Cooperative Spectrum Sensing over Correlated Rayleigh Fading Channels in Cognitive Radio using Factor Graph

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***Abstract*—The cooperative spectrum sensing in cognitive radio (CR) environments is generally modelled as a distributed binary hypothesis testing problem. However, modelling of the system in presence of correlated multipath-fading is an important issue. It deals with complex dependencies of large number of random variables. Earlier developments on decision fusion consider correlated local decisions [1]–[3], correlated shadowing [4]–[6], spatial correlation [7] and independent Rayleigh fading [8] channels.**

This paper presents a new likelihood ratio test (LRT) based fusion rule using normal factor graph (NFG) and Sum-Product-Algorithm (SPA) model for cooperative spectrum sensing in presence of correlated multipath-fading channels. The proposed approach requires exact channel statistics instead of instantaneous channel state information (CSI). It leads to Neyman-Pearson (N-P) criteria based optimal sensing. We assume SUs are aligned in a linear fashion and each channel is correlated only with its adjacent channels with identical fading correlation coefficients. In this respect, we derive a new tri-variate Rayleigh probability density function using Miller's [9] approach. Moreover, closed form solutions for local probabilities of detection and false alarm are also derived by assuming that all SUs perform energy detection, experience identical signal-to-noise ratios (SNRs) which make real-time computations simple. The decision fusion is performed using SPA as message passing strategy over the NFG. Simulation results are provided to validate the performance of the proposed approach.

I. Introduction

In fixed spectrum allocation process, a particular frequency band is restricted to the licensed users of the band. However, it has been observed that the most licensed spectrum is often under-utilized (sparse in frequency), even one frequency band is not used continuously (sparse in time) and simultaneously in all geographical locations (sparse in space) [10]. These facts may be exploited in spectrum crisis situations. Cognitive radio network (CRN) [11], based on *Dynamic Spectrum Access* (DSA) technique, is emerging as a possible solution to the problem of inefficient use of allocated licensed spectrum. In this method, the unlicensed or secondary users (SUs) are allowed to sense the spectrum periodically, identify the spectrum holes i.e. absence of PUs in that band and opportunistically utilize it. It is also called *opportunistic spectrum access*.

In *cooperative sensing*, information from multiple SUs are jointly used to detect the absence of primary user's (PU) signal (spectrum holes). It combats many random factors like multipath, shadowing by exploiting the spatial diversity among the CRs. It also enhances the accuracy, reliability, and performance of spectrum sensing at the cost of complexity [6], [12]. However, spatial correlations among SUs (or CRs) affect the performance of cooperative sensing. Correlations among the CRs may also vary due to their mobility. They may come close enough to perform sensing over correlated fading scenarios due to the non-isotropic scattering model, since the distribution of Angle of Arrival (AoA) is more likely to be non-uniform [13]. Spatially close SUs are likely to be affected by the same environmental conditions and may suffer from common errors. Therefore sensing over spatially correlated fading channels is quite practical.

In Centralized Cooperative Spectrum Sensing (CCSS), a central unit called fusion centre (FC) collects hard-sensing (local decisions) or soft-sensing observations (local test statistics) from CRs, identifies the available spectrum (white space) and broadcasts this information to all CRs or directly controls CR traffic. It is simple to implement but not immune to node failure. The work presented in this paper is concentrated on CCSS over correlated fading in white space of radio spectrum.

CCSS may be viewed as distributed detection (using multiple detectors) with an FC. A detailed survey on distributed detection schemes has been presented in [14]. Though LRT rule is optimal for data fusion, there is no closed form solution available for coupled local best thresholds with global optimal decision [15]. However, if prior knowledge about PU's signal is not available, the energy detection (ED) method is optimal for detecting zero-mean constellation signals [16]. Therefore energy detection based local sensing with identical decision rules is a popular model for getting global optimal solutions and frequently used in many distributed detection problems.

LRT based data fusion at FC is implemented using either *Neyman-Pearson* (N-P) criterion (maximization of probability of detection subject to a constraint on probability of false alarm) or *Bayes* criterion (minimization of probability of error) [17]. Other known sub-optimal fusion rules are AND, OR, VOTING [18] for ideal SU-FC (reporting) channels, Chair-Varshney rule for high SNR [19], equal gain combiner (EGC) for medium SNR, maximum ratio combining (MRC) for low SNR [20]. However, all the sub-optimal fusion rules generally assume independent fading channels with statistically independent local decisions.

