

# Speeded Multi-Attribute Decision Making under Certainty

Ioannis Karvelas<sup>1</sup>[0000-0003-2502-2903]

<sup>1</sup> University College Dublin, Belfield, Dublin 4, Dublin, Ireland  
ioannis.karvelas@ucdconnect.ie

**Abstract.** Research regarding decision making is a main and active part of cognitive sciences. There are many attempts to formalize the decision process in different contexts. A popular theory that has successfully encapsulated many known context effects is the accumulation of evidence through time. Within this frame, information from the environment is integrated into distinct dimensions and integrated into preference states iteratively. The determining factor of reaching a decision is the imposing of a decision criterion prior to the decision task and is mainly affected by the available time to make the decision. In this sense, there seems to be no distinction between knowing from the start the time constraint of the decision task. To test this, 32 individuals were recruited to participate to an online psychological experiment that featured decision tasks in both contexts. Results showed no significant difference between the confidence that participants had about their choice, but there was a slight difference between the contexts regarding the time that participants required to express their preference.

**Keywords:** Decision Making, Reasoning, Cognition.

## 1 Introduction

### 1.1 General Information about Decision Making

Decisions play a fundamental role in everyday life. Very often we are required to make choices that will somehow affect us (and possibly others), either in a positive or a negative manner. Investigating how people think during decision making and ultimately uncovering the fundamental cognitive mechanisms that are orchestrated during the deliberation process of landing on a choice are very important aspects of understanding human cognition.

The field of research around decision making was and remains to this day very active. Undeniably, the first step towards formalizing decision processes was taken by von Neumann and Morgenstern who established the Expected Utility Theory (EUT). According to this theoretical frame, individuals choose strategies and actions that maximize the expected utility, following the rationality principle [1]. Maximizing policies however, imply an absolutism in calculating the optimal solution (in our case the decision). Experiments throughout the years have shown that human behavior is neither rational nor absolute when it comes to decision-making, and many paradoxes that violate

the rationality principle have been observed (see [2] for a review of common paradoxes in choice under risk). As a result, many researchers have since attempted to capture the “true” nature of the human decision processes by inventing new theories and models that incorporate these paradoxes.

In research regarding decision making, it is commonly mentioned that there are three environment archetypes in which people are called to decide. These archetypes are distinguished based on the amount of information that decision makers hold prior to the decision process. They are commonly referred to as decisions under uncertainty, risk and certainty [3]. In decisions under uncertainty, decision makers are ignorant of the outcomes of their actions and the probabilities of occurrence of each of the outcomes in every action. Decisions under risk stand as the most commonly studied setting, where decision makers are aware only of the probabilities of the outcomes of the available actions. In decisions under certainty, each action produces a single outcome which is known by the decision maker. This setting is the simplest of the three categories and one could probably say it where most trivial everyday decisions belong. This study is focused on the last setting.

## **1.2 Understanding the Components of Making a Decision**

Bussemeyer and Townsend in their famous work on Decision Field Theory (DFT) [4] said that when humans face a choice, they attempt to make an evaluation of all the available outcomes. In the real world, the number of possible outcomes can be overwhelming and thus, it takes time for a person to finally decide. During this elapsed time, many different processes of information retrieval, comparison and integration happen and a final choice is expressed when the preference associated with that particular choice is dominant.

A very popular theoretical approach to studying decision making, that stemmed out of DFT, are accumulator models. Within this approach, evidence is continuously gathered through time by the decision maker until a decision is made. There are numerous variations of accumulator models in the literature, such as the Multi-alternative Decision Field Theory (MDFT, an extension of DFT) [5], the leaky competing accumulator model (LCA) [6], the Multi-attribute Linear Ballistic Accumulator (MLBA) [7], the Associative Accumulation Model [8] and many more. Almost all these theories of evidence accumulators maintain the representational structure of a connectionist network. These models differ in their technical implementation and in some structural properties, however the main components which they utilize in order to capture the decision process are very similar.

I will continue by outlining and discussing the three major components of decision processes that are shared among accumulator models: (a) retrieval of information related to the choice alternatives of a decision, (b) the integration of attribute values into preference states given the attention shifting, and (c) the comparison of these values with the choice criterion.

