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NONOSCILLATORY PROPERTIES FOR SOLUTION OF NONLINEAR NEUTRAL DIFFERENCE EQUATIONS OF SECOND ORDER WITH POSITIVE AND NEGATIVE COEFFICIENTS

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ABSTRACT

This research discussed nonoscillatory properties for resolve of nonlinear neutral differences equation of second order with positive and negative coefficients. The various new conditions which is ensure that all nonoscillatory solutions tend to zero or infinity like $n \to \infty$ *are given two examples are illustrate the ordinary results.*

Keywords: Oscillation**,** Non Oscillation, Neutral Difference Equations, Difference Equations.

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INTRODUCTION

The paper deal with the following second order nonlinear neutral difference equations with positive and negative coefficients:

$$\Delta^2(y_n - p_n y_{n-m}) + q_n G(y_{n-k}) - r_n G(y_{n-l}) = f_n \qquad (1)$$

where Δ is the forward difference operator given by $\Delta y_n = y_{n+1} - y_n$, p_n, q_n, r_n and f_n are infinite sequences of real numbers, $q_n > 0, r_n > 0$, m, k, l are positive integers, $G \in (R, R)$,

$y_n G(y_n) > 0$for $y_n \neq 0$. Throughout, we suppose that the following assumptions are satisfied:
(H_1) $0 < p_n \leq p < \infty$, $p$is constant ;
(H_2) $\sum_{j=n_1}^{\infty} \sum_{i=j-l+k}^{j-1} q_i < \infty$;
(H_3)There exists asequence$\{F_n\}$such that $\Delta^2 F_n = f_n$ and$\lim_{n\to\infty} F_n = 0$;
(H_4)There exist positive constant β_1 such that $G(y) \geq \beta_1 y$;
(H_5)There exist positive constant β_2 such that$G(y) \leq \beta_2 y$.

By a solution of equation (1) we mean a sequence y which satisfies equation (1) for each large$n$.

A solution y is said to be a nonoscillatory if it is eventually positive or eventually negative; otherwise, it is called oscillatory [1]. Oscillatory and asymptotic behavior solution of nonlinear neutral difference equations in many studies others, see for examples and the references cited therein[2-10]. The authors studied the existence of nonoscillatory solution of second order nonlinear neutral difference equations with positive and negative coefficients[11].

$$\Delta(r_n(\Delta y_n + p y_{n-m})) + q_n f(y_{n-k}) - r_n g(y_{n-l}) = 0$$

Inthe researchers obtained the oscillation and non oscillation criteria of second order nonlinear neutral difference equations with positive and negative coefficients[12].

$$\Delta^2(y_n + p_n y_{n-m}) + q_n G(y_{n-k}) - r_n H(y_{n-l}) = 0$$

In this researchers established some new sufficient conditions for oscillation and asymptotic behavior solution of the equation [13].

$$\Delta^m(y_n - p_n y_{n-s}) + q_n G(y_{n-k}) - r_n H(y_{n-l}) = f_n$$

The purpose of this paper is to obtain new sufficient conditions for then nonoscillation of all solutions of eq. (1). An example is provided to illustrate the main result.

MAIN RESULT

The next results provide some sufficient conditions for the nonoscillation of all solutions of eq. (1)
Let $y = y_n$ be a nonoscillatory solution of (1) for$n \geq N$.
Define for $n \geq n_0$

$$Z_n = y_n - p_n y_{n-m} \quad (2)$$

Assuming (H_2)hold, we define for $n \geq n_0$

$$T_n = \sum_{j=n}^{\infty} \sum_{i=j-l+k}^{j-1} q_i G(y_{i-k}) \quad (3)$$

And$\Delta^2 T_n = -\left(q_n G(y_{n-k}) + q_{n-l+k} G(y_{n-l})\right)$, $l > k$
Set

$$W_n = Z_n - T_n - F_n (4)$$

The following theorem based on Theorem 7.6.1, pp. 184 [14]:
Theorem 2.1. ([14], pp.182)
Assume that $\{p_n\}$ is a nonnegative sequence of real numbers and let k bea positive integer. Suppose that

$$\liminf_{n\to\infty}\sum_{i=n-k}^{n-1} p_i > \frac{k^{k+1}}{(k+1)^{k+1}}$$

Then

i. The difference inequality$y_{n+1} - y_n + p_n y_{n-k} \le 0,\ \ n = 0,1,2,\cdots$

Cannot have eventually positive solutions.

ii. The difference inequality$y_{n+1} - y_n + p_n y_{n-k} \ge 0,\ n = 0,1,2,\cdots$

Cannot have eventually negative solutions.

