Total Energy-Shaping IDA–PBC Control of the 2D-SpiderCrane

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Abstract— The objective of this paper is to further extend the application of the Interconnection and Damping Assignment Passivity-Based Control (IDA-PBC) technique to the 2D-SpiderCrane mechanism with both kinetic and potential energy being shaped. An earlier effort had focused exclusively on potential energy alone. By shaping the total energy and by adding appropriate damping we achieve almost global asymptotic stability of the desired equilibrium.

I. INTRODUCTION

Many different types of cranes are used in various industries for carrying load using cables and pulleys. These cranes are structurally similar to parallel actuated robots but with a fundamental difference – the cable can only pull the end effector but not push it. Also, because of its inherent underactuation properties, the payload is free to swing in a pendulum-like motion resulting in several performance and safety concerns. Such features make the modeling and feedback control laws for such systems more challenging than their rigid robot counterparts. For these reasons, cable-suspended crane mechanisms have received much attention in the recent past.

Motivated by the desire to achieve precise payload positioning with minimum swing several researchers have examined the control problem for an overhead crane system [6], [13], [9], [10]. A simple proportional-derivative (PD) controller is proposed by Fang et al. [5] to asymptotically regulate the overhead crane system. It was shown that a PD feedback loop at the gantry creates an artificial spring/damper system which absorbs the payload energy and thus minimizes the swing. In [16], the load motions of a container crane is divided into five sections and an optimal control strategy is proposed for each section. In [11], Kiss et al. developed a PD controller for a vertical crane-winch system that only requires the measurement of winch angle and its derivative rather than a cable angle measurement. In [12], the overhead crane that exhibits double-pendulum dynamics is investigated by Weiping et al. Dynamics and control related issues are discussed in [18] with reference to planar translational cable driven robots. As opposed to the previous approaches on single cable mechanisms, an attempt on multiple cable mechanisms is the SpiderCrane at EPFL [3], [4], where a jet scheduling controller is proposed with a dynamical system, called the jet-scheduler, providing the derivatives (the jets) of an ideal stabilizing flat-output trajectory instead of explicitly specifying the desired flat-output trajectory.

On the other hand, the Interconnection and damping assignment passivity-based control (IDA-PBC) is a controller design methodology introduced in [14] to achieve stabilization of mechanical systems shaping the total energy of the system; see [15] for a recent survey. The stabilization of a gantry crane system modeled with pulley dynamics leading to a holonomic constraint using IDA–PBC is described in [2] and [7]. IDA-PBC based stabilization and point-to-point control of a 2D SpiderCrane changing its interconnection structure is reported in [8]. In all these papers only the potential energy was shaped. The main contribution of this paper is to prove that IDA–PBC can be applied to shape also the kinetic energy achieving a better performance than the designs where only potential energy is changed.

The paper is organized as follows: Section II presents the dynamic model of the 2D SpiderCrane and formulates the problem. The payload and pulley dynamics are separated to arrive at a Port-Hamiltonian Model for the former. We then apply a partial linearization technique using Spong’s Normal Form [17] in order to simplify the dynamic equations. In Section III we briefly recall the IDA-PBC methodology and apply it to the 2D SpiderCrane mechanism. The control strategy obtained by the IDA-PBC methodology has been assessed using simulations and the results are presented in Section IV. Finally, we wrap up the paper with some concluding remarks and future work in Section V.

Notation: Throughout the paper it is assumed that all functions and mappings are \( C^\infty \) and define the row vector \( \nabla_x(\cdot) \triangleq \left( \frac{\partial (\cdot)}{\partial x} \right)^T \). Moreover, \( I_n \) denotes the \( n \times n \) identity matrix while \( e_i \) denotes the standard \( i \)-th basis vector.

II. THE 2D-SPIDERCRANE

The problem with the classical cranes is that the large inertia of the boom or the gantry limits rapid acceleration and deceleration since these may give rise to large inertial forces. For applications demanding fast weight handling, a new crane design has been proposed by the Laboratory of Automatic Control at École Polytechnique Fédérale de Lausanne (EPFL), see [3]. Its main feature is the absence of heavy mobile components. The heavy elements of the mechanical structure are fixed and the positioning is done by cables that carry the load. As a result, this crane can work at considerably higher speeds which makes it an ideal choice as a fast weight handling equipment. The 2D SpiderCrane studied here was inspired by the 3D version at EPFL and essentially captures all control-theoretic challenges of the larger system.
A. Port-Hamiltonian Model