**Information Retrieval of Alternatives as a Composition of Attributes.** Naturally, in order for a decision process to take place, there must be two or more choice alternatives at question. These alternatives can be products, items, actions, situations etc. Alternatives are considered to be strongly connected with associative memory mechanisms. The perceptual image of an item can greatly vary among individuals, according to their beliefs and experiences. Every alternative has properties, distinguished characteristics that are either defined explicitly, by the real world or implicitly, by the decision maker. Thus, we can choose to view an alternative  $x_i$  as a multidimensional vector of  $M$  attributes:

$$x_i = (x_{i1}, x_{i2}, \dots, x_{iM}) \quad (1)$$

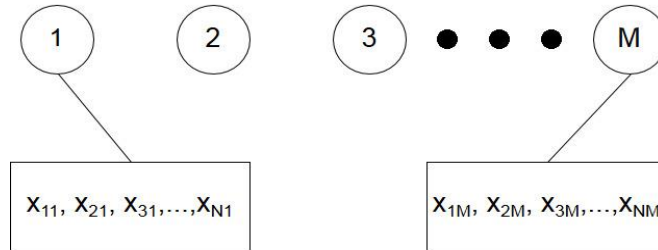
In this formula,  $x_{ij}$  represents the amount of attribute  $j$  in alternative  $i$ , while a set of available alternatives  $X$  can be written as:

$$X = (x_1, x_2, \dots, x_N) \quad (2)$$

The manner in which input information is represented and passed in accumulator models slightly varies between studies, as it is firmly bound to the strict structure of each model. However, in this study a certain degree of abstraction and generalization will be kept in order not to unnecessarily stray from its main purpose. In general, the input information of the accumulator network is the attributes of the available alternatives in vectorized form. This information is passed into an attribute layer (see Fig. 1). Theory of concept formation also suggests that saliency of information is ordered according to the form in which is presented. For example, if a list of items is presented in vertical form, then information on the top will be more salient than information presented under it [9]. This information is then used in the main decision process as described in the next section.

**Integration of Attribute Values in Accord with Attention.** Information in the attribute layer is transferred to the accumulation layer (described in the next section) in order to reach a decision. To accomplish this, most accumulator models have adopted a dynamic, stochastic method called sequential sampling [2]. According to this method, events (in this case attributes) are attended iteratively, according to some probability rating  $p_i$ . By introducing this method, the decision model automatically assimilates the factor of time. In this sense, the deliberation process can be distinguished into different discrete timestamps  $D(t)$  and in every timestamp an attribute is attended. Sequential sampling is assumed to approximate how human attention shifts between elements. Under fair conditions,  $n$  attributes would be assigned  $(n + 1)^{-1}$  probability each, because there would be  $n$  cases where some attribute  $i$  would be attended and one case where no attribute would be attended at all. In reality however, attributes are not attended at the same rate, as visual fixation studies suggest, and attributes are assigned with probability weights that are firmly dependent on the context of each decision [6].

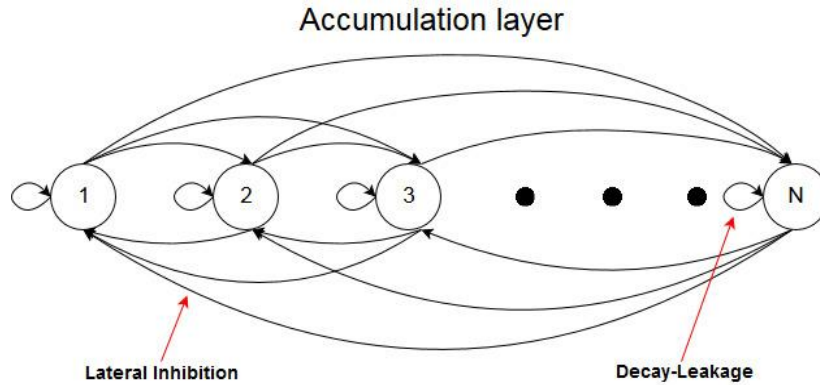
### Attribute layer



**Fig. 1.** A visual representation of the attribute layer inside an accumulator network. Each numbered node refers to one of the  $M$  attributes and encapsulates information for each of the  $N$  available alternatives regarding that attribute.