Theorem 2.2. ([15], Lemma 2.1- Lemma 2.2, pp.477-478)

i. Suppose that $0 < p_n \le \delta < 1$ for some positive constant $\delta, n \ge n_0$. Let x_n be a nonoscillatory solution of a functional inequality

$x_n[x_n - p_n x_{n-m}] < 0$In a neighbourhood of infinity, where$m > 0$, then$\lim_{n\to\infty} x_n = 0$.

ii. Suppose that $1 < \delta \le p_n$ for some positive constant $\delta,\ n \ge n_0$. Let x_n be a nonoscillatory solution of a functional inequality

$x_n[x_n - p_n x_{n+m}] > 0$In a neighbourhood of infinity, where$m > 0$, then$\lim_{n\to\infty} x_n = 0$.

Theorem 2.3. Suppose that $r_n \le q_{n-l+k}$, if$(H_1) - (H_5)$ hold, in addition to

$$\liminf_{n\to\infty} \sum_{j=n-(l-\rho-m)}^{n-1} \sum_{i=j}^{j+\rho} \frac{|r_i - q_{i-l+k}|}{p_{i-l+m}} > \frac{(l-\rho-m)^{l-\rho-m+1}}{\beta_1\,(l-\rho-m+1)^{l-\rho-m+1}} \qquad (5)$$

$l > \rho + m$.Then every nonoscillatory solution of equation (1) goes to infinity as$n \to \infty$.

Proof. Let y_n be an eventually positive solution of (1) for$n \ge n_0$, we may assume that there exist a positive ntegern_0, and $y_{n-m} > 0, y_{n-k} > 0, y_{n-l} > 0$,for$n \ge n_0 \ge$ N.Set Z_n, T_n, W_n as in (2),(3) and (4) then (1) it become

$$\Delta^2 W_n - (r_n - q_{n-l+k})G(y_{n-l}) = 0, \qquad (6)$$

Hence $\Delta^2 W_n \le 0$,thus there exist $n_1 \ge n_0$such that $\Delta W_n < 0$ or $\Delta W_n > 0$ for $n \ge n_1 \ge n_0$.

Now let $\Delta W_n < 0$ for $n \ge n_1$.Thus implies that $W_n < 0$ for $n \ge n_2 \ge n_1$ and$W_n \to -\infty$as $n \to \infty$. We claim that $y_n \to \infty$as $n \to \infty$, otherwise there exist $n_2 \ge n_1$and$\alpha > 0$such that $y_n \le \alpha$,so (H_5) is hold then (3) is become

$$T_n \le \alpha\beta \sum_{j=n}^{\infty} \sum_{i=j-l+k}^{j-1} q_i \qquad (7)$$

Substitution (7) in (4) we obtained

$$W_n \ge -p_n y_{n-m} - \alpha\beta \sum_{j=n}^{\infty} \sum_{i=j-l+k}^{j-1} q_i - F_n$$

$$W_n > -p_n y_{n-m} + \varepsilon$$

Implies that

$$W_n \ge -p_n y_{n-m} \qquad (8)$$

This is a contradiction. Then $y_n \to \infty$as $n \to \infty$.

Now If $\Delta W_n > 0$ for $n \geq n_1$, then$W_n > 0$ or $W_n < 0$for $n \geq n_2 \geq n_1$.

First, suppose that $W_n > 0$, from (4) we can conclude that $W_n \leq Z_n - F_n$then $W_n < Z_n + \varepsilon$ for $\varepsilon > 0$, hence$y_n Z_n > 0$, By Theorem2.2-ii we get $\lim_{n\to\infty} y_n = 0$, then$\lim_{n\to\infty} Z_n = \lim_{n\to\infty} W_n = 0$,which is contradiction since W_n is positive increasing. Next if$W_n < 0$, by taking the summation from n to$n + \rho$,$\rho > l$for both sides of (4) yield

$$\Delta w_{n+\rho+1} - \Delta W_n = \sum_{i=n}^{n+\rho} (r_i - q_{i-l+k}) G(y_{i-l})$$

$$-\Delta W_n \leq \sum_{i=n}^{n+\rho} (r_i - q_{i-l+k}) G(y_{i-l}) \qquad (9)$$

So by(H_4) is hold then (9) become

$$-\Delta W_n \leq \beta_1 \sum_{i=n}^{n+\rho} (r_i - q_{i-l+k}) y_{i-l} \qquad (10)$$

Now (8) can be written in the $-\frac{1}{p_n} W_n \leq y_{n-m}$.Hence

$$-\frac{1}{p_{n-l+m}} W_{n-l+m} \leq y_{n-l}$$

Substituting the last inequality in (10) we obtain

$$-\Delta W_n \leq -\beta_1 \sum_{i=n}^{n+\rho} \frac{(r_i - q_{i-l+k})}{p_{n-l+m}} W_{i-l+m}$$

$$-\Delta W_n \leq \beta_1 \left(\sum_{i=n}^{n+\rho} \frac{|r_i - q_{i-l+k}|}{p_{i-l+m}} \right) W_{n+\rho-l+m}$$

$$\Delta W_n + \beta_1 \left(\sum_{i=n}^{n+\rho} \frac{|r_i - q_{i-l+k}|}{p_{i-l+m}} \right) W_{n-(l-\rho-m)} \geq 0$$

By theorem 2.1-ii and in virtue of (5) it follows that the last inequality cannot has eventually negative solution, which is a contradiction.