Consider the planar 2D SpiderCrane mechanism as shown in Figure 1. The positioning of the load is done by adjusting the lengths $l_1$ and $l_2$. The model represents the underactuation of degree one and is subject to two holonomic constraints. Here, the position of the load is given by $(x_p, y_p)$ with the load mass being $m$. The positions of the two motors are $(x_a, y_a)$ and $(x_b, y_b)$ with the corresponding rotary inertias taken as $I_a$ and $I_b$. The ring has mass $M$ and the position $(x_r, y_r)$. The load is attached to the ring using a cable with fixed length of $L_3$. In this mechanism the ring functions as the cart to carry the load as in the conventional overhead gantry crane system. The difference here is, the ring is allowed to move in a vertical plane whereas in the conventional crane system the gantry is allowed to move only along one axis. For the purpose of this study, we make the following assumptions:

i) The cable is massless and inelastic

ii) Dissipative forces on the system are negligible

iii) Both the pylons are assumed to be at the same height.

The dynamic equations of the 2D-SpiderCrane [8] are described by a Hamiltonian model of the form

$$\begin{bmatrix} \dot{q} \\ \dot{\dot{p}} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla_q \hat{H}^T \\ \nabla_p \hat{H}^T \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{G}(q) \end{bmatrix} v, \quad (1)$$

with the configuration variables

$$q = \begin{bmatrix} x_r & y_r & \theta & l_1 & l_2 \end{bmatrix}^T,$$

with $x_r, y_r$ the horizontal and vertical ring position, $\theta$ the swing angle, $l_1, l_2$ the cable lengths. The form of the total energy of the system is given as

$$\hat{H}(q, p) = \frac{1}{2} p^T \hat{M}^{-1} q + V(q), \quad (2)$$

where $\hat{M} = \hat{M}^T > 0$ the mass matrix, given by $\hat{M}(q_3) =$

$$\begin{bmatrix} m_r + m & 0 & mL_3 \cos(q_3) & 0 & 0 \\ 0 & m_r + m & mL_3 \sin(q_3) & 0 & 0 \\ mL_3 \cos(q_3) & mL_3 \sin(q_3) & mL_3^2 & 0 & 0 \\ 0 & 0 & 0 & l_1 & 0 \\ 0 & 0 & 0 & 0 & l_2 \end{bmatrix},$$

with $m_r$ the ring mass, $m$ the mass of the load and $L_3$ the (fixed) length of the cable attached to the load. The potential energy is given by

$$\hat{V}(q_2, q_3) = (m_r + m)gq_2 - mgL_3 \cos(q_3),$$

and the input matrix is $\hat{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$. We refer to [8] for a detailed description of the modeling issues. We can see that the gantry part is decoupled from the pulley mechanism, as shown in Figure 2 and Figure 3. Hence, we can concentrate on the gantry part and our objective is to position the payload, which is suspended by a cable from the ring mass $m_r$ on which two actuated forces $u = \text{col}(u_1, u_2)$ act. The (reduced) inertia matrix is then

$$\tilde{M}(q_3) = \begin{bmatrix} m_r + m & 0 & mL_3 \cos(q_3) \\ 0 & m_r + m & mL_3 \sin(q_3) \\ mL_3 \cos(q_3) & mL_3 \sin(q_3) & mL_3^2 \end{bmatrix},$$

with the (reduced) input force matrix

$$\hat{G} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$ 

B. Partial feedback linearized model

In order to simplify the dynamic equations we apply a partial linearization that transforms the system into Spong’s Normal Form, see [17] for details. To this end we need to look at the equivalent Euler-Lagrange representation of
mechanical systems. The Euler-Lagrange equations have the familiar form
\[
\frac{d}{dt} \nabla_q^\top \dot{L}(q, \dot{q}) - \nabla_q^\top \ddot{L}(q, \ddot{q}) = \tilde{G}(q)v,
\]
where \( \ddot{L}(q, \ddot{q}) \) is called the Lagrangian and for simple mechanical systems is given by \(^1\)
\[
\ddot{L}(q, \ddot{q}) = \frac{1}{2} \dot{M}(q)\ddot{q} - \tilde{V}(q).
\]
In this case the equations take the well-known form
\[
\begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
-1 & 0 & 0
\end{bmatrix},
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{bmatrix}.
\]

For the 2D-SpiderCrane we can state the following.

