	0.86	0.86	0.81	0.58	0.5	0.5	0.5	0.5	0.5	0.31	0	0	0	0	0	0	3.72
9	1	0.95	0.81	0.58	0.31	0	0	0	0	0	0	0	0	0	0	0	1.52

Similarly, the second sub-relation R_{21} would be the Cartesian product of the second input (width) and the output (speed) and is given by:

$$R_{11} = (\text{width})_{\substack{\text{tiny}\\ \text{small}\\ \text{medium}\\ \text{large}}} \times (\text{speed value})_{\substack{\text{fast}\\ \text{regular}\\ \text{slow}\\ \text{minimum}}}$$

Note that sub-relation R_{21} is equivalent to sub-relation R_{11} which represents the following relation:

Width rule	*Speed universe* 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	*Average*
0	0	0	0	0	0	0	0	0	0	0	0	0.31	0.58	0.81	0.95	1	13.48
1	0	0	0	0	0	0	0.31	0.5	0.5	0.5	0.5	0.5	0.58	0.81	0.86	0.86	11.2
2	0	0	0	0	0	0	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.5	0.5	10.4
3	0	0	0	0	0	0	0.31	0.58	0.81	0.95	1	0.95	0.81	0.58	0.31	0	10
4	0	0.31	0.5	0.5	0.5	0.5	0.5	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0	8.05
5	0	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.5	0.5	0.5	0.5	0.5	0.31	0	6.95
6	0	0.31	0.58	0.81	0.95	1	0.95	0.81	0.58	0.31	0	0	0	0	0	0	5
7	0.5	0.5	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0	0	0	0	0	0	4.51
8	0.86	0.86	0.81	0.58	0.5	0.5	0.5	0.5	0.5	0.31	0	0	0	0	0	0	3.72
9	1	0.95	0.81	0.58	0.31	0	0	0	0	0	0	0	0	0	0	0	1.52

Assuming that the depth and the width carry an equal importance influencing the output speed, then the importance factor for the depth and the width would be:

ε_1 = importance level of the depth = 0.5
ε_2 = importance level of the width = 0.5

Applying Equation 1, the final required value of the speed can be found:

$$\text{speed value} = \varepsilon_1 \times [\text{av}(\text{depth} \circ R_{11})] + \varepsilon_2 \times [\text{av}(\text{width} \circ R_{21})]$$

For example, if the depth of the cavity has a grey level of

5 and the width of the cavity is 2 mm then the speed value should be:

$$\text{speed value} = 0.5 \times 6.95 + 0.5 \times 10.49 = 8.72\ \text{cm min}^{-1}$$

and the final speed should be:

$$8.72 + 20 = 28.72\ \text{cm min}^{-1}$$

All possible values of the inputs and the inferred speed output are shown in Table 5 and the control surface of the final speed of the robot is shown in Fig. 7.

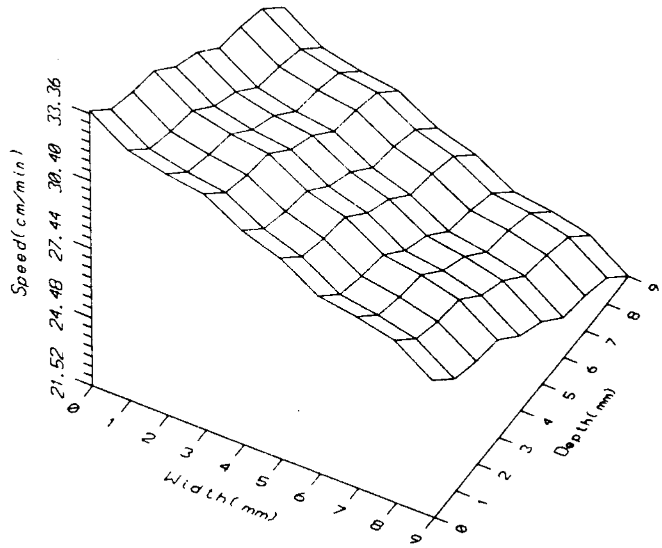

Fig. 7. Control surface using proposed technique.

3.4. *Comparison of the two techniques*

Comparing Figs 6 and 7, it can be noticed that the proposed technique is more linear than Zadeh's technique. It is expected that the traditional fuzzy logic technique is more successful for non-linear systems than the proposed one. However, the chosen welding process example is more or less considered as linear behaviour. In order to maintain the validity of the comparison, both techniques assume fuzzy singletons (a fuzzy singleton is a fuzzy set that has a unity membership value for a specific level and zero membership for any other level) as inputs and the centre of gravity was adopted as the method of defuzzifying the fuzzy output. It was decided to measure the inferred output speed (not the final speed) as the basis of comparison following a simple percentage of deviation formula:

$$\%\ \text{deviation} = \frac{(\text{inferred output})_{\text{Zadeh's}} - (\text{inferred output})_{\text{proposed}}}{(\text{inferred output})_{\text{Zadeh's}}} \times 100$$

For example, for a cavity size of 2-mm width and a grey level of 5, then the deviation of the proposed technique to that of Zadeh's would be (from Tables 4 and 5):

$$\%\ \text{deviation} = \frac{8.22 - 8.72}{8.22} \times 100 = -6.10\%$$

Table 6 shows the percentage deviations between the two techniques which are also shown schematically in Fig. (8).

4. The cornering process

This section describes another implementation of the proposed multivariable technique to a robotic manufacturing system. It is an imitation of the skilled human operator controlling a task termed as the 'cornering process' which has no model and depends completely on human intuition. Cornering is one stage of force transducer (load cell) manufacture, where the objective is to achieve an equal electric output related only to the force (load) exerted to the transducer, irrespective of the position of the load. By this the effect of torsion or moment on the cell that could be interpreted as force (load) is eliminated, i.e. assuming the load application can be represented in Equation 2:

$$\text{Load} = (A \times F) + (B \times M) + (C \times T) \qquad (2)$$

where

A = force coefficient
B = moment coefficient
C = torque coefficient
F = force exerted on the load-cell
M = moment resulting from application of force F
T = torque resulting from application of force F

Then, the objective of cornering is to achieve null value for the moment and torque coefficients ($B = C = 0$) and hence achieve an output value related only to the exerted force on the cell. This is done by geometrically changing the nominal axes of the transducer by filing one or more corners of the transducer and hence it is termed the 'cornering process'. A human's action to achieve acceptable cornering performance consists of two distinct operations:

(1) Checking whether the load readings are within specification by applying load via a moment arm; and
(2) filing one or more corners to attain (1) above.

The cycle of checking and filing is repeated continuously

Table 5. Fuzzy model using the proposed technique with importance factor of 0.5 for both inputs

Inputs	*Speed level* 0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	*Average value*	*Final speed*
Level = 0 →	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.95	1.00	13.48	33.48
Level = 1 →	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.50	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	11.28	31.28
Level = 2 →	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	10.49	30.49
Level = 3 →	0.00	0.00	0.00	0.00	0.00	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	10.00	30.00
Level = 4 →	0.00	0.31	0.50	0.50	0.50	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	8.05	28.05
Level = 5 →	0.00	0.31	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.50	0.31	0.00	6.95	26.95
Level = 6 →	0.00	0.31	0.58	0.81	0.95	1.00	0.95	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	5.00	25.00
Level = 7 →	0.50	0.50	0.58	0.81	0.86	0.86	0.86	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	4.51	24.51
Level = 8 →	0.86	0.86	0.81	0.58	0.50	0.50	0.50	0.50	0.50	0.31	0.00	0.00	0.00	0.00	0.00	0.00	3.72	23.72
Level = 9 →	1.00	0.95	0.81	0.58	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.52	21.52