The problem with correlated local decisions was already studied in different forms. Aalo and Viswanathan [1] consider correlated noise and derived the fusion rule. Drakopoulos and Lee [2] derive a fusion rule based on correlated local decision rules. Kam et al. [3] presents another generalized form of the optimal fusion rule for correlated local decisions. [6] suggests a suboptimal fusion rule termed as linear-quadratic (LQ) strategy by assuming dependent PU-SU channels and received signals at SUs follow correlated log-normal distribution. [21] considers spatial correlation coefficient based on (given or estimated) location information. It is shown in [4], [5] that large number of closely located SUs may be less effective than a small number of farther located SUs due to correlated fading and shadowing. Spatial correlation based user selection in both multipath and shadow fading is presented in [7] by modifying the space-time correlation model of [22] which assumes that distance between the users are fixed. However, they have not derived any closed form solutions for probabilities of detection and false alarm.

Inference over probabilistic models [23] is often used for computationally expensive, high dimension wireless communication systems such as cooperative spectrum sensing. It is an easier way of representing complex dependencies among large number of random variables and finding the exact marginal. Good message-passing algorithms like Pearl's Belief-Propagation (BP) algorithm, Sum-Product-Algorithm (SPA) [24] with suitable graphical models are used for drawing inference even when exact solution is intractable (NP-hard). There are several well-known graphical models [23]: Bayesian Network (BN), Markov Random Field (MRF), Tanner graph (TG), and Factor graph (FG). FG/SPA setting [25] is efficient for computing exact solutions of "marginalize product of functions (MPF)" problems, where others fail and suitable for model based design. According to Hammersley-Clifford theorem [26], only joint distribution in the form of Gibbs distribution (e.g. Gaussian, exponential) may form the associated MRF. There is no such restriction on FG. Besides, FGs are more general than others (any BN, MRF or TG can be encoded as FG with no increase in its representation size [27]). Therefore we adopt FG, precisely Forney-style (normal) factor graph (NFG) [24], as our probabilistic graphical model. NFG is a simplified version of general FG, where functions and variables are represented by nodes and edges respectively.

The main challenge here is proper *modelling* of correlated channels without using the location information of CRs to get the closed form analytical solutions for performance evaluation. It results in multivariate analysis. Joint *pdf* of bivariate Nakagami-m and Rayleigh distributions are given in [28]. Multivariate Nakagami-m distribution with exponential correlation matrix is discussed in [29] with the help of [9]. Joint *pdf* of trivariate Nakagami-m distribution with arbitrary covariance matrix is presented in [30], [31]. [31] also extends the concept to quadrivariate case and state that end-around case (SU_1 is also correlated with SU_K) is intractable. Multivariate Rayleigh distribution is also investigated extensively in [32]. It involves conditional and unconditional independence of large number of random variables in a large CRN. Therefore message passing based inference algorithms over probabilistic graphical model may be a better approach for complex networks. FG/SPA based approach to cooperative spectrum sensing was first addressed by [33]. The probability of detection and LRT statistic are reformulated in [8] by considering exact channel statistics over independent Rayleigh fading channels. In this paper, we develop analytical models for CCSS scheme by assuming PU-SU (sensing) as spatially correlated and SU-FC (reporting) channels as independent multipath fading channels. We compute the inferences using SPA over the normal factor graph. Therefore we try to connect two active research area, namely inference methods in graphical models and distributed