**Proposition 1:** Applying partial feedback linearization to the 2D-SpiderCrane mechanism with the control
\[
v = \frac{1}{m_r + m} \left[ mL_3 \cos(q_3)\dot{q}_2 - mL_3 \sin(q_3)\dot{q}_2^2 + mL_3 \cos(q_3)\dot{q}_2^2 \right] + u
\]
yields the Hamiltonian dynamics
\[
(\Sigma_{snf}) : \begin{cases}
\dot{p} = \dot{q} \\
\dot{q} = \frac{\partial}{\partial q} \sin(q_3)e_3 + Gu,
\end{cases}
\]
with inertia matrix \( M = I_3 \), potential energy \( V = \frac{m}{2} \cos(q_3) \), \( \epsilon \triangleq -L_3 \) and new input force matrix \( G \), along with its left full rank annihilator \( G^\perp \), given by
\[
G = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
\cos(q_3) & \sin(q_3)
\end{bmatrix},
G^\perp = \left[ \cos(q_3), \sin(q_3), -\epsilon \right].
\]

Based on the model (5), our objective is to derive an IDA–PBC law that achieves stabilization of the desired equilibrium \( (q^*_1, q^*_2, 0, 0, 0, 0) \) by appropriately shaping both kinetic and potential energy functions.

III. IDA–PBC of the 2D-SpiderCrane

The main result of the IDA–PBC technique for underactuated mechanical systems is contained in the following proposition, see for example [15],[14].

**Proposition 2:** Assume that there is a matrix \( M_d = M_d^\top \in \mathbb{R}^{n\times n} \) and a function \( V_d(q) \) that satisfy the PDEs

**Kinetic Energy:**
\[
G^\perp \left\{ \nabla_q (p^\top M^{-1} p) - M_d M^{-1} \nabla_q (p^\top M_d^{-1} p) + 2J_2 M_d^{-1} p \right\} = 0,
\]

**Potential Energy:**
\[
G^\perp \left\{ \nabla_q V - M_d M^{-1} \nabla_q V_d \right\} = 0,
\]
for some \( J_2(q, p) = -J_2^\top(q, p) \in \mathbb{R}^{n\times n} \) and a full rank left annihilator \( G^\perp \in \mathbb{R}^{(n-m)\times n} \). Then, the system
\[
\begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
0 & I_n \\
-I_n & 0
\end{bmatrix}
\begin{bmatrix}
\nabla_q H^\top \\
\nabla_p H^\top
\end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u,
\]
where \( H(q, p) = \frac{1}{2}p^\top M^{-1}(q)p + V(q) \), in closed loop with the IDA–PBC control
\[
u = (G^\top G)^{-1} G^\top \left( \nabla_q H - M_d M^{-1} \nabla_q H_d + 2J_2 M_d^{-1} p \right) - K_v G^\top \nabla_p H_d,
\]
with \( 0 < K_v = K_v^\top \in \mathbb{R}^{n\times n} \), takes the Hamiltonian form
\[
(\Sigma_d) : \begin{bmatrix}
\dot{q} \\
\dot{p}
\end{bmatrix} =
\begin{bmatrix}
0 & M_d^{-1} M_d \\
-M_d M^{-1} J_2 - G K_v G^\top
\end{bmatrix}
\begin{bmatrix}
\nabla_q H_d^\top \\
\nabla_p H_d^\top
\end{bmatrix}
\]
where the new total energy is \( H_d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R} \)
\[
H_d(q, p) = \frac{1}{2} p^\top M_d^{-1}(q)p + V_d(q).
\]
Further, if \( M_d \) is positive definite in a neighborhood of \( q^* \) and
\[
q^* = \arg \min V_d(q)
\]
than \( (q^*, 0) \) is a stable equilibrium of (10) with Lyapunov function \( H_d \). This equilibrium is asymptotically stable if it is locally detectable from the output \( G^\top(q)M_d^{-1}(q)p \).

Clearly, the success of IDA–PBC relies on the possibility of solving the PDEs (7) and (6). For the case of underactuation degree one systems, \( i.e., m = n - 1 \), it was shown in [1] that the PDEs can be explicitly solved, provided the inertia matrix and the force induced by the potential energy (on the unactuated coordinate) are independent of the unactuated coordinate.