	Inferred speed output	*Actual speed*
Width level = 0, Depth level = 0 →	0.5 * 13.48 + 0.5 * 13.48 = 13.48	33.48
Width level = 0, Depth level = 1 →	0.5 * 13.48 + 0.5 * 11.28 = 12.38	32.38
Width level = 0, Depth level = 2 →	0.5 * 13.48 + 0.5 * 10.49 = 11.98	31.98
Width level = 0, Depth level = 3 →	0.5 * 13.48 + 0.5 * 10.00 = 11.74	31.74
Width level = 0, Depth level = 4 →	0.5 * 13.48 + 0.5 * 8.05 = 10.76	30.76
Width level = 0, Depth level = 5 →	0.5 * 13.48 + 0.5 * 6.95 = 10.22	30.22
Width level = 0, Depth level = 6 →	0.5 * 13.48 + 0.5 * 5.00 = 9.24	29.24
Width level = 0, Depth level = 7 →	0.5 * 13.48 + 0.5 * 4.51 = 9.00	29.00
Width level = 0, Depth level = 8 →	0.5 * 13.48 + 0.5 * 3.72 = 8.60	28.60
Width level = 0, Depth level = 9 →	0.5 * 13.48 + 0.5 * 1.52 = 7.50	27.50
Width level = 1, Depth level = 0 →	0.5 * 11.28 + 0.5 * 13.48 = 12.38	32.38
Width level = 1, Depth level = 1 →	0.5 * 11.28 + 0.5 * 11.28 = 11.28	31.28
Width level = 1, Depth level = 2 →	0.5 * 11.28 + 0.5 * 10.49 = 10.89	30.89
Width level = 1, Depth level = 3 →	0.5 * 11.28 + 0.5 * 10.00 = 10.64	30.64
Width level = 1, Depth level = 4 →	0.5 * 11.28 + 0.5 * 8.05 = 9.67	29.67
Width level = 1, Depth level = 5 →	0.5 * 11.28 + 0.5 * 6.95 = 9.12	29.12
Width level = 1, Depth level = 6 →	0.5 * 11.28 + 0.5 * 5.00 = 8.14	28.14
Width level = 1, Depth level = 7 →	0.5 * 11.28 + 0.5 * 4.51 = 7.90	27.90
Width level = 1, Depth level = 8 →	0.5 * 11.28 + 0.5 * 3.72 = 7.50	27.50
Width level = 1, Depth level = 9 →	0.5 * 11.28 + 0.5 * 1.52 = 6.40	26.40
Width level = 2, Depth level = 0 →	0.5 * 10.49 + 0.5 * 13.48 = 11.98	31.98
Width level = 2, Depth level = 1 →	0.5 * 10.49 + 0.5 * 11.28 = 10.89	30.89
Width level = 2, Depth level = 2 →	0.5 * 10.49 + 0.5 * 10.49 = 10.49	30.49
Width level = 2, Depth level = 3 →	0.5 * 10.49 + 0.5 * 10.00 = 10.24	30.24
Width level = 2, Depth level = 4 →	0.5 * 10.49 + 0.5 * 8.05 = 9.27	29.27
Width level = 2, Depth level = 5 →	0.5 * 10.49 + 0.5 * 6.95 = 8.72	28.72
Width level = 2, Depth level = 6 →	0.5 * 10.49 + 0.5 * 5.00 = 7.74	27.74
Width level = 2, Depth level = 7 →	0.5 * 10.49 + 0.5 * 4.51 = 7.50	27.50
Width level = 2, Depth level = 8 →	0.5 * 10.49 + 0.5 * 3.72 = 7.10	27.10
Width level = 2, Depth level = 9 →	0.5 * 10.49 + 0.5 * 1.52 = 6.00	26.00
Width level = 3, Depth level = 0 →	0.5 * 10.00 + 0.5 * 13.48 = 11.74	31.74
Width level = 3, Depth level = 1 →	0.5 * 10.00 + 0.5 * 11.28 = 10.64	30.64
Width level = 3, Depth level = 2 →	0.5 * 10.00 + 0.5 * 10.49 = 10.24	30.24
Width level = 3, Depth level = 3 →	0.5 * 10.00 + 0.5 * 10.00 = 10.00	30.00
Width level = 3, Depth level = 4 →	0.5 * 10.00 + 0.5 * 8.05 = 9.02	29.02
Width level = 3, Depth level = 5 →	0.5 * 10.00 + 0.5 * 6.95 = 8.48	28.48
Width level = 3, Depth level = 6 →	0.5 * 10.00 + 0.5 * 5.00 = 7.50	27.50
Width level = 3, Depth level = 7 →	0.5 * 10.00 + 0.5 * 4.51 = 7.26	27.26
Width level = 3, Depth level = 8 →	0.5 * 10.00 + 0.5 * 3.72 = 6.86	26.86
Width level = 3, Depth level = 9 →	0.5 * 10.00 +,0.5 * 1.52 = 5.76	25.76
Width level = 4, Depth level = 0 →	0.5 * 8.05 + 0.5 * 13.48 = 10.76	30.76
Width level = 4, Depth level = 1 →	0.5 * 8.05 + 0.5 * 11.28 = 9.67	29.67

Table 5. (Contd). Fuzzy model using the proposed technique with importance factor of 0.5 for both inputs