**Comparison of Alternatives Given the Choice Criterion.** Many accumulator models follow the structure of an auto-associative connectionist network, where both input and output nodes represent the available choice alternatives. In the output layer, the preferential state of the decision is encoded in terms of accumulated values for each of the  $N$  alternatives. These values are aggregated (usually linearly) numerical transformations of the values in the attribute layer, weighted by an additional attentional weight. Earlier theories in decision-making also suggest the existence of a more direct competition between alternatives and also decay in the information held for each alternative. Thus, in the output layer of most accumulators, nodes are connected by self-feedback loops and links of lateral inhibition (see Fig. 2). As it was mentioned before, accumulator theories suggest that evidence is gradually gathered through time. The final form of each of the alternative preferences at a given point in time  $t$  is a set of functions that contain the sum of (a) the current activation value of the node, (b) the activation of the node in the  $t - 1$  point in time, weighted by a decay parameter and (c) the negative sum of the activation values of the rest of the alternative nodes, weighted by a lateral inhibition parameter. In this manner, by aggregating preferential values into nodes representing alternatives, accumulator models are able to predict both the direction and the strength of choice.

When evidence about an available alternative reaches or surpasses a certain threshold  $\theta$ , the decision process is terminated and the direction of the decision is stabilized in favor of that specific alternative [11]. The decision criterion for each of the alternatives is set prior to the decision task and it is an increasing function of time that is available. This criterion is linked with the decision makers' choice certainty and it relies on evidence and decision time [12]. There have been many suggestions in previous research about how exactly judgements of certainty are affected by preference values of the different alternatives, such as that the distance between a criterion and an actual preference value or that the magnitude of differences between accumulated evidence for every alternative are factors.



**Fig. 2.** A visual representation of the accumulation layer. The curved arrows above and below the nodes declare the bidirectional lateral inhibitory relationship between each pair of alternatives. The self-feedback loop in every node represents the information leakage over time.

## 2 Hypothesis and Experimental Design

### 2.1 A Possibility of an Additional Dimension in the Factor of Time

In the previous section, the decision process, as described by common accumulator models, was discussed. As shown, preferential states evolve in a dynamic manner as a function of elapsed time. At any given time, we can view the accumulated preference values by “pausing” the process. So far, there has been no discussion in the literature about the decision maker’s knowledge of time constraining regarding the decision. There are many studies that focus on different imposed time constraints and there is huge amount of research done on the speed-accuracy tradeoff. However, there has not been any work towards investigating the exact difference between situations where there is prior knowledge of the time-restriction and situations of “sudden death”, meaning suddenly interrupted decisions. If knowledge of the time constraints of a decision has a significant effect on the decision process, then in this sense, there should not be a significant effect on the outcome of the decision. To investigate this difference, I designed and conducted an experiment that focused on exploring the relation of confidence and response time, as well as on choice reversals in environments under certainty in both time-restraint knowledge conditions.

### 2.2 Experimental Design

This experiment was granted an exemption from full ethical review by the UCD School of Computer Science Taught Masters Research Ethics Committee (TM-REC-SCS).

**Participants.** 32 male and female adults between ages of 20 and 40 years old were recruited online via social media to participate in the study. There were no specific

prerequisites established in order to participate, apart from having normal visual health and proficiency in the English language.

**Materials.** The experiment was implemented using the Labvanced online experiment platform [13]. Participants used their own devices, however the web server purposely allowed access only to tabletop and laptop computers and not smartphones or tablet devices for consistency reasons related to response timing when selecting the answers. Prior to commencing the experiment, individuals were prompted to use an external mouse and not a built-in, in case they were using laptops. The experiment ran in full-screen display mode throughout the whole procedure. Each trial featured a decision between two alternatives. There were four different decision scenarios and each scenario featured two back-to-back trials, one with knowledge of the time constraint and one without.

In the apartments scenario, the decision maker was asked to choose between two apartments to rent, as he or she supposedly moved to a new city after being hired by a company and was planning to live alone. The given attributes for every apartment were (1) the monthly expenses for renting and utilities, in euro currency, (2) the distance between the apartment and the workplace, in kilometers, (3) the neighborhood quality in which the apartment was located, which was presented in a 0 to 5 score, (4) the size of the apartment in square meters and (5) the year when the apartment was built.