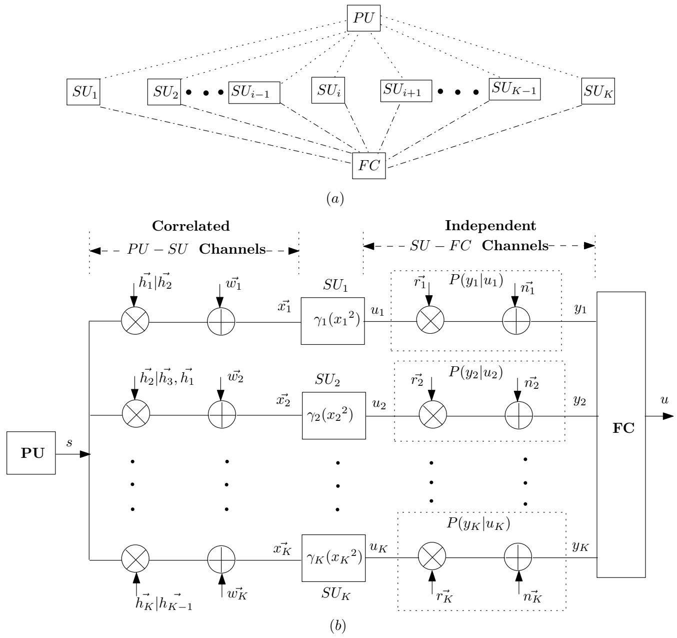

Fig. 1. Block diagram for system model of Centralized Cooperative Spectrum Sensing (CCSS) technique with K secondary users (sensors) and one primary user. Final decision is taken by the fusion centre.

detection methods over correlated multipath fading channels in CR scenario. Main contributions in the present paper are,

- Factor graph based modelling of cooperative spectrum sensing in spatially correlated multipath fading channels.
- Analysis of cooperative spectrum sensing for multipath fading environment by assuming correlation between adjacent channels (in a linear alignment of SUs).
- Derivation of a new trivariate Rayleigh pdf.
- Derivation of a LRT based new fusion rule over correlated Rayleigh fading channels.

This paper is structured as follows: Section II explains the notations and assumptions used in the article, the basic system model, and problem formulation of CCSS over correlated fading channels. The system analysis, analytical solutions i.e. derivation of local probability of detection, probability of false alarm, LR-based fusion rule are presented in Section III. Simulation results are presented in Section IV. Finally, section V concludes the paper with future research path directions.

II. CCSS System Model Over Correlated Fading

Imperfect channel knowledge affects the performance of cooperative spectrum sensing scheme. Therefore proper system modelling is very important for its practical applications. The block diagram of CCSS system is shown in Fig.1. The system consists of one PU, K secondary users and one FC at one place. All SUs are simultaneously sensing the state of PU. We assume, SUs are located in 1-D linear alignment.

A. Notations and Assumptions

Throughout this paper, K is a positive integer, v_1^K denotes the set $\{v_1, ..., v_K\}$, upper-scored variable $\vec{v}_i$ denotes vector $\{v_{i_1}, ..., v_{i_N}\}$ over N sampling intervals for i-th SU, $\sim u_i$

denotes inversion of binary symbol u_i, $P(x)$ denotes probability density function (pdf), and $F(x)$ denotes cumulative distribution function (CDF). Channel coefficients h_i, r_i are Rayleigh and noise samples w_i, n_i are Gaussian distributed random variables (RVs) respectively. CN denotes complex Gaussian and no distinction is considered between RVs and their values. f_D denotes maximum Doppler frequency.

We are interested in TV band (500-700 MHz). In this band, coherence bandwidth $(B_c) >$ signal bandwidth (B_s) and therefore frequency-selective fading is less likely. Hence, only slow, flat-fading channels are considered throughout this article. It is assumed that the sensing duration (T_s) is much shorter than the average *busy-to-idle* and *idle-to-busy* state transition periods of the PU. Otherwise, sensing outcomes will not be meaningful for the corresponding utilization period. It is also assumed that the correlation among the mobile CR users do not change significantly during the sensing process i.e. $T_s < T_c \approx \frac{0.5}{f_D}$, where, $T_c =$ coherence time for isotropic/non-isotropic scattering [34]. We assume that PU is memory-less and no prior knowledge is available about PU's traffic characteristic i.e. *a priori* probabilities of transmitted signal s is unknown. Also consider, SUs are transmitting at same power relative to PU (as interweave DSA model) and energy detector is used at each SU for local decision. Throughout the article, it is considered that each SU is performing *hard-local sensing*. The local decisions are arriving at FC in a *synchronous* manner i.e. they are based on a common clock and arrive at some predetermined instants. As a special case we formulate our problem by assuming that *sensing* channels are spatially correlated only with its immediate neighbours and *reporting* channels are assumed to be statistically independent.