	Inferred speed output	*Actual speed*
Width level = 4, Depth level = 2 →	0.5 * 8.05 + 0.5 * 10.49 = 9.27	29.27
Width level = 4, Depth level = 3 →	0.5 * 8.05 + 0.5 * 10.00 = 9.02	29.02
Width level = 4, Depth level = 4 →	0.5 * 8.05 + 0.5 * 8.05 = 8.05	28.05
Width level = 4, Depth level = 5 →	0.5 * 8.05 + 0.5 * 6.95 = 7.50	27.50
Width level = 4, Depth level = 6 →	0.5 * 8.05 + 0.5 * 5.00 = 6.52	26.52
Width level = 4, Depth level = 7 →	0.5 * 8.05 + 0.5 * 4.51 = 6.28	26.28
Width level = 4, Depth level = 8 →	0.5 * 8.05 + 0.5 * 3.72 = 5.88	25.88
Width level = 4, Depth level = 9 →	0.5 * 8.05 + 0.5 * 1.52 = 4.78	24.78
Width level = 5, Depth level = 0 →	0.5 * 6.95 + 0.5 * 13.48 = 10.22	30.22
Width level = 5, Depth level = 1 →	0.5 * 6.95 + 0.5 * 11.28 = 9.12	29.12
Width level = 5, Depth level = 2 →	0.5 * 6.95 + 0.5 * 10.49 = 8.72	28.72
Width level = 5, Depth level = 3 →	0.5 * 6.95 + 0.5 * 10.00 = 8.48	28.48
Width level = 5, Depth level = 4 →	0.5 * 6.95 + 0.5 * 8.05 = 7.50	27.50
Width level = 5, Depth level = 5 →	0.5 * 6.95 + 0.5 * 6.95 = 6.95	26.95
Width level = 5, Depth level = 6 →	0.5 * 6.95 + 0.5 * 5.00 = 5.98	25.98
Width level = 5, Depth level = 7 →	0.5 * 6.95 + 0.5 * 4.51 = 5.73	25.73
Width level = 5, Depth level = 8 →	0.5 * 6.95 + 0.5 * 3.72 = 5.33	25.33
Width level = 5, Depth level = 9 →	0.5 * 6.95 + 0.5 * 1.52 = 4.24	24.24
Width level = 6, Depth level = 0 →	0.5 * 5.00 + 0.5 * 13.48 = 9.24	29.24
Width level = 6, Depth level = 1 →	0.5 * 5.00 + 0.5 * 11.28 = 8.14	28.14
Width level = 6, Depth level = 2 →	0.5 * 5.00 + 0.5 * 10.49 = 7.74	27.74
Width level = 6, Depth level = 3 →	0.5 * 5.00 + 0.5 * 10.00 = 7.50	27.50
Width level = 6, Depth level = 4 →	0.5 * 5.00 + 0.5 * 8.05 = 6.52	26.52
Width level = 6, Depth level = 5 →	0.5 * 5.00 + 0.5 * 6.95 = 5.98	25.98
Width level = 6, Depth level = 6 →	0.5 * 5.00 + 0.5 * 5.00 = 5.00	25.00
Width level = 6, Depth level = 7 →	0.5 * 5.00 + 0.5 * 4.51 = 4.76	24.76
Width level = 6, Depth level = 8 →	0.5 * 5.00 + 0.5 * 3.72 = 4.36	24.36
Width level = 6, Depth level = 9 →	0.5 * 5.00 + 0.5 * 1.52 = 3.26	23.26
Width level = 7, Depth level = 0 →	0.5 * 4.51 + 0.5 * 13.48 = 9.00	29.00
Width level = 7, Depth level = 1 →	0.5 * 4.51 + 0.5 * 11.28 = 7.90	27.90
Width level = 7, Depth level = 2 →	0.5 * 4.51 + 0.5 * 10.49 = 7.50	27.50
Width level = 7, Depth level = 3 →	0.5 * 4.51 + 0.5 * 10.00 = 7.26	27.26
Width level = 7, Depth level = 4 →	0.5 * 4.51 + 0.5 * 8.05 = 6.28	26.28
Width level = 7, Depth level = 5 →	0.5 * 4.51 + 0.5 * 6.95 = 5.73	25.73
Width level = 7, Depth level = 6 →	0.5 * 4.51 + 0.5 * 5.00 = 4.76	24.76
Width level = 7, Depth level = 7 →	0.5 * 4.51 + 0.5 * 4.51 = 4.51	24.51
Width level = 7, Depth level = 8 →	0.5 * 4.51 + 0.5 * 3.72 = 4.11	24.11
Width level = 7, Depth level = 9 →	0.5 * 4.51 + 0.5 * 1.52 = 3.02	23.02
Width level = 8, Depth level = 0 →	0.5 * 3.72 + 0.5 * 13.48 = 8.60	28.60
Width level = 8, Depth level = 1 →	0.5 * 3.72 + 0.5 * 11.28 = 7.50	27.50
Width level = 8, Depth level = 2 →	0.5 * 3.72 + 0.5 * 10.49 = 7.10	27.10
Width level = 8, Depth level = 3 →	0.5 * 3.72 + 0.5 * 10.00 = 6.86	26.86
Width level = 8, Depth level = 4 →	0.5 * 3.72 + 0.5 * 8.05 = 5.88	25.88
Width level = 8, Depth level = 5 →	0.5 * 3.72 + 0.5 * 6.95 = 5.33	25.33
Width level = 8, Depth level = 6 →	0.5 * 3.72 + 0.5 * 5.00 = 4.36	24.36
Width level = 8, Depth level = 7 →	0.5 * 3.72 + 0.5 * 4.51 = 4.11	24.11
Width level = 8, Depth level = 8 →	0.5 * 3.72 + 0.5 * 3.72 = 3.72	23.72
Width level = 8, Depth level = 9 →	0.5 * 3.72 + 0.5 * 1.52 = 2.62	22.62
Width level = 9, Depth level = 0 →	0.5 * 1.52 + 0.5 * 13.48 = 7.50	27.50
Width level = 9, Depth level = 1 →	0.5 * 1.52 + 0.5 * 11.28 = 6.40	26.40
Width level = 9, Depth level = 2 →	0.5 * 1.52 + 0.5 * 10.49 = 6.00	26.00
Width level = 9, Depth level = 3 →	0.5 * 1.52 + 0.5 * 10.00 = 5.76	25.76
Width level = 9, Depth level = 4 →	0.5 * 1.52 + 0.5 * 8.05 = 4.78	24.78
Width level = 9, Depth level = 5 →	0.5 * 1.52 + 0.5 * 6.95 = 4.24	24.24
Width level = 9, Depth level = 6 →	0.5 * 1.52 + 0.5 * 5.00 = 3.26	23.26
Width level = 9, Depth level = 7 →	0.5 * 1.52 + 0.5 * 4.51 = 3.02	23.02
Width level = 9, Depth level = 8 →	0.5 * 1.52 + 0.5 * 3.72 = 2.62	22.62
Width level = 9, Depth level = 9 →	0.5 * 1.52 + 0.5 * 1.52 = 1.52	21.52

Table 6. Percentage deviation of inferred output from Zadeh's technique

Width	*Depth*	*% Deviation*	*Width*	*Depth*	*% Deviation*
0	0	0.00	5	0	2.60
0	1	−6.62	5	1	−2.94
0	2	−5.93	5	2	−6.10
0	3	−2.77	5	3	−9.47
0	4	2.74	5	4	−0.00
0	5	2.60	5	5	0.00
0	6	7.60	5	6	10.05
0	7	−7.64	5	7	6.71
0	8	−11.05	5	8	3.96
0	9	0.00	5	9	−7.71
1	0	−6.62	6	0	7.60
1	1	0.00	6	1	2.58
1	2	1.64	6	2	−0.02
1	3	3.85	6	3	0.00
1	4	−2.33	6	4	10.10
1	5	−2.94	6	5	10.05
1	6	2.58	6	6	0.00
1	7	1.88	6	7	−5.42
1	8	−0.00	6	8	−10.83
1	9	11.79	6	9	8.85
2	0	−5.93	7	0	−7.64
2	1	1.64	7	1	1.88
2	2	0.00	7	2	−0.00
2	3	2.33	7	3	0.02
2	4	−4.66	7	4	7.40
2	5	−6.10	7	5	6.71
2	6	−0.02	7	6	−5.42
2	7	−0.00	7	7	0.00
2	8	−2.18	7	8	−4.62
2	9	9.62	7	9	18.20
3	0	−2.77	8	0	−11.05
3	1	3.85	8	1	−0.00
3	2	2.33	8	2	−2.18
3	3	0.00	8	3	−3.24
3	4	−7.99	8	4	4.24
3	5	−9.47	8	5	3.96
3	6	0.00	8	6	−10.83
3	7	0.02	8	7	−4.62
3	8	−3.24	8	8	0.00
3	9	−15.21	8	9	22.68
4	0	2.74	9	0	−0.00
4	1	−2.33	9	1	11.79
4	2	−4.66	9	2	9.62
4	3	−7.99	9	3	−15.21
4	4	0.00	9	4	−6.06
4	5	−0.00	9	5	−7.71
4	6	10.10	9	6	8.85
4	7	7.40	9	7	18.20
4	8	4.24	9	8	22.68
4	9	−6.06	9	9	0.00

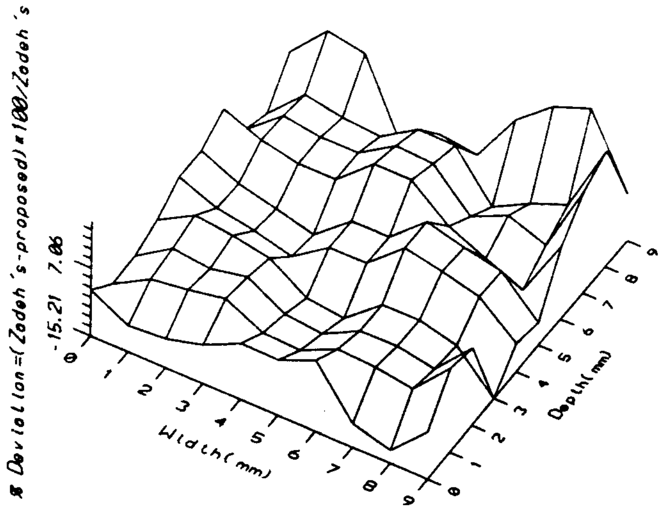

Fig. 8. Percentage deviation of proposed technique to Zadeh technique.

until the load-cell produces a set of four corner readings which are within specification. A robotic cornering system was built to imitate the human actions in the cornering process. The test requires that the robot moves the load to the four corners via a peg and hole mechanism with two slotted infrared sensors used to sense the location at any one time (Fig. 9). A deburring tool also attached to the end effector acts as a metal removal mechanism (Fig. 10). A FANUC robot was utilized for this task which has been interfaced with an OLIVETTI M20 running under PC-DOS. The computer was the master controller of the process, using a BASIC program capable of controlling the process, handshaking with the robot, and interfaced with a PREMA digital voltmeter. The strategic flow chart of the robotic cornering system is shown in Fig. 11 where the proposed simplified multivariable fuzzy logic technique was used to imitate the human action to achieve the cornering performance.