Theorem 2.5. Suppose that$r_n \geq q_{n-l+k}$, and $(H_1) - (H_5)$ are hold, then every nonoscillatory bounded solution of equation (1) converge to zero as $n \to \infty$.

Proof. Let y_n be an eventually positive solution of (1) for $n \geq n_0$,we may assume that there exist a positive integer n_0, and $y_{n-m} > 0, y_{n-k} > 0, y_{n-l} > 0$,for $n \geq n_0 \geq N$,since from (6) we have $\Delta^2 W_n \leq 0$ for $n \geq n_0$,thus there exist a $n_1 \geq n_0$ such that $\Delta W_n > 0$ or $\Delta W_n < 0$ for $n \geq n_1 \geq n_0$.

Next suppose that $\Delta W_n > 0$ for $n \geq n_1$.Thus implies that $W_n < 0$or $W_n > 0$ for $n \geq n_2 \geq n_1$ and $W_n \to \infty$as $n \to \infty$. So for (4) we get $W_n \leq y_n - F_n$, It implies that $y_n \to \infty$ as $n \to \infty$, which is a contradiction ,hence $\Delta W_n > 0$ is not possible .

Now If $\Delta W_n > 0$ for $n \geq n_1$, then$W_n > 0$ or $W_n < 0$for $n \geq n_2 \geq n_1$.

First, suppose that $W_n < 0$.Then there exists$n_3 \geq n_2, \mu > 0$ such that $W_n \leq -\mu < 0$, for $n \geq n_2$. Since y_n is bounded, then$\limsup_{n\to\infty} y_n = a < \infty$,and there exists a subsequence $\{s_j\}_{j=1}^{\infty}$ such that$y_{s_j} \to \infty$as$s_j \to \infty$,and $y_{\delta_j} = \max\{y_s: s_0 \leq \varepsilon \leq s_j\}$, $\limsup_{n\to\infty} y_{\delta_j} \leq a$
Since y_n is bounded then by (7) and (4) we get

$$y_{s_j} \leq -\mu + p_{s_j} y_{s_j-m} + \alpha\beta \sum_{j=n}^{\infty} \sum_{i=j-l+k}^{j-1} q_i + F_{s_j}$$

$$< -\mu + p_{s_j} y_{\delta_j} + \varepsilon \quad \varepsilon > 0$$

So $y_{s_j} \leq -\mu + \limsup_{n\to\infty} y_{\delta_j}asj \to \infty$, we get from the last inequality $a \leq -\mu + a$ which is a contradiction.

Finally if$W_n > 0$,from (4) we get $W_n \leq Z_n - F_n$,implies that$0 < W_n \leq Z_n$ for large enough n , so $y_n Z_n > 0$ and by Theorem2.3-ii,it follows that $\lim_{n\to\infty} y_n = 0$.

PRACTICAL METHOD AND CONSIDERATIONS

In this section, we illustrate the main results by giving some examples

Example 1: Consider the difference equation:

$$\Delta^2(y_n - \left(1 + \left(\tfrac{1}{2}\right)^n\right) y_{n-1}) + \tfrac{5}{2}\left(\tfrac{1}{2}\right)^n y_{n-2} - \left(\tfrac{1}{2}\right)^n y_{n-3} = \tfrac{1}{2}\left(1 + \left(\tfrac{1}{2}\right)^n\right) 2^n \qquad (11)$$

Where $m = 1, k = 2, l = 3, \rho = 1$, $G(y_n) = y_n$, $\beta_1 = \beta_2 = 2$

One can find that all conditions of theorem 2.4 hold as follows:

$$\boldsymbol{p_n} = \mathbf{1} + \left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\boldsymbol{n}} = 1 \ \ as\ n \to \infty$$

$$\sum_{j=n_1}^{\infty} \sum_{i=j-l+k}^{j-1} q_i < \sum_{j=n_1}^{\infty} \left(5\left(\frac{1}{2}\right)^j\right) < \infty$$

$$r_n - q_{n-l+k} = \left(\frac{1}{2}\right)^n - \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = -18 < 0 \quad , n \to \infty$$

$$\liminf_{n\to\infty} \sum_{j=n-(l-\rho-m)}^{n-1} \sum_{i=j}^{j+\rho} \frac{|r_i - q_{i-l+k}|}{p_{i-l+m}} > \frac{2^3}{3^2}$$

So, according to theorem 2.4 every solution of (11) is goes to infinity as $n \to \infty$, for instance $y_n = 2^n$is such a solution.