A. Kinetic energy shaping

Under some assumptions it was shown in [1] that the kinetic energy PDE (KE-PDE) can be written as
\[
\sum_{j=1}^n \gamma_j \frac{dM_d}{dq_j} = -\left( \mathcal{J}(q)A^\top(q) + A(q)\mathcal{J}^\top(q) \right),
\]
where we defined
\[
\mathcal{J} \triangleq \left[ \alpha_1 \alpha_2 \ldots \alpha_n \right] \in \mathbb{R}^{n\times n},
\]
with the vector functions \( \alpha_k \in \mathbb{R}^n \), \( n_o \triangleq \frac{1}{2}(n-1) \), being free parameters and the matrix
\[
A \triangleq \left[ W_1(G^\perp)^\top \ldots \ldots W_n(G^\perp)^\top \right] \in \mathbb{R}^{n\times n},
\]
with the skew-symmetric matrices \( W \) defined, in the case \( n = 3 \), as
\[
W_1 \triangleq \begin{bmatrix} 0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 0 \end{bmatrix},
W_2 \triangleq \begin{bmatrix} 0 & 0 & 1 \\
0 & 0 & 0 \\
-1 & 0 & 0 \end{bmatrix},
W_3 \triangleq \begin{bmatrix} 0 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0 \end{bmatrix}.
\]

\(^1\)Although \( \tilde{V} \) doesn’t change we wrote \( \tilde{V} = \dot{V} \) to keep a homogeneous notation.
Additionally, for simplicity of presentation we defined
\[ \gamma = \text{col}(\gamma_1, \ldots, \gamma_n) = M^{-1}M_d(G^\perp)^\top. \] (16)

However, instead of the previous form of the kinetic energy we will consider the PDE
\[ \sum_{j=1}^n \gamma_j \frac{dM_d}{dq_j} = -2A(q_j)J^\top(q). \] (17)
As noted in [1], the set of matrices \( M_d \) that satisfies (17) is strictly contained in the set that satisfies (13), which in fact characterizes all solutions of (6). At this point it is convenient to state the main result on kinetic energy shaping of [1].

**Proposition 3**: Let Assumptions A.1-A.3 of [1] be satisfied. Under these conditions, for all desired positive definite matrices of the form
\[ M_d(q_3) = \int_{q_3^*}^{q_3} G(\mu)\Psi(\mu)G^\top(\mu)dm + M_d^0, \] (18)
where the matrix function \( \Psi = \Psi^\top \in \mathbb{R}^{(n-1)\times(n-1)} \) and the constant matrix \( M_d^0 \in \mathbb{R}^{(n-1)\times(n-1)} \), may be arbitrarily chosen, there exists a \( J_2 \) such that the kinetic PDE (6) holds in a neighborhood of \( q_3^* \).

However, in order to find a solution to (17) we propose an alternative parametrization of the set of assignable energy functions similar to the one proposed in [1] for the VTOL example. First we see that for the 2DSpiderCrane model given in (5), Assumptions A.1-A.3 of [1] are clearly satisfied with \( i = 3 \). Moreover, Assumption A.4 is satisfied with \( s = g \sin(q_3) \), which is defined as
\[ s = G^\perp \nabla_q V, \] (19)
and Assumption A.7 is also satisfied since
\[ G^\perp M^{-1}e_3 = -\epsilon \] (20)
Now, since the matrix \( J \) is free and imposing that \( M_d \) be a function of \( q_3 \) only, we can express (17) as
\[ \frac{dM_d}{dq_3}e_j \in ImA, j = 1, 2, 3 \]
with
\[ A = \begin{bmatrix} -\sin(q_3) & \epsilon & 0 \\ \cos(q_3) & 0 & \epsilon \\ 0 & \cos(q_3) & \sin(q_3) \end{bmatrix}. \] (21)

By fixing \( \gamma_3 = \gamma_3^0 \), a positive constant, and since \( \gamma_3 = G^\perp M_d e_3 \) we restrict the third column of \( M_d \) to have a constant projection along \( G^\perp \). Then, our objective is to define the other two columns in such a way that \( M_d \) be positive definite, with derivative living in the range space of \( A \). The following lemma characterizes all admissible vectors \( x = M_d e_3 \), see [1] for the proof.

**Lemma 1**: Fix a constant \( \gamma_3^0 \neq 0 \) and define the set
\[ \{x = \text{col}(x_1, x_2, x_3) : \mathbb{R} \to \mathbb{R}^3 \mid G^\perp x = \gamma_3^0 \text{ and } \frac{dx}{dq_3} \in ImA\}. \]
All elements of the set are generated as
\[ x_1 = (\gamma_3^0 + \epsilon x_3) \cos(q_3), \quad x_2 = (\gamma_3^0 + \epsilon x_3) \sin(q_3), \quad x_3 \text{ an arbitrary differentiable function}. \]