4.1. *Cornering process fuzzy variables*

After testing the load-cell, the operator calculates the error between the maximum and minimum of the readings. Depending upon this error and the change of error compared with the previous cycle, the operator adjusts the pressure applied by the file. Thus, two input fuzzy variables are identified, namely the error (E) and the change of error (CE) as inputs to the system. The tool chosen for the automatic cornering system was a pneumatic deburring tool. The greater the tool tip entry into the load-cell the greater the amount of material removed and hence this was chosen to be the first fuzzy output (LI). Also as the tool advances, more metal

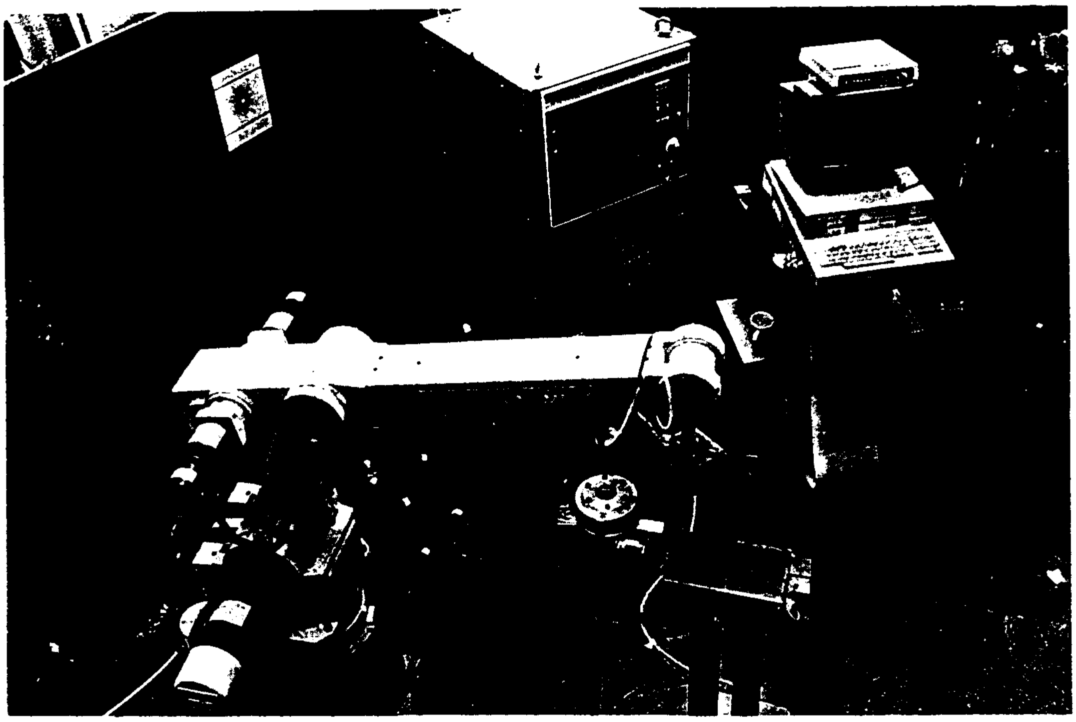

Fig. 9. Peg/hole load shifting mechanism.

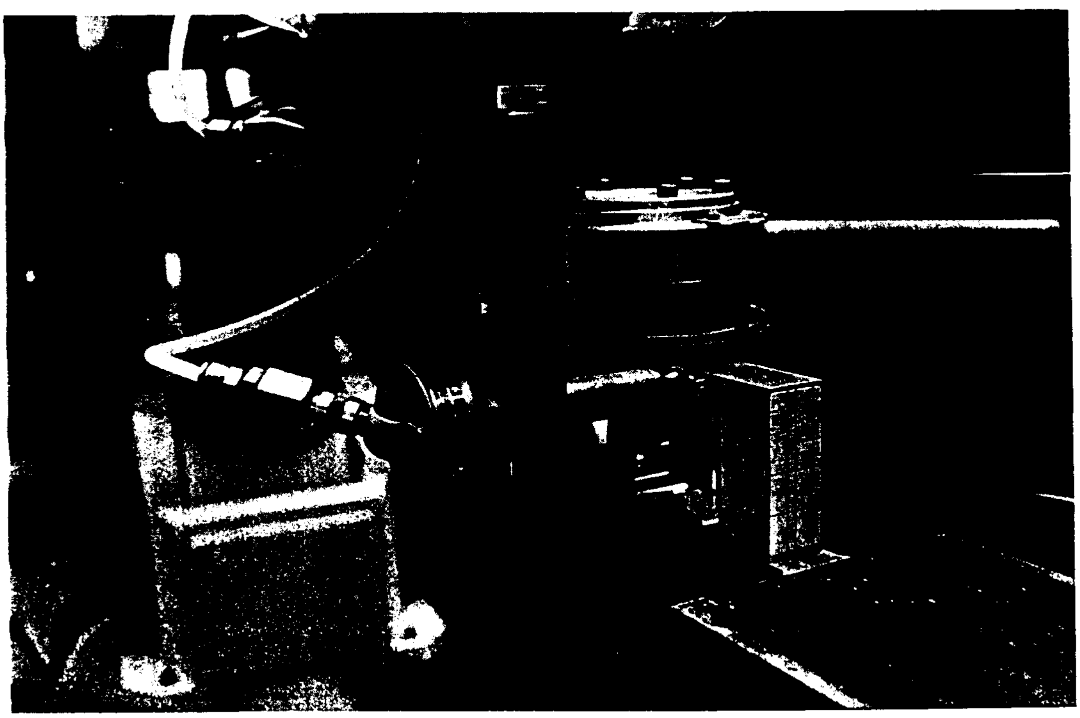

Fig. 10. Metal removal mechanism.

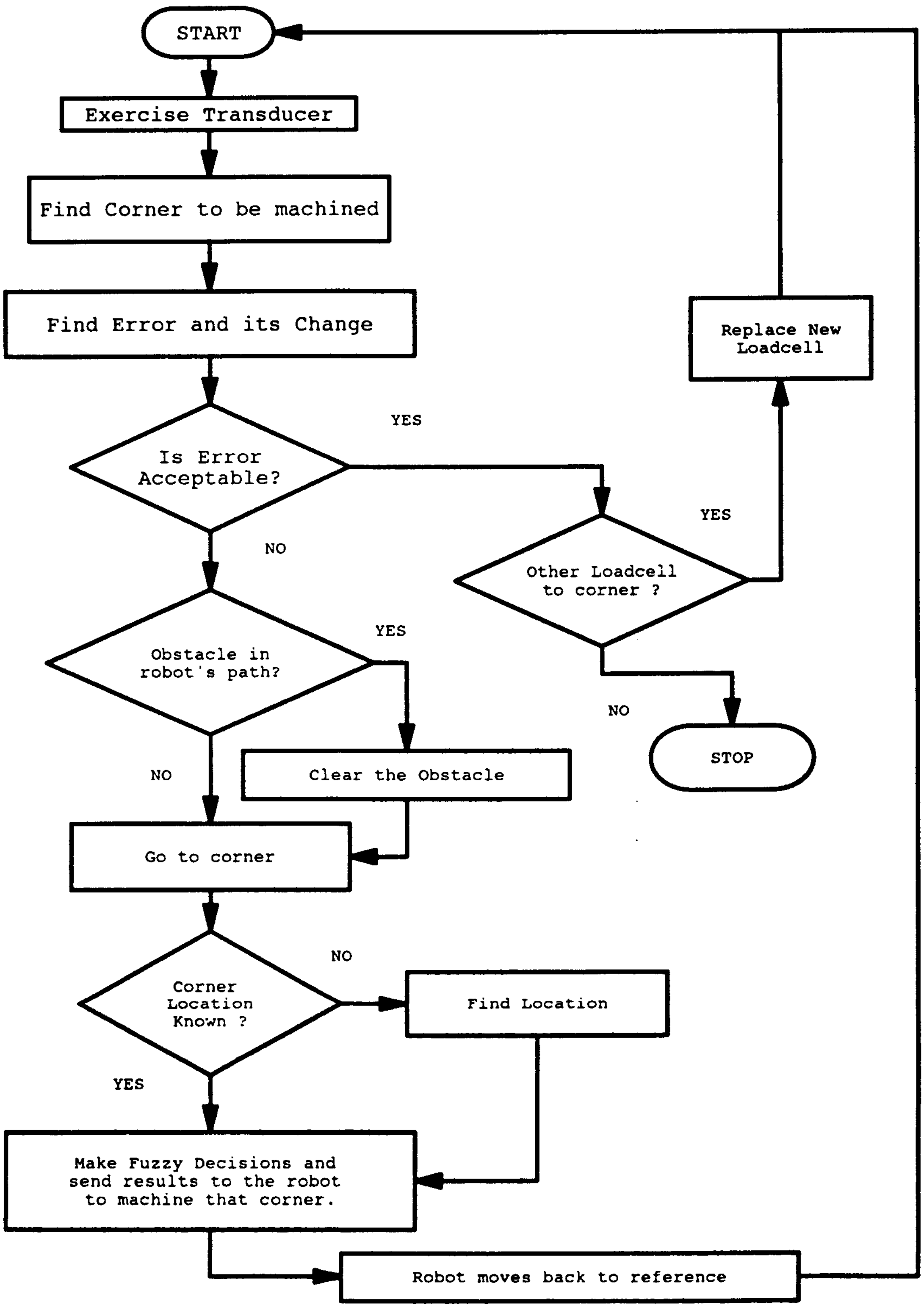

Fig. 11. Flow chart of automatic cornering process.

removal occurs which describes the second fuzzy output (*MI*) of the system.