B. Problem Formulation

According to the CCSS model of Fig.1, all SUs are monitoring the same frequency at which PU is transmitting. The baseband-equivalent signal (s), where $s \in \{0,1\}^N$, transmitted by the PU and propagated to the i-th SU over a correlated (only with immediate neighbours), frequency non-selective, slow Rayleigh fading channel ($\vec{h}_i|\vec{h}_{i-1}, \vec{h}_{i+1}$), where h_i is Rayleigh distributed RV with parameter σ_i i.e. $h_i \sim R(\sigma_i)$ and σ_i^2 is the variance of component Gaussian random variables from which the Rayleigh variable (h_i) is generated. Hence, the i-th SU observes a complex baseband equivalent signal ($\vec{x_i}$) over correlated fading channels as,

$$\vec{x_i} = (\vec{h_i}|\vec{h}_{i-1}, \vec{h}_{i+1})s + \vec{w_i} \tag{1}$$

where $w_i \sim CN(0, \sigma_{w_i}^2)$ i.e. $\vec{w}_k$ is statistically independent, circularly symmetric complex Gaussian (CSCG) distributed. At i-th SU, the signal is mapped onto local decision u_i. The mapping employs energy detector ($\gamma_i(\vec{x_i}^2)$) to compute the total symbol energy $= \sum_{n=1}^{N} \|x_i(n)\|^2$ for symbol-by-symbol detection or average symbol energy $= \frac{1}{N}\sum_{n=1}^{N} \|x_i(n)\|^2$ for sequence detection. $\|.\|$: denotes Euclidean norm. All u_i's are transmitted to a fusion center (FC) where the final decision (u) is derived. The channel between each SU and FC is assumed to be noisy. It may be binary symmetric channel (BSC) or additive white Gaussian noise (AWGN) or Rayleigh faded channel and characterized by $P(y_i|u_i)$, where y_i denotes the received signal at FC from i-th SU. Therefore over i-th BSC with crossover probability (α_i), the received signal at FC is,

$$\left.\begin{aligned} y_i = u_i \text{ with probability } & \ 1-\alpha_i \\ = \sim u_i \text{ with probability } & \ \alpha_i \end{aligned}\right\} \tag{2}$$

where, $u_i \in \{0, 1\}$. In i-th AWGN channel with independent additive noise $n_i \sim CN(0, \sigma_{n_i}^2)$, the received signal at FC is,

$$y_i = u_i + n_i \tag{3}$$

Consequently, in i-th independent Rayleigh fading with $r_i \sim R(\sigma_{r_i})$, the received signal at FC is,

$$y_i = u_i r_i + n_i \tag{4}$$

Therefore the spectrum sensing problem may be modelled as binary hypothesis testing with null and alternative hypotheses,

H_0: *Primary user is idle or not active* i.e. $s = 0$ *and*
H_1: *Primary user is busy or active* i.e. $s = 1$

Alternatively, it becomes a binary hypothesis testing problem to decide whether or not the mean received power at SU is higher than the expected power (threshold). It is assumed that SUs use the spectrum whenever they detect a spectral hole (white space). The constraint in our system is the probability of interference with PU's transmission i.e. the probability of erroneous decision about the presence of PU. Therefore for efficient utilization of spectrum, the system design needs to minimize the probability of missed detection (P_m) or maximize the probability of detection (P_d) subject to the constraint that probability of false alarm (P_f) $\leq$ pre-defined threshold (τ). It becomes simple, one-side trial, *Neyman-Pearson* hypothesis testing problem [17].

III. System Analysis of CCSS over Spatially Correlated Multipath Fading Channels

It is well recognized that proper analysis of a system depends on its exact probabilistic model. The CCSS model defined in section-II largely depends on different channel conditions considered between PU-SU and SU-FC. In many practical CR applications (e.g. vehicular and pedestrian) fading channels may be correlated when CRs come close enough. Therefore consideration of correlated fading channel is very relevant to the context of spectrum sensing in CR network. We consider multivariate Rayleigh distribution [9] to model the correlated scenario. As discussed earlier, PU-SU channels are suffering from slow, frequency-flat, correlated Rayleigh fading i.e. one channel is correlated with only its immediate neighbours. Moreover, the received signals (y_i) at FC from i-th SU may be written as eq.(2) for BSC, eq.(3) for AWGN, or eq.(4) for Rayleigh faded SU-FC channels respectively.