For simplicity we choose \( x_3 \) to be a constant, \( x_3 = k_2 \). This choice yields
\[ M_d e_3 = \begin{bmatrix} k_1 \cos(q_3) \\ k_1 \sin(q_3) \\ k_2 \end{bmatrix}, \] (22)
with \( k_1 \neq \gamma_3^0 + \epsilon k_2 \). In order to satisfy Assumption A.6 we require \( \gamma_3^0 \) to be positive and hence, \( k_1 - \epsilon k_2 > 0 \). In order to decide for the two remaining columns of \( M_d \), we search for functions \( m_{k1}, k, l \) denoting the \( kl \)-th element of \( M_d \) such that
\[ \begin{bmatrix} \frac{d\gamma_{11}}{dq_3} \\ \frac{d\gamma_{12}}{dq_3} \\ \frac{d\gamma_{13}}{dq_3} \\ \frac{d\gamma_{22}}{dq_3} \\ \frac{d\gamma_{23}}{dq_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \in \text{Im} A. \] (23)

Doing some straightforward computations yields the following ODEs
\[ \begin{align*}
\frac{dm_{11}}{dq_3} &= -b(q_3) \sin(q_3) \\
\frac{dm_{12}}{dq_3} &= b(q_3) \cos(q_3) - k_1 \epsilon \\
\frac{dm_{13}}{dq_3} &= c(q_3) \cos(q_3) \\
\frac{dm_{22}}{dq_3} &= -c(q_3) \sin(q_3) + k_1 \epsilon,
\end{align*} \] (24)
for some functions \( b(q_3) \) and \( c(q_3) \). Equating the right hand sides for the second and fourth equations we have that a simple choice is to take \( b = 2k_1 \epsilon \cos(q_3), \quad c = 2k_1 \epsilon \sin(q_3) \). Substituting the above choices into (24) and integrating yields the desired inertia matrix
\[ M_d(q_3) = \begin{bmatrix} k_1 \cos^2(q_3) + k_3 & k_1 \epsilon \sin(q_3) \cos(q_3) & k_1 \cos(q_3) \\
-\epsilon k_1 \cos(q_3) & k_1 \cos(q_3) & k_1 \sin(q_3) \\
-\epsilon k_1 \cos(q_3) & k_1 \sin(q_3) & k_2 \end{bmatrix} \] (25)
with \( k_3 > 0 \) an integration constant added to ensure positive definiteness of \( M_d \).

**B. Potential energy shaping**

The last step of the IDA-PBC approach consists in finding an appropriate desired potential energy function. To this end we make use of the following proposition [1].

**Proposition 4**: Let Assumptions A.1-A.5 of [1] be satisfied and \( M_d \) be given by (18). Under these conditions, all solutions of the potential PDE (7) are given by
\[ V_d(q) = \int_0^{q_3} s(\mu) \gamma_1(\mu) \mu + \Phi(z(q)), \] (26)
with \( \gamma, s \) given in (16), (19), respectively, and \( z \in \mathbb{R} \) defined as
\[ z(q) = q - \int_0^{q_3} \gamma(\mu) \gamma_1(\mu) \mu, \] (27)
with \( \Phi \) an arbitrary differentiable function.

For simplicity we select the function \( \Phi \) to have quadratic form \( i.e., \Phi(z(q)) = \frac{1}{2}(z(q) - z(q^*))^\top P(z(q) - z(q^*)), \)
with a symmetric positive definite matrix $P$, although other choices are possible, see [8]. For the 2D-SpiderCrane we have
\[
\gamma = \begin{bmatrix}
    k_3 \sin(q_3) & (k_3 - \epsilon k_1) \sin(q_3) & k_1 - \epsilon k_2
\end{bmatrix}^T,
\]
\[
z(q) = \begin{bmatrix}
    q_1 - \frac{k_3}{k_3 - \epsilon k_1} \left(1 - \cos(q_3)\right) \\
    q_2 - \frac{k_3}{k_3 - \epsilon k_1} \left(1 - \cos(q_3)\right) \\
    0
\end{bmatrix}.
\] (28)

Thus, we are now in place to state the following proposition.

**Proposition 5:** A set of energy functions of the form (11) assignable via IDA-PBC to the 2D-SpiderCrane (5) is characterized by the globally positive definite and bounded inertia matrix (25) with $k_1$ an arbitrary number and positive $k_2$, $k_3$ verifying $k_3 > 2|k_1\epsilon|$, $k_2 > \frac{k_3}{2}$, and desired potential energy
\[
V_d(q) = -\frac{g}{k_1 - \epsilon k_2} \cos(q_3) + \Phi(z(q)),
\] (29)

with $z(q)$ given in (28), that satisfies (7) for all $P = P^T > 0$.