The error is the most critical input to the system where if the amount of material removed is more than required according to the input, then the error will be transferred to the opposite corner. Thus a range between zero and nine {0,9} μV was chosen for the input universe of the error and its change, which has been partitioned using the concept of fuzzimetric arcs as described in Section 3.1 into the following fuzzy sets:

$$PO = 1/0 + 0.866/1 + 0.5/2 + 0/3 + 0/4 + 0/5 + 0/6 + 0/7 + 0/8 + 0/9$$
$$PS = 0/0 + 0.5/1 + 0.866/2 + 1/3 + 0.866/4 + 0.5/5 + 0/6 + 0/7 + 0/8 + 0/9$$
$$PM = 0/0 + 0/1 + 0/2 + 0/3 + 0.5/4 + 0.866/5 + 1/6 + 0.866/7 + 0.5/8 + 0/9$$
$$PB = 0/0 + 0/1 + 0/2 + 0/3 + 0/4 + 0/5 + 0/6 + 0.5/7 + 0.866/8 + 1/9$$

where the linguistic hedges are defined as:

$$PO^{1/2} = 1/0 + 0.93/1 + 0.7/2 + 0/3 + 0/4 + 0/5 + 0/6 + 0/7 + 0/8 + 0/9$$
$$PO^2 = 1/0 + 0.75/1 + 0.25/2 + 0/3 + 0/4 + 0/5 + 0/6 + 0/7 + 0/8 + 0/9$$
$$PS^{1/2} = 0/0 + 0.7/1 + 0.93/2 + 1/3 + 0.93/4 + 0.7/5 + 0/6 + 0/7 + 0/8 + 0/9$$
$$PS^2 = 0/0 + 0.25/1 + 0.75/2 + 1/3 + 0.75/4 + 0.25/5 + 0/6 + 0/7 + 0/8 + 0/9$$
$$PM^{1/2} = 0/0 + 0/1 + 0/2 + 0/3 + 0.7/4 + 0.93/5 + 1/6 + 0.93/7 + 0.7/8 + 0/9$$
$$PM^2 = 0/0 + 0/1 + 0/2 + 0/3 + 0.25/4 + 0.75/5 + 1/6 + 0.75/7 + 0.25/8 + 0/9$$
$$PB^{1/2} = 0/0 + 0/1 + 0/2 + 0/3 + 0/4 + 0/5 + 0/6 + 0.7/7 + 0.93/8 + 1/9$$
$$PB^2 = 0/0 + 0/1 + 0/2 + 0/3 + 0/4 + 0/5 + 0/6 + 0.25/7 + 0.75/8 + 1/9$$

4.2. *Cornering fuzzy algorithms*

At the time that the fuzzy algorithm was constructed it was decided to split it into two parts: the first part describes the relation between both of the inputs and only one of the outputs; the second part describes the relation between both of the inputs and the second output only. Hence, it can be seen that the sub-algorithm described previously was not followed exactly as a one-input–one-output relation. However, this experimental application gives a good demonstration of the technique. From the knowledge elicitation of the cornering process, the control rules of the process have been divided into two parts; the first algorithm describes the control rules output of *MI* from the effect of the inputs *E* and *CE*; these are:

IF E=PO AND CE=PO THEN MI=PO
IF E=PO AND CE=PS THEN MI=PO
IF E=PO AND CE=PM THEN MI=PO^2
IF E=PO AND CE=PB THEN MI=PO^2
IF E=PS AND CE=PO THEN MI=PS^2
IF E=PS AND CE=PS THEN MI=PS^2
IF E=PS AND CE=PM THEN MI=$PS^{1/2}$
IF E=PS AND CE=PB THEN MI=PM
IF E=PM AND CE=PO THEN MI=$PM^{1/2}$
IF E=PM AND CE=PS THEN MI=PM
IF E=PM AND CE=PM THEN MI=PM^2
IF E=PM AND CE=PB THEN MI=$PB^{1/2}$
IF E=PB AND CE=PO THEN MI=PB^2
IF E=PB AND CE=PS THEN MI=PB^2
IF E=PB AND CE=PM THEN MI=PB
IF E=PB AND CE=PB THEN MI=PB

these rules are shown in tabular format in Table 7.

Table 7. Control rule output for metal insert (*MI*)

	CE			
E	*PO*	*PS*	*PM*	*PB*
PO	PO	PO	PO^2	PO^2
PS	PS^2	PS^2	$PS^{1/2}$	PM
PM	$PM^{1/2}$	PM	PM^2	$PB^{1/2}$
PB	PB^2	PB^2	PB	PB

Table 8. Control rule output for load-cell insert (*LI*)

	CE			
E	*PO*	*PS*	*PM*	*PB*
PO	PS	PS	PS^2	$PO^{1/2}$
PS	PM	PM	PM^2	$PS^{1/2}$
PM	PB^2	PB^2	PM	PM^2
PB	$PB^{1/2}$	PB	PB^2	PM

Similarly, the control rules of the second output (*LI*) as shown in Table 8 are:

IF E=PO AND CE=PO THEN LI=PS
IF E=PO AND CE=PS THEN LI=PS
IF E=PO AND CE=PM THEN LI=PS^2
IF E=PO AND CE=PB THEN LI=$PO^{1/2}$
IF E=PS AND CE=PO THEN LI=PM
IF E=PS AND CE=PS THEN LI=PM
IF E=PS AND CE=PM THEN LI=PM^2
IF E=PS AND CE=PB THEN LI=$PS^{1/2}$
IF E=PM AND CE=PO THEN LI=PB^2
IF E=PM AND CE=PS THEN LI=PB^2
IF E=PM AND CE=PM THEN LI=PM
IF E=PM AND CE=PB THEN LI=PM^2
IF E=PB AND CE=PO THEN LI=$PB^{1/2}$
IF E=PB AND CE=PS THEN LI=PB
IF E=PB AND CE=PM THEN LI=PB^2
IF E=PB AND CE=PB THEN LI=PM

4.3. *Rules combination and fuzzy relations*

From the first fuzzy sub-algorithm (Table 7) that relates the first input (the error) with the first output (metal insert), the sub-relation between them can be developed, which may be denoted as R_{11} which is the union of Cartesian products of all control rules of the first input (error) and the first output (*MI*):

$$R_{11} = \text{error}_{\substack{PO\\PS\\PM\\PB}} \times \text{metal insert}_{\substack{PO \text{ or } PO^2\\PS^2 \text{ or } PS^{1/2} \text{ or } PM\\PM^{1/2} \text{ or } PM \text{ or } PM^2 \text{ or } PB^{1/2}\\PB^2 \text{ or } PB}}$$

Table 9. Fuzzy sub-relation $R_{11} = E \times MI$

	MI									
E	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0	1	0.866	0.5	0	0	0	0	0	0	0
1	0.866	0.866	0.7	0.7	0.7	0.7	0.5	1.5	0.5	0
2	0.5	0.7	0.93	0.93	0.93	0.866	0.866	0.866	0.5	0
3	0	0.7	0.93	1	0.93	0.866	1	0.866	0.5	0
4	0	0.7	0.93	0.93	0.93	0.866	0.866	0.866	0.7	0.7
5	0	0.7	0.7	0.7	0.7	0.93	0.93	0.93	0.7	0.93
6	0	0	0	0	0.7	0.93	1	0.93	0.93	1
7	0	0	0	0	0.7	0.93	0.93	0.93	0.7	0.93
8	0	0	0	0	0.7	0.7	0.7	0.7	0.93	0.75
9	0	0	0	0	0	0	0	0.25	0.75	1