A. Probability Model of the Detection Problem

In this section, we try to solve this distributed detection problem by passing messages over NFG using SPA. The complex envelope of the received signal at input of the

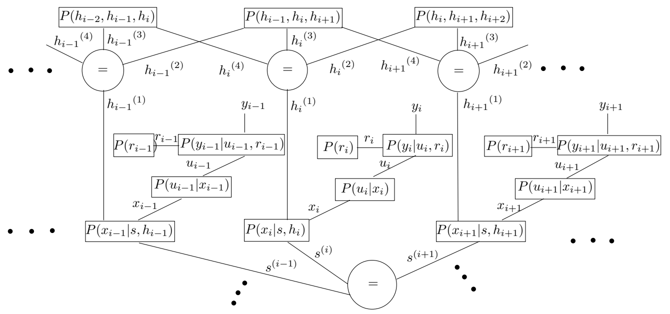

Fig. 2. NFG for joint distribution $P(h_1^K, x_1^K, u_1^K, r_1^K, y_1^K|s)$, where adjacent PU-SU channels are correlated and *hard local decisions* are employed. The graph is shown for only three PU-SU-FC channels.

demodulators of SUs are corrupted by multiplicative Rayleigh fading ($\vec{h}_i$) and AWGN ($\vec{w}_i$). Now, we have to find likelihood functions, $P(y_1^K|s=0)$ and $P(y_1^K|s=1)$, where $y_1^K = \{y_1, ..., y_K\}$, to compute the LRT statistic. Therefore a detection problem is mapped to Bayesian inference problem by finding the likelihoods. The joint probability distribution, $P(s, h_i, h_{i-1}, h_{i+1}, x_i, u_i, r_i, y_i)$ represents the correlated CCSS model for i-th PU-SU-FC channel of Fig.1. The likelihood function, $P(y_i|s)$, may be computed as follows,

$$\int P(s, h_i, h_{i-1}, h_{i+1}, x_i, u_i, r_i, y_i) dr_i du_i dx_i dh_{i-1}^{i+1} = P(s, y_i)$$

$$P(y_i|s) = \int P(h_i, h_{i-1}, h_{i+1}, x_i, u_i, r_i, y_i|s) dr_i du_i dx_i dh_{i-1}^{i+1} \quad (5)$$

where, $P(h_i, h_{i-1}, h_{i+1}, x_i, u_i, r_i, y_i|s)$ is the joint distribution of interest and it may be further factorized as,

$$P(r_i)P(y_i|u_i, r_i)P(u_i|t_i)P(t_i|s, h_i)$$
$$P(h_i|h_{i-1}, h_{i+1})P(h_{i-1}, h_{i+1}) \quad (6)$$

where, $t_i = |\vec{x}_i|^2$, h_i already depends on h_{i-1},h_{i+1} and all are independent of s. Therefore for *hard* local decisions,

$$P(h_1^K, x_1^K, u_1^K, r_1^K, y_1^K|s) = \prod_{i=1}^{K} P(y_i|u_i, r_i)P(u_i|t_i)P(t_i|s, h_i)$$
$$P(h_i|h_{i-1}, h_{i+1})P(h_{i-1}, h_{i+1})P(r_i) \quad (7)$$

Fig.2 shows the factor graph, while two adjacent PU-SU channels are correlated. It contains cycles. Therefore it does not guarantee the convergence of SPA [24]. The graph shown in Fig.2 can be free from cycle by exploiting the assumption of correlation between adjacent channels. The Fig.3 is the cycle free version of Fig.2. It greatly simplifies our operation for obtaining exact marginals using SPA. We cannot run SPA over the FG of Fig.3 as long as we know the distribution of $P(h_i|h_{i-1}, h_{i+1})$ and $P(h_{i-1}, h_{i+1})$ in eq.(7). Let us consider the following assumptions for multivariate Rayleigh distribution based modelling of correlated fading scenario. The covariance matrix is considered in the form of Toeplitz matrix. Let, h_{i-1}, h_i and h_{i+1} are jointly Rayleigh distributed random variables with covariance matrix,

$$M = \begin{bmatrix} \sigma_{i-1}^2 & \rho\sigma_{i-1}\sigma_i & 0 \\ \rho\sigma_{i-1}\sigma_i & \sigma_i^2 & \rho\sigma_2\sigma_{i+1} \\ 0 & \rho\sigma_i\sigma_{i+1} & \sigma_{i+1}^2 \end{bmatrix}$$