Example 2:Consider the difference equation

$$\Delta^2(y_n - \left(1 + \left(\frac{1}{e}\right)^n\right) y_{n-m}) + \frac{(e^{-1} + e)}{e^2} e^{-n} y_{n-2} - (-2e^{-1} + 2)e^{-3} y_{n-3} = f_n (12)$$

Where$m = 1, k = 2, l = 3, \rho = 1$, $G(y_n) = y_n$, $\beta_1 = \beta_2 = 2$,

$$f_n = \left(\frac{1}{e}\right)^n (e^{-2} - 3e^{-1} - e^1) + e^{-2n}(-e^{-3} + 3e^{-1})$$

One can find that all conditions of theorem 2.5 hold as follows:

$$p_n = 1 + \left(\frac{1}{e}\right)^n = 1 \ as\, n \to \infty$$

$$\sum_{j=n_1}^{\infty} \sum_{i=j-l+k}^{j-1} q_i = \frac{(\mathrm{e}^{-1} + \mathrm{e})}{\mathrm{e}} \sum_{\mathrm{j=n_1}}^{\infty} \mathrm{e}^{-\mathrm{j}} < \infty$$

$$r_n - q_{n-l+k} = 0.06294 > 0$$

So, according to theorem 2.5 every solution of (12) has nonoscillatory solution tend to zero as$n \to \infty$, for instance $y_n = e^{-n}$is such a solution.

CONCLUSION

In this paper some necessary and sufficient conditions are get it to ensure the nonoscillatory for all solutions of second order nonlinear neutral delay difference equations with positive and negative coefficients. One sequences are used for this suggest.

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REFERENCES

[1] M.Migda,J.Migda, Nonoscillatory Solution to Second –Order Neutral Difference Equations,Symmetry.2018,(10).207:1-14.

[2] K.S.Vidhyaa,C.Dharuman, Asymptotic Behavior of Third Order Delay Difference Equations with Positive and Negative Terms, international Journal of Pure and Applied Math.2017.(113).10:157-165.

[3] R.Arul,M.Angayarkanni,Oscillatory and Asymptotic Behavior of Solution of Second Order Neutral Delay Difference Equations with "Maxima", Advance in Pure Mathematics,2015.5:71-81.

[4] Hasanögunmez,ÖzkanÖcalan: Oscillations of difference equations with several positive and negative coefficients. Fasciculi mathematic, 2013.51: 115–122.

[5] B.Selvaraj, G.Jawahar, Nonoscillatory Solution of Second Order Neutral Delay Difference Equations,J.Comp&Math.Sci, 2010.1(5):566-751.

[6] C.Jayakumar, G.Tamilselvi, Oscillatory Behavior of Solution of Second Order Neutral Delay Difference Equations, IJSART, 2018.4(4):10-13.

[7] E.Thandapani, M.Vijaya, Classifications of Solutions of Second Order Nonlinear Neutral Delay Difference Equations with positive and negative coefficients, SARAJEVO Journal of Math.2011.7 (19):57-66.

[8] X. H. Tang, J. S. Yu, D. H. Peng,Oscillations and non- oscillations of neutral difference equations with positiveand negative coefficients. Computers and mathematics application, 2000. 39: 169 – 181.

[9] H. A. Mohamad; and H. M. Mohi: Oscillations of Neutral Difference Equations of Second Order with Positive and Negative Coefficients,Pure and Applied Mathematics Journal, February 2016, (5), 1: 9-14

[10] H. A. Mohamad; and I. Z.Mushtt: Oscillations and asymptotic behavior of Second Order non-Linear Neutral Difference Equations,Journal of College of Education, April 2017(1):47-56.

[11] Y.Tian, Y.Cai,T.Li,Existence of Nonoscillatory Solution of Second Order Neutral Difference Equations ,Nonlinear Sci.Appl.2015.(8):884-892.

[12] A. K. Tripathy, S. Panigrahi: Oscillations in nonlinear neutral difference equation with positive and negative coefficients: International Journal of Difference Equations, 2010,(5), 2: 251 – 265.

[13] R.N.Rath, B.L.Barik,S.K.Rath, Oscillation of Higher Oder Neutral Functional Difference Equations with Positive and Negative Coefficients. Math.Slovaco.2010.60 (3):361-384.

[14] I. Gyori, G. Ladas, Oscillation theory of delay differential equations with applications. Oxford Clarendon Press, 1991.

[15] Jaroš J and Kusano T, on a class of first order nonlinear functional differential equations of neutral type, Czechoslovak Mathematical Journal, 1990. (40), 3: 475-490.