Table 10. Fuzzy sub-relation $R_{21} = CE \times MI$

	MI									
CE	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0	1	0.866	0.75	1	0.75	0.93	1	0.93	0.93	1
1	0.866	0.866	0.75	0.75	0.75	0.93	0.93	0.93	0.93	0.93
2	0.866	0.866	0.75	0.75	0.75	0.866	0.866	0.866	0.866	0.866
3	1	0.866	0.75	1	0.75	0.866	1	0.866	0.866	1
4	0.866	0.866	0.75	0.75	0.75	0.866	0.866	0.866	0.866	0.866
5	0.75	0.75	0.93	0.93	0.93	0.75	0.75	0.75	0.866	0.866
6	1	0.75	0.93	1	0.93	0.75	1	0.75	0.866	1
7	0.75	0.75	0.93	0.93	0.93	0.75	0.75	0.75	0.866	0.866
8	0.75	0.75	0.7	0.7	0.7	0.866	0.866	0.866	0.93	0.93
9	1	0.75	0.25	0	0.5	0.866	1	0.866	0.93	1

where the logical OR represents the maximum of membership values between the fuzzy variables. Relation R_{11} is tabulated in Table 9. Also, the union of the Cartesian product of all the control rules of the second input (change of error) and the first output (*MI*) is denoted by R_{21} and can be developed as shown in Table 10:

$$R_{21} = \text{error change}_{\begin{array}{l}\text{PO}\\ \text{PS}\\ \text{PM}\\ \text{PB}\end{array}} \times \text{metal insert}_{\begin{array}{l}\text{PO or PS}^2\text{ or PM}^{1/2}\text{ or PB}^2\\ \text{PO or PS}^2\text{ or PM or PB}^2\\ \text{PO}^2\text{ or PS}^{1/2}\text{ or PM}^2\text{ or PB}\\ \text{PO}^2\text{ or PM or PB}^{1/2}\text{ or PB}\end{array}}$$

Bearing in mind that the same universe is used for all of the fuzzy variables and that a fuzzy singleton is assumed to be a specific input, a final single relation may be used for both of the inputs with one of the outputs. This is described schematically in Fig. 12 where each of the inputs has an individual sub-relation with each of the outputs and the final fuzzy sub-relation R_1 as shown in Fig. 13 is the minimum of these relations (Table 11), i.e.

$$\mu_{R_1} = \min(\mu_{R_{11}}, \mu_{R_{21}}) \tag{3}$$

Note: this approximation is not part of the proposed technique and it is valid only for this particular case where the objects are of the same dimensionality.

Similarly, from Table 7 the relation R_{12} between the first input E (the error) and the second output LI (load-cell insert) may be developed (Table 12) as:

$$R_{12} = \text{error}_{\begin{array}{l}\text{PO}\\ \text{PS}\\ \text{PM}\\ \text{PB}\end{array}} \times \text{load-cell insert}_{\begin{array}{l}\text{PS or PS}^2\text{ or PO}^{1/2}\\ \text{PS}^{1/2}\text{ or PM}^2\text{ or PM}\\ \text{PB}^2\text{ or PM or PM}^2\\ \text{PB}^{1/2}\text{ or PB or PB}^2\text{ or PM}\end{array}}$$

and the fuzzy relation R_{22} between the second input (CE) and the second output (LI) can be developed (Table 13) as:

$$R_{22} = \text{error change}_{\begin{array}{l}\text{PO}\\ \text{PS}\\ \text{PM}\\ \text{PB}\end{array}} \times \text{load-cell insert}_{\begin{array}{l}\text{PS or PM or PB}^2\text{ or PB}^{1/2}\\ \text{PS or PM or PB}^2\text{ or PB}\\ \text{PS}^2\text{ or PM or PM}^2\text{ or PB}^{1/2}\\ \text{PO}^{1/2}\text{ or PS}^{1/2}\text{ or PM}^2\text{ or PM}\end{array}}$$

Then the final relation R_2 shown schematically in Fig. 14 between the combined inputs (the error E and its change

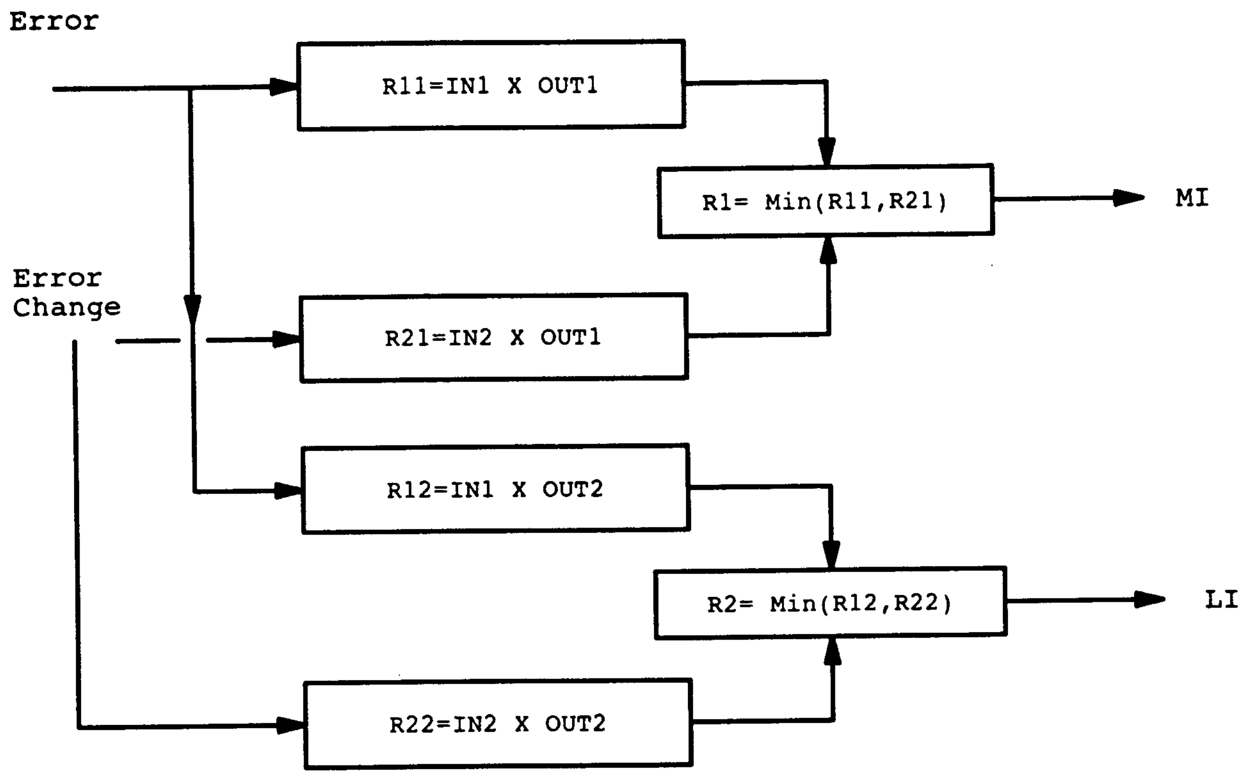

Fig. 12. Fuzzy relations of automated cornering system.