In the restaurants scenario, the decision maker was asked to choose between two restaurants to have dinner in. The given attributes for every restaurant were (1) the average price per person, in euro currency, (2) the estimated amount of time until the dinner is served from the moment of the decision, in minutes, (3) the service quality, which was presented in a 0 to 10 score, (4) the overall impression of the restaurant's atmosphere, in a 0 to 20 score and (5) the food quality, given in a 0 to 5 score.

In the cars scenario, the decision maker was asked to choose between two cars to buy for daily transportation. The given attribute for every car were (1) the selling price of the car in euro currency, (2) the ease of handling regarding the car's operation in a 0 to 5 score, (3) the frequency in which the car should be serviced on average in months, (4) how stylish is the car, in a 0 to 10 score and (5) how comfortable is the riding experience, in a 0 to 20 score.

In the universities scenario, the decision maker was asked to choose between two universities located in the same city, in which he or she was supposed to do a Master's degree. The attributes that were given for every university were (1) the annual fees for the degree, in euro currency, (2) the university's facilities, in a 0 to 10 score, (3) the percentage of graduates that found employment in the same field as their studies after graduating, (4) the faculty's reputation in academia and industry, in a 0 to 5 score, and (5) the university's world ranking compared to other very well know universities.

In every decision task, alternatives were presented in terms of 5 attributes. Most of the types of attributes were created based on aspects that people often consider when asked to choose in those contexts. Prior to creating the materials, popular websites related to the four scenarios (real estate sites, restaurant critic sites, automobile critic sites and university ranking sites) were visited. Attributes were presented using distinct metric values in each scenario, in order to avoid confusion and in random order from top

to bottom. The values of the attributes were created in order to introduce an intense trade-off between the alternatives to the decision procedure and thus, there was no dominating alternative in any of the trials. Additionally, the ratio between alternative values was kept constant for both trials in the same decision scenario. Specifically, for every scenario a baseline set of two alternatives was created and then in order to create an additional set all values were multiplied by 1.2 (which was assumed not to affect the difference between attributes significantly).

The time constraint for all the trials was set to 17 seconds, as preliminary experiments showed that people required at least 25 seconds to reach a quick decision and 50 to 60 seconds to decide without haste.

Before every trial, an instruction page informed the participants about the context of the decision and the nature of the attributes that were to be presented regarding each alternative. Additionally, the nature of the imposed time constraint for every trial was given. Participants were either informed that they had 17 seconds or that they would not be told how much time they had. In the trials where participants knew how much time they had left there was an on-screen countdown timer that began counting the moment the frame of each trial appeared, starting from 17 seconds down to 0 seconds. When the available time had passed, a pop-up message informed the participants that they had to submit their answer. In the trials where participants had no knowledge of the available time, there was no countdown and participants were informed that the available time had passed by the same pop-up message as in the previous case.

Participants submitted their choice by clicking one of two boxes that contained the alternative they preferred. Once the choice had been submitted, a new page appeared and participants had to answer two questions before proceeding to the next trial. The first question asked them about their confidence regarding their choice (in a 10-point scale), and in the second question, they had to provide a brief reasoning description for choosing their answer in free-form text. The alternatives during answering these questions were not displayed in order to avoid looking at the attributes and altering the participants' beliefs.

Apart from recording the participants' choice, confidence and reason behind their choice in every trial, two more variables were recorded, one being the timestamp that the pop-up message emerged on the screen, and the other being the timestamp when participants expressed by clicking the appropriate box. Both variables were recorded as the server's UNIX timestamp measurements (in milliseconds). The Labvanced servers have reported a median offset across all time-measurements in past experiments as being around 20ms and a median standard deviation around 12ms [14].

### **3 Results**

The variables that were analyzed in this study were (a) the response time, which was created by calculating the difference between the two UNIX timestamps that were recorded during the experiment, (b) the confidence level of each choice, which was given directly from the participant, and (c) the choice reversal, a binary variable that indicated if choice between trials of the same scenario remained the same. The analysis was done

in R and in Microsoft Excel. The average response rate for all participants in all of the trials was 2195.852ms (SD=1756.154) and the median was 1113.5ms (see Table 1 for means of measurements across scenarios). The median response times in trials with prior knowledge (“KN”) and trials with no knowledge (“UNKN”) of time constraint were 943ms and 1215.5ms respectively. The median confidence level in both KN and UNKN was 8 out of 10 (see Table 2 for means of measurements under both conditions analytically and Fig. 3 for the distribution of the confidence levels).