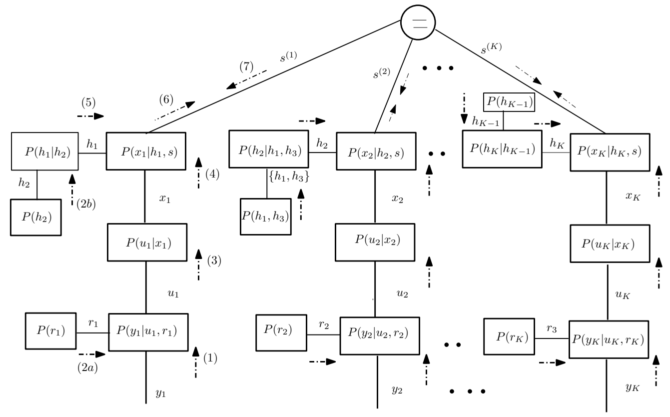

Fig. 3. Normal factor graph after removing the cycles. The graph is shown only for $K = 3$ PU-SU-FC channels.

where, σ_i^2 is the variance of the component Gaussian RVs of $i-$th Rayleigh RV and ρ is the identical correlation coefficient between any two adjacent Rayleigh RVs. According to [[9], eq.(2.1)], with appropriate substitution of the parameters, the *tri-variate* Rayleigh pdf may be written as, (steps are omitted due to space limit)

$$P(h_{i-1}, h_i, h_{i+1}) = \frac{8h_{i-1}h_ih_{i+1}}{\Omega_{i-1}\Omega_i\Omega_{i+1}\alpha} e^{\left\{-\frac{{h_i}^2}{\alpha\Omega_i} - \frac{(1-\rho^2)}{\alpha}\left(\frac{{h_{i-1}}^2}{\Omega_{i-1}} + \frac{{h_{i+1}}^2}{\Omega_{i+1}}\right)\right\}}$$
$$I_0\left[\left(\frac{2\rho}{\alpha\sqrt{\Omega_{i-1}\Omega_i}}\right)h_{i-1}h_i\right] I_0\left[\left(\frac{2\rho}{\alpha\sqrt{\Omega_i\Omega_{i+1}}}\right)h_ih_{i+1}\right] \quad (8)$$

where, $\Omega_i = 2\sigma_i^2$, $\alpha = (1-2\rho^2)$, and $I_0(.)$ is the modified Bessel function [35] of first kind with zero-th order. Marginal distribution of (h_{i-1}, h_{i+1}) may be obtained from eq.(8) using [[36], 2.15.20, eq.(8)] as,

$$\begin{aligned} P(h_{i-1}, h_{i+1}) &= \int_0^\infty P(h_{i-1}, h_i, h_{i+1})\ dh_i \\ &= D_i \int_0^\infty h_i e^{-\frac{{h_i}^2}{C}} I_0(A_ih_i)\, I_0(B_ih_i)\ dh_i \\ &= \frac{CD_i}{2} e^{\left(\frac{C(A_i^2+B_i^2)}{4}\right)} I_0\left(\frac{A_iB_iC}{2}\right) \end{aligned} \quad (9)$$

Marginal distribution of $P(h_i)$ is obtained using [[36], 2.15.5, eq.(4)] with appropriate substitution of the parameters as,

$$\begin{aligned} P(h_i) &= \int_0^\infty P(h_{i-1}, h_i, h_{i+1})\ dh_{i-1}\ dh_{i+1} \\ &= \frac{8h_ie^{-\frac{h_i^2}{\alpha\Omega_i}}}{\alpha\Omega_{i-1}\Omega_i\Omega_{i+1}} \int_0^\infty h_{i+1} I_0(B_{i+1}h_{i+1})\, e^{-ph_{i+1}^2}\ dh_{i+1} \\ &\times \int_0^\infty h_{i-1} I_0(A_{i-1}h_{i-1})\, e^{-qh_{i-1}^2}\ dh_{i-1} \\ &= \frac{2\alpha h_i}{\Omega_i(1-\rho^2)^2} e^{-\left\{\frac{1-3\rho^2}{\alpha\Omega_i(1-\rho^2)}\right\}h_i^2}\ \text{, as } {}_1F_1\left(1;1;Kz^2\right) = e^{(Kz^2)} \end{aligned} \quad (10)$$