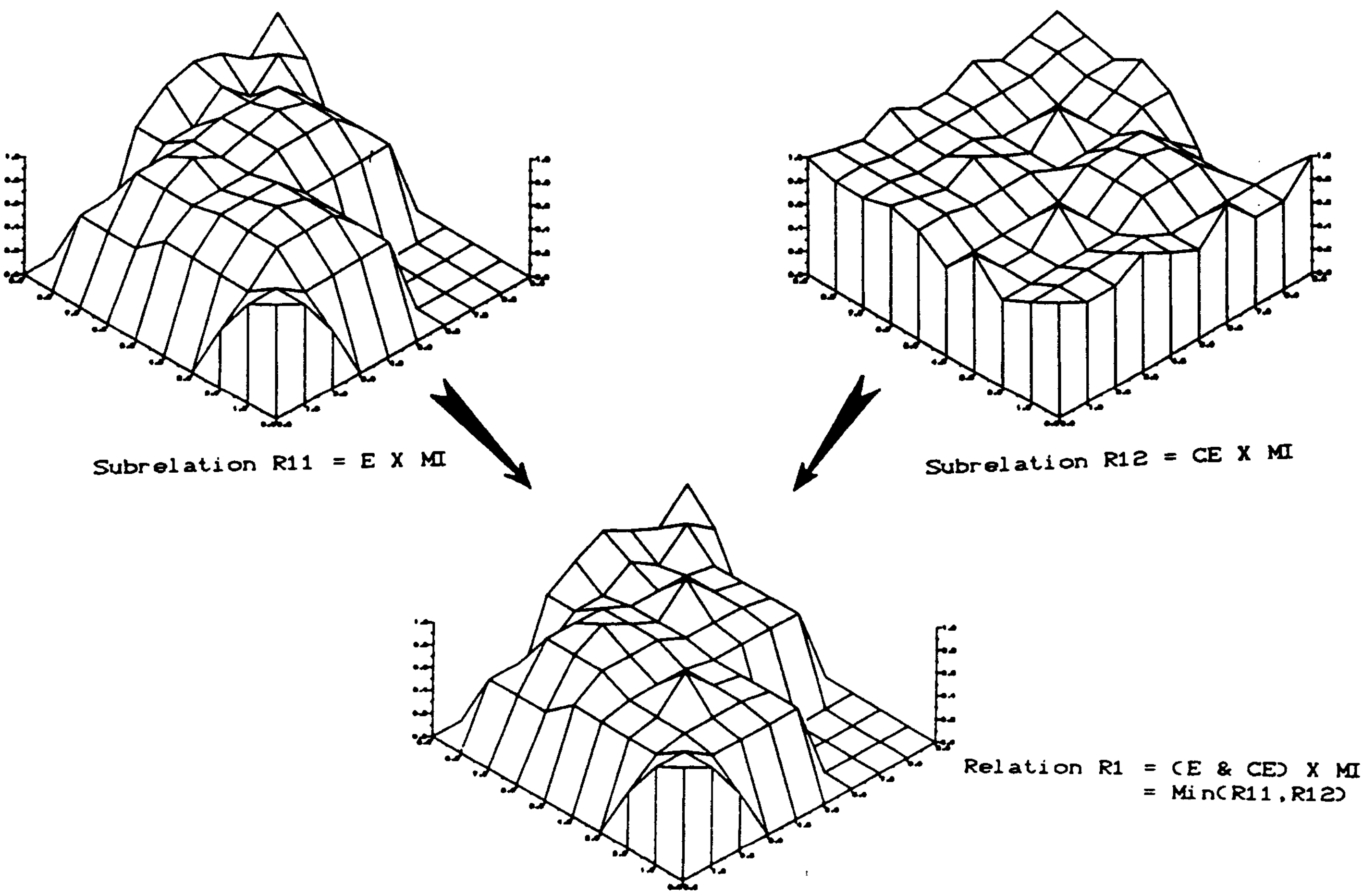

Fig. 13. Schematic diagram of the development of relation R_1.

Table 11. Fuzzy relation $R_1 = \min(R_{11}, R_{21})$

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Average*
0	1	0.866	0.5	0	0	0	0	0	0	0	0.788
1	0.866	0.866	0.7	0.7	0.7	0.7	0.5	0.5	0.5	0	3.5
2	0.5	0.7	0.75	0.75	0.75	0.866	0.866	0.866	0.5	0	4.12
3	0	0.7	0.75	1	0.75	0.866	1	0.866	0.5	0	4.55
4	0	0.7	0.75	0.75	0.75	0.866	0.866	0.866	0.7	0.7	5.028
5	0	0.7	0.7	0.7	0.7	0.75	0.75	0.75	0.7	0.866	5.123
6	0	0	0	0	0.7	0.75	1	0.75	0.866	1	6.65
7	0	0	0	0	0.7	0.75	0.75	0.75	0.7	0.866	6.575
8	0	0	0	0	0.7	0.7	0.7	0.7	0.93	0.75	6.6
9	0	0	0	0	0	0	0	0.25	0.75	1	8.375

Table 12. Fuzzy sub-relation $R_{12} = E \times LI$

	LI									
E	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0	1	0.93	0.866	1	0.866	0.5	0	0	0	0
1	0.93	0.93	0.866	0.866	0.866	0.5	0.5	0.5	0.5	0
2	0.7	0.7	0.93	0.93	0.93	0.866	0.866	0.866	0.5	0
3	0	0.7	0.93	1	0.93	0.866	1	0.866	0.5	0
4	0	0.7	0.93	0.93	0.93	0.866	0.866	0.866	0.5	0.25
5	0	0.7	0.7	0.7	0.7	0.866	0.866	0.866	0.75	0.75
6	0	0	0	0	0.5	0.866	1	0.866	0.75	1
7	0	0	0	0	0.5	0.866	0.866	0.866	0.75	0.75
8	0	0	0	0	0.5	0.866	0.866	0.866	0.93	0.93
9	0	0	0	0	0.5	0.866	1	0.866	0.93	1

Table 13. Fuzzy sub-relation $R_{22} = CE \times LI$

	LI									
CE	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
0	0	0.5	0.866	1	0.866	0.866	1	0.866	0.93	1
1	0	0.5	0.866	0.866	0.866	0.866	0.866	0.866	0.93	0.93
2	0	0.5	0.866	0.866	0.866	0.866	0.866	0.866	0.866	0.866
3	0	0.5	0.866	1	0.866	0.866	1	0.866	0.866	1
4	0	0.5	0.866	0.866	0.866	0.866	0.866	0.866	0.866	0.866
5	0	0.25	0.75	0.75	0.75	0.866	0.866	0.866	0.75	0.75
6	0	0.25	0.75	1	0.75	0.866	1	0.866	0.75	1
7	0.7	0.7	0.75	0.75	0.75	0.866	0.866	0.866	0.5	0
8	0.93	0.93	0.93	0.93	0.93	0.866	0.866	0.866	0.5	0
9	1	0.93	0.93	1	0.93	0.866	1	0.866	0.5	0