**Table 1.** General statistics of the experiment’s results.

	Avg. Response Time in ms (SD)	Avg. Confidence Level (SD)	Number of Choice Reversals (%)
Total	2195.852 (2922.486)	7.706349 (1.488407)	34 (26.5625%)
Apartments	2004.719 (2516.411)	7.546875 (1.29819)	9 (28.125%)
Restaurants	2013.422 (2160.579)	7.453125 (1.638689)	6 (18.75%)
Cars	1880.578 (2527.342)	7.904762 (1.519339)	7 (21.875%)
Universities	2884.688 (4016.479)	7.934426 (1.412692)	12 (37.5%)

**Table 2.** Average response time and confidence in known (KN) and unknown (UNKN) time-constraints.

	Avg. Response Time in ms in KN (SD)	Avg. Confidence Level in KN(SD)	Avg. Response Time in ms in UNKN (SD)	Avg. Confidence Level in UNKN(SD)
Total	1726(2228.5)	7.71(1.4)	2665.7(3416.8)	7.71(1.57)
Apartments	1046.3(980.2)	7.44(1.3)	2963.1(3141.1)	7.66(1.29)
Restaurants	1946.2(2095.6)	7.59(1.5)	2080.6(2221.6)	7.31(1.76)
Cars	1965.3(2948)	7.87(1.4)	1795.8(2017.4)	7.94(1.66)
Universities	1946.1(2281.4)	7.94(1.4)	3823.3(5029.6)	7.93(1.46)

**Table 3.** Average difference between confidences in choice reversals and choice sustaining (positive shows greater confidence in the trial where time-constraint was known and vice versa)

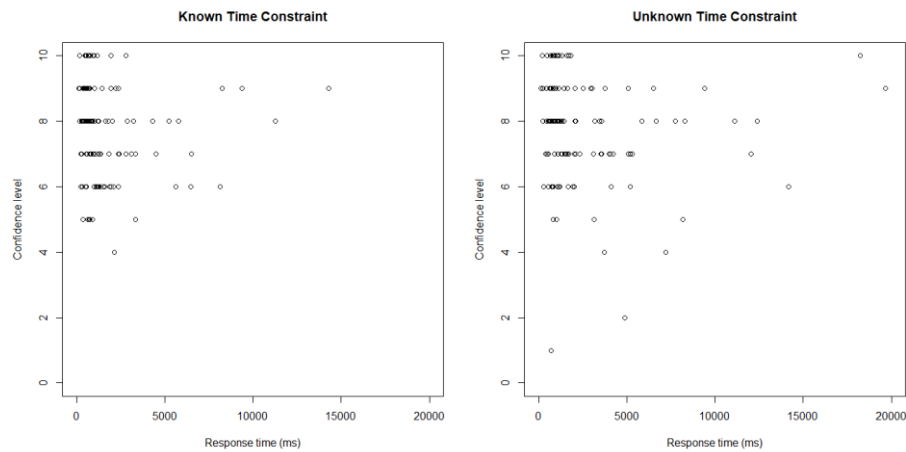
	Avg. Difference in Confidence in Choice Reversals	Avg. Difference in Confidence in Choice sustaining
Total	0.016	0.011
Apartments	0.11	-0.35
Restaurants	0.67	0.19
Cars	-0.71	-0.2
Universities	0	0.4

I carried out a Saphiro-Wilk test for normality in R for the response times and found that the data do not follow a normal distribution. Thus, the statistical significance of the median difference response time between known and unknown time-constraint trials was tested using a non-parametric test: A two-sample paired Wilcoxon signed rank test. The test showed a statistically significant difference between the median response times



on the KN and UNKN trials on the  $\alpha=0.05$  level of significance ( $p\text{-value} < .001$ ); the 95% confidence interval for the difference is  $(-815.5, -235.0)$ .

In a total of 128 pairs of trials of the same scenario, there were 34 reversals of choice (26.6%). However, the average confidence levels in KN and UNKN did not show any clear patterns between the two cases (see Table 3).



**Fig. 3.** Confidence levels (vertical axis) with relation to response time (horizontal axis) in trials of known (left) and unknown (right) time constraints.