Moreover, the resulting IDA-PBC control law (9) ensures almost global asymptotic stability of the desired equilibrium $(q_1^*, q_2^*, 0, 0, 0)$.

**Proof:** The condition $k_1 > \epsilon k_2$ is imposed by the potential energy shaping. We now only need to verify the positive definiteness of $M_d$ since Assumptions A.1-A.6 have been shown to be satisfied. First, we observe that we should have $k_2 > 0$. Next we decompose $M_d$ as $M_d = M_d^1 + M_d^2$ with
\[
M_d^1 = \begin{bmatrix}
    d & 0 & k_1 \cos(q_3) \\
    0 & d & k_1 \sin(q_3) \\
    k_1 \cos(q_3) & k_1 \sin(q_3) & k_2
\end{bmatrix},
\]
\[
M_d^2 = \begin{bmatrix}
    k_1 \epsilon \cos^2(q_3) + a & k_1 \epsilon \sin(q_3) \cos(q_3) & 0 \\
    k_1 \epsilon \sin(q_3) \cos(q_3) & -k_1 \epsilon \cos^2(q_3) + a & 0 \\
    0 & 0 & 0
\end{bmatrix},
\]
and wrote $k_3 = a + d$. By Sylvester’s criterion we know that for $M_d^1$ to be positive definite it is necessary and sufficient that all of the leading principal minors need to be positive. This gives $d > 0$ and $d > |k_1\epsilon|$. Finally, we have that $M_d^2$ is positive semi-definite if and only if $a > |k_1\epsilon|$. Almost global asymptotic stability of the desired equilibrium can be concluded by using similar arguments as for the VTOL example in [1].

![Fig. 4. Transient performance for $\epsilon = -1$ and initial conditions $z(0) = [0.7, 0.7, \pi/18, 0, 0, 0]$.](image1)

![Fig. 5. Transient performance for $\epsilon = -1$ and initial conditions $z(0) = [0.7, 0.7, \pi/6, 0, 0, 0]$.](image2)

![Fig. 6. Transient performance for $\epsilon = -1$ and initial conditions $z(0) = [1.7, 1.7, \pi/2, 0, 0, 0]$.](image3)

**IV. SIMULATIONS**

We carried out extensive simulations to show the transient performance of the 2D-SpiderCrane dynamics in closed loop with the proposed IDA-PBC control law. The desired equilibrium is chosen to be $(q_1^*, q_2^*, q_3^*) = (0.5, 1, 0)$ while the values of the simulation parameters are summarized in Table I where we defined the damping injection matrix $K_v$ as
\[
K_v = \begin{bmatrix}
    K_v^1 & K_v^2 & K_v^3
\end{bmatrix}.
\]

Figure 4 depicts the simulation results for cable length $L_3 = 1m$, initial swing angle of $\pi/18$ rad and initial position of the ring $(q_{10}, q_{20}) = (0.7m, 0.7m)$. The performance is

![Figure 4](image4)

![Figure 5](image5)

![Figure 6](image6)
very satisfactory with smooth responses, exhibiting small overshoot and small settling time. Similar results are shown also in Figure 5 for initial swing angle $\pi/6$ rad. Finally, we demonstrate the controller performance in the case of initial swing angle of $\pi/2$ rad and initial position of the ring $(q_{10}, q_{20}) = (1.7m, 1.7m)$, while keeping the same values of the adjustable gains and parameters. The once more fast and smooth transient behavior of the closed loop system is shown in Figure 6.

V. CONCLUSIONS - FUTURE WORK

In this paper we have presented the IDA-PBC based control strategy for stabilization and point-to-point control of the 2D SpiderCrane gantry mechanism. The 2D SpiderCrane is a novel mechanism involving multiple-cables to transport loads. After partial feedback linearization using Spong’s Normal Form, it was observed that the 2D SpiderCrane dynamics are similar to the VTOL dynamics. Besides ensuring almost global asymptotic stability of the desired equilibrium, the IDA-PBC based control law improves transient performance. The total energy shaping was performed by the appropriate selection of the inertia matrix and the potential energy function.

For simplicity we have chosen a quadratic function for the potential energy shaping. Although it results in a smooth control law, some other function may also be selected. Providing such a degree of freedom is the essence of the IDA-PBC. The future work is aimed at testing the proposed controller experimentally and comparing it with the existing control strategies.

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