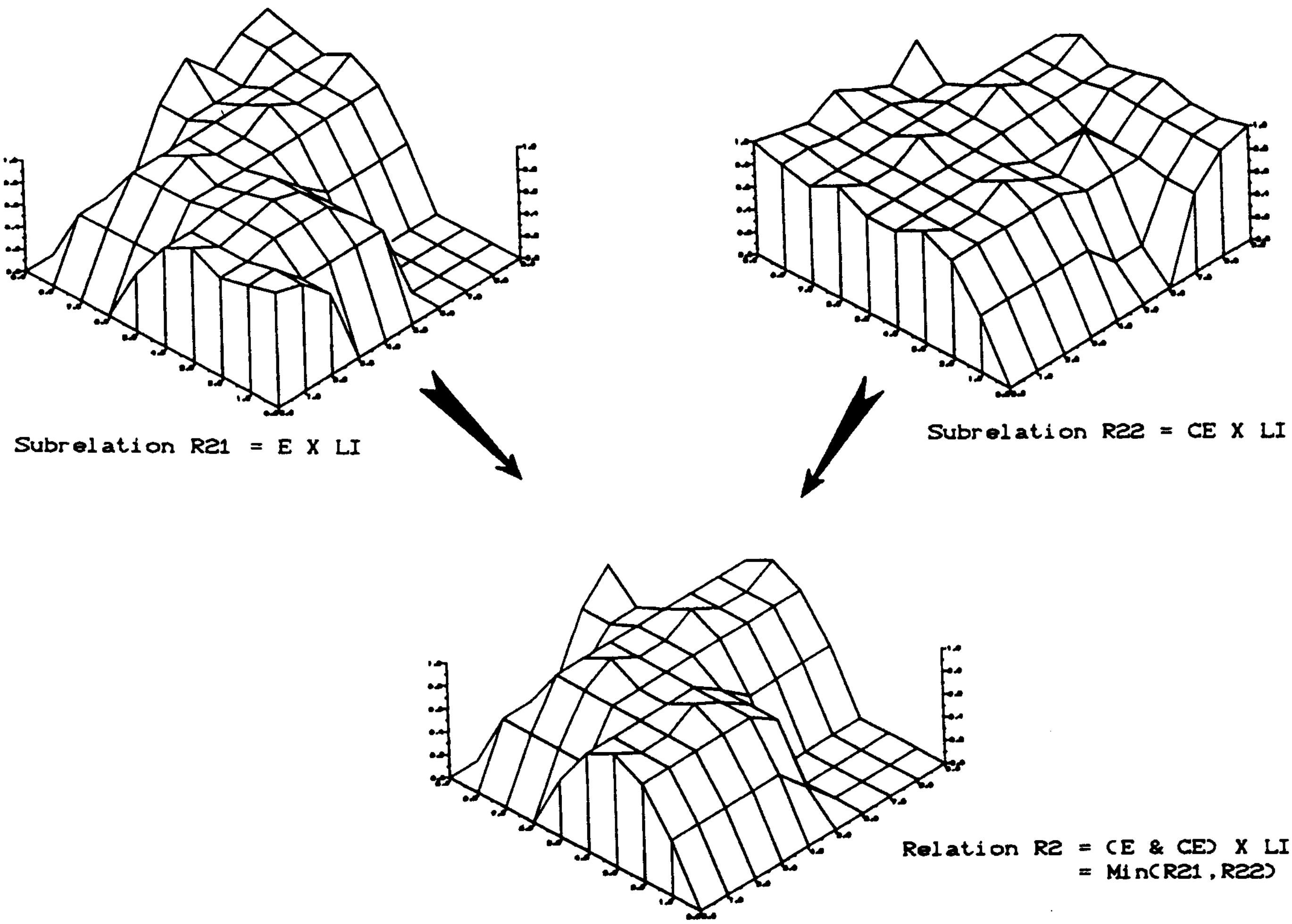

Fig. 14. Schematic diagram of the development of relation R_2.

CE) and the second output *LI* (load-cell insert) can be represented by the minimum of R_{12} and R_{22} (Table 14) as:

$$\mu_{R_2} = \min(\mu_{R_{12}}, \mu_{R_{22}}) \tag{4}$$

Note: this approximation is not part of the proposed technique and it is valid only for this particular case where the objects have the same dimensionality.

Replacing Equations 3 and 4 into Equation 1 would result in the final outputs *MI* and *LI* of the system being given by:

$$MI = \{\overline{E \circ R_1}\} \times \varepsilon_1 + \{\overline{CE \circ R_1}\} \times \varepsilon_2 \tag{5}$$

$$LI = \{\overline{E \circ R_2}\} \times \varepsilon_1 + \{\overline{CE \circ R_2}\} \times \varepsilon_2 \tag{6}$$

Later it was noticed that the error (E) has the full importance level of unity, i.e. $\varepsilon_1 = 1$ and thus the change of error (CE) has the null importance level of zero, i.e. $\varepsilon_2 = 0$. It should be noted, however, that the influence of the change of error is already included in the final relation due to the fact that the minimum operator was used to combine the relations. Hence in this special case, the final outputs *MI* (metal insert) and *LI* (load-cell insert) of the cornering process reduce to:

$$MI = \overline{E \circ R_1} \tag{7}$$

$$LI = \overline{E \circ R_2} \tag{8}$$

4.4. *Results and observation*

Satisfactory results were achieved where an average of 8 cycles were required to corner the load-cell as shown in Fig. 15a. However, two extreme cases have been noticed, with the load-cell being completed in 3 cycles in some cases (Fig. 15b), and requiring up to 14 cycles to complete (Fig. 15c) in the other extreme. In one specific case the load-cell cornering was not completed even after

Table 14. Fuzzy relation $R_2 = \min(R_{12}, R_{22})$

	0	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*Average*
0	0	0.5	0.866	1	0.866	0.5	0	0	0	0	3.00
1	0	0.5	0.866	0.866	0.866	0.5	0.5	0.5	0.5	0	4.175
2	0	0.5	0.866	0.866	0.866	0.866	0.866	0.866	0.5	0	4.5
3	0	0.5	0.866	1	0.866	0.866	1	0.866	0.5	0	4.5
4	0	0.5	0.866	0.866	0.866	0.866	0.866	0.866	0.5	0.25	4.674
5	0	0.25	0.7	0.7	0.7	0.866	0.866	0.866	0.75	0.75	5.44
6	0	0	0	0	0.5	0.866	1	0.866	0.75	1	7.056
7	0	0	0	0	0.5	0.866	0.866	0.866	0.5	0	6.598
8	0	0	0	0	0.5	0.866	0.866	0.866	0.5	0	6.195
9	0	0	0	0	0.5	0.866	1	0.866	0.5	0	6

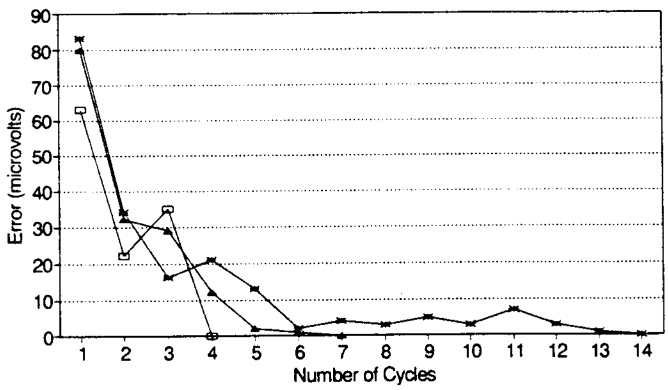

Fig. 15. Cornering process results. (Sample of cornering three transducers.) (a) ▲ Typical cycle; (b) □ short cycle; (c) ★ long cycle.

17 cycles at which point the process was abandoned. This is not due to any inadequacy in the fuzzy data developed but most probably due to other factors influencing the load-cell performance such as the poor adhesive bonding of the strain gauges, out of tolerance frame manufacture, misalignment of strain gauges, etc. This also explains the difference in sensitivity from one load-cell to another and the different behaviour of the cornered load-cells when removing the same amount of metal for the same error.

Initially the robot was controlled in Cartesian mode in order to achieve the *MI* and *LI* outputs, however the minimum resolution of movement in Cartesian mode was too coarse in certain stages for metal removal. In order to achieve higher resolution, it was decided to simulate the Cartesian movement using joint mode, allowing robot control at the pulse level of the servo-motors.

5. Conclusion

A simplified multivariable fuzzy system has been proposed in Section 2 that not only simplifies the technique, but also avoids the possible memory overload for some industrial computers. In order to clarify the technique, it was demonstrated in the form of a welding example in Section 3, where it has also been compared with Zadeh's technique. The analysis of comparison was presented as a percentage of deviation from Zadeh's technique where a deviation of the inferred output by 22.68% maximum and −15.21% minimum was found. The suitability of using the proposed technique for a particular application has to be evaluated by the system builder which is dependent on the linearity/non-linearity of the system. It was noticed that the proposed technique is more suitable for linear systems. In any case, the proposed strategy provides a simple fuzzy technique that is capable of avoiding memory overload for multivariable systems. Section 4 described an experiment using the technique on a transducer manufacturing process termed the 'cornering process' which has been built on a normal PC without the need for sophisticated hardware (high RAM capacity) or a sophisticated inference structure. The proposed system also closely simulates a human's behaviour who subjectively weighs the importance of each individual input and hence concludes the output(s). However, it is very important to note that just like human behaviour, the importance level factor is set initially only as part of knowledge elicitation of the process and modified/tuned later by experimentation. Eventually, it might be possible for some system to automate this tuning process of the importance level factor. The applicability of the fuzzy logic to the automated cornering system shows that it was adequate as a simulation of human skills and the variance in the number of cycles required to finish each component (load-cell) was consistent with that expected of manual human actions. However, although fuzzy logic was used as a method to deal with uncertainty and imprecision in the automated cornering system, it was noticed that the higher system resolution would lead to a higher fuzzy system efficiency. Conversely, the lower system resolution would lead to an inefficient fuzzy system and hence, a minimum level of resolution of the available hardware would be required before the implementation of fuzzy technique in any manufacturing environment.

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