Therefore conditional distribution,

$$P(h_i|h_{i-1},h_{i+1}) = \frac{P(h_{i-1},h_i,h_{i+1})}{P(h_{i-1},h_{i+1})} = \frac{2h_i}{C} e^{\left\{-\frac{C(A_i^2+B_i^2)}{4} - \frac{h_i^2}{C}\right\}} \frac{I_0(A_ih_i)\, I_0(B_ih_i)}{I_0\left(\frac{A_iB_iC}{2}\right)} \quad (11)$$

where, $p{=}\frac{1-\rho^2}{\alpha\Omega_{i+1}}$, $q{=}\frac{1-\rho^2}{\alpha\Omega_{i-1}}$, $A_{i-1}{=}\frac{2\rho h_i}{\alpha\sqrt{\Omega_{i-1}\Omega_i}}$, $A_i{=}\frac{2\rho h_{i-1}}{\alpha\sqrt{\Omega_{i-1}\Omega_i}}$, $A_{i+1} = A_ih_i = A_{i-1}h_{i-1}$, $B_{i-1} = B_ih_i = B_{i+1}h_{i+1}$, $B_i = \frac{2\rho h_{i+1}}{\alpha\sqrt{\Omega_i\Omega_{i+1}}}$, $B_{i+1} = \frac{2\rho h_i}{\alpha\sqrt{\Omega_i\Omega_{i+1}}}$, $C = \rho\Omega_i$ and $D_i = \frac{8h_{i-1}h_{i+1}}{\alpha\Omega_{i-1}\Omega_i\Omega_{i+1}} exp\left\{-\frac{1-\rho^2}{\alpha}\left(\frac{h_{i-1}^2}{\Omega_{i-1}} + \frac{h_{i+1}^2}{\Omega_{i+1}}\right)\right\}$, and ${}_1F_1(.;.;.)$ is confluent hypergeometric function [[35], 13.1].

B. Computation of Messages in Factor Graph Model

In NFG (probability) functions are represented by nodes and variables or group of variables are represented by associated edges of the nodes. Therefore, number of computations become approximately half of general FG. Now applying the SPA as message computation rule [24], intermediate messages are computed and passed between the nodes of the graph. The message propagation follows single step natural scheduling. The desired likelihoods $P(y_1^K|s = 0)$ and $P(y_1^K|s = 1)$, are computed as marginals by executing SPA over the factor graph of Fig.3. According to SPA, as the graph has no cycle, computation of messages starts from leaf nodes ($P(h_{i-1},h_{i+1}), P(r_i)$) and half edge ($y_i$) (edge connected to only one node) and proceed from node to node. The message ($M_{P(r_i)\to r_i}$) from leaf node ($P(r_i)$) to the connecting edge (variable), r_i, is the marginal value of that function (node) w.r.t. the variable. For half edge, the message ($M_{y_i\to P(y_i|u_i,r_i)}$) from the edge to the node is initialized with 1. Every message is computed only once. First compute all incoming messages to any intermediate node. Then, multiply incoming messages with the marginal value of that node w.r.t. the variable associated with the outgoing edge (sum of the functions for all associated variables except the outgoing). It is the outgoing message from the function node to the variable edge. Following are the messages, indexed $(1,2,...,6,7)$, with dotted arrows on the corresponding edges of the graph in Fig.3. Doted arrows show the flow of the messages for computing the marginal values. Therefore intermediate and final messages for correlated channel model are shown in Appendix A.