## 4 Conclusion

My prior assumption for the experiment was that the tasks were equal in terms of difficulty. In general, response times and confidence levels across all four scenarios were very close, a fact that probably supports this assumption. Additionally, participants very rarely reported the varying differences across trials of the same scenario when asked to explain their reason of choice, meaning that the multiplication of attributes by 1.2 did not alter the difference between alternatives in a vivid manner.

The statistical analysis of the response times showed that there is a difference regarding response time between prior knowledge and ignorance of the time constraint (with ignorance requiring more time in order to reach the same confidence level), but still this difference is very small (under 1 second). Confidence levels in both conditions showed no variation, with the majority of responses being between 6 and 10. The latter finding supports current theories about the choice criterion being a function of time (same amount of time resulted in same choice certainty on average, regardless of the time-constraint knowledge).

The high percentage of cases where there was reversal in the choice and the fact that there was no significant shift in confidence levels between the two time-contexts were unexpected. It will be worthwhile to investigate this further, by focusing on choice

reversals and choice sustaining on bigger samples. A further step towards providing more solid evidence for the effect of prior knowledge of time would be to focus on various timepoints during a decision process instead of focusing on one, as the current study did. It is possible that after many tasks requiring the same amount of time, the decision maker will get accustomed to the specific time constraint and learn to adapt under pressure. Also, using a more intuitive and continuous method instead of discrete scaling systems for participants to express their certainty judgement could prove more useful for the analysis as it might lead to more accurate results. Finally, in this study there were some outliers that could have been avoided if participants were not allowed unlimited time to decide after the pop-up message. This was done on purpose, however there were participants who found out and exploited that.

## References

1. Bossaerts, P., Murawski, C.: Computational complexity and human decision-making. *Trends in Cognitive Sciences*, 21(12), 917-929 (2017). doi:10.1016/j.tics.2017.09.005
2. Bhatia, S.: Sequential sampling and paradoxes of risky choice. *Psychonomic Bulletin & Review*, 21(5), 1095-1111 (2014). doi:10.3758/s13423-014-0650-1
3. Busemeyer, J. R.: Decision making under uncertainty: A comparison of simple scalability, fixed-sample, and sequential-sampling models. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 11(3), 538-564 (1985). doi:10.1037/0278-7393.11.3.538
4. Busemeyer, J. R., Townsend, J. T.: Decision field theory: A dynamic-cognitive approach to decision making in an uncertain environment. *Psychological Review*, 100(3), 432-459 (1993). doi:10.1037/0033-295X.100.3.432
5. Roe, R. M., Busemeyer, J. R., Townsend, J. T.: Multialternative decision field theory: A dynamic connectionist model of decision making. *Psychological Review*, 108(2), 370-392 (2001). doi:10.1037/0033-295X.108.2.370
6. Usher, M., McClelland, J. L.: The time course of perceptual choice: The leaky, competing accumulator model. *Psychological Review*, 108(3), 550-592 (2001). doi:10.1037/0033-295X.108.3.550
7. Trueblood, J. S., Brown, S. D., Heathcote, A.: The multiattribute linear ballistic accumulator model of context effects in multialternative choice. *Psychological Review*, 121(2), 179-205 (2014). doi:10.1037/a0036137
8. Bhatia, S.: Associations and the accumulation of preference. *Psychological Review*, 120(3), 522-543 (2013). doi:10.1037/a0032457
9. Trabasso, T., Bower, G. H.: *Attention in learning: Theory and research*. Wiley, London; New York (1968).
10. Kim, B. E., Seligman, D., Kable, J. W.: Preference reversals in decision making under risk are accompanied by changes in attention to different attributes. *Frontiers in Neuroscience*, 6, 109 (2012). doi:10.3389/fnins.2012.00109
11. Diederich, A.: MDFT account of decision making under time pressure. *Psychonomic Bulletin & Review*, 10(1), 157-166 (2003). doi:10.3758/BF03196480
12. Kiani, R., Corthell, L., Shadlen, M.: Choice certainty is informed by both evidence and decision time. *Neuron*, 84(6), 1329-1342 (2014). doi:10.1016/j.neuron.2014.12.015
13. LABVANCED Homepage, <https://www.labvanced.com>, last accessed 2018/08/06.
14. LABVANCED FAQ page, [https://www.labvanced.com/faq\\_eng.html](https://www.labvanced.com/faq_eng.html), last accessed 2018/08/08.