The marginal of s on $i-$th branch, i.e. $g(s^{(i)})$, is computed from the final messages of interest as follows:

$$g(s^{(i)}) = \int_{t_i}\int_{h_i} M_{P(t_i|s,h_i)\to s} \times M_{s\to P(t_i|s,h_i)} dh_i\ dt_i = P(y_i|0)\int_0^{\tau_i} P(t_i|s)dt_i + P(y_i|1)\int_{\tau_i}^{\infty} P(t_i|s)dt_i \quad (12)$$

where, τ_i = local threshold. Therefore the desired likelihood function in cooperative spectrum sensing scenario are obtained from the marginal distributions of s and computed as the accumulated final messages (product over all the branches) over the edge s from eq.(12) as follows,

$$P(y_1^K|s) = \prod_{i=1}^{K} g(s^{(i)}) \quad (13)$$

C. Analytical Solutions for Correlated Rayleigh Channels

This section presents detail analytical explanations for computing the probability of detection, probability of false alarm and LRT statistics for local sensors as well as system level over the above correlated channel model. Given, $x_i = (h_i|h_{i-1},h_{i+1})s + w_i$, $t_i = |x_i|^2$ where, $w_i \sim CN(0,\sigma_{w_i}^2)$ and, h_i is defined previously with correlation coefficient ρ.

Now, under $H_0 : x_i = w_i$ as $s = 0$ and therefore, $x_i \sim CN(0,\sigma_{w_i}^2)$. Thus x_i is complex Gaussian under H_0. Consider N number of samples as sensing interval. We may conclude that, the unconditional pdf of t_i under H_0 follows *chi-square* distribution with $2N$ degrees of freedom [35] and corresponding CDF of t_i under H_0 with local threshold τ_i is,

$$F(t_i|H_0) = \int_{t_i<\tau_i} P(t_i|H_0)dt_i = \int_0^{\tau_i} \frac{{t_i}^{N-1}\times e^{-\frac{t_i}{2\sigma_{w_i}^2}}}{(\sigma_{w_i})^{2N}2^N\Gamma(N)}dt_i = \frac{1}{\Gamma(N)}\int_0^{\tau_i}\frac{{t_i}^{N-1}\times e^{-\frac{t_i}{2\sigma_{w_i}^2}}}{\left(2\sigma_{w_i}^2\right)^N}dt_i = \frac{\gamma(N,\frac{\tau_i}{2\sigma_{w_i}^2})}{\Gamma(N)} \quad (14)$$

where, $\gamma(N,x){=}\int_0^x t^{N-1}e^{-t}dt$ and $\Gamma(N){=}(N-1)!$ are incomplete gamma and complete gamma functions respectively [35]. The probability of false alarm may be written as, $P_{f_i} =$

$$\int_{t_i>\tau_i} P(t_i|H_0)dt_i = 1 - \int_{t_i<\tau_i} P(t_i|H_0)dt_i = 1 - \frac{\gamma(N,\frac{\tau_i}{2\sigma_{w_i}^2})}{\Gamma(N)} \quad (15)$$

According to eq.(12), message under H_0 may be written as,

$$M_{P(t_i|\vec{h_i},s)\to s} = P(y_i|u_i=0)(1-P_{f_i}) + P(y_i|u_i=1)P_{f_i} = P(y_i|1) + [P(y_i|0) - P(y_i|1)]\frac{\gamma(N,\frac{\tau_i}{2\sigma_{w_i}^2})}{\Gamma(N)} = P(y_i|H_0) \quad (16)$$

Otherwise, under $H_1 : x_i = h_is + w_i = h_i + w_k$, [setting, $s = 1$], where $P(h_i)$ follows eq.(10). The unconditional pdf of x_i under H_1 may be obtained using Bayesian approach of [37] i.e. marginalizing the conditional pdf, $P(x_i|h_i;H_1)$, of x_i over unknown parameter h_i. Therefore pdf of x_i under H_1 is computed as,

$$P(x_i|H_1) = \int_{-\infty}^{\infty} P(x_i|h_i;H_1)P(h_i)dh_i = \frac{2\alpha}{\Omega_i\sqrt{2\pi\sigma_{w_i}^2}(1-\rho^2)^2}\int_0^{\infty} h_i e^{-\left\{\frac{(h_i-x_i)^2}{2\sigma_{w_i}^2}+\frac{h_i^2}{L}\right\}} dh_i = \frac{2\sigma_{w_i}}{RS\sqrt{2\pi}} e^{-\frac{x_i^2}{2\sigma_{w_i}^2}} + \left\{1 + erf\left(\frac{x_i}{\sigma_{w_i}\sqrt{2R}}\right)\right\}\frac{x_i}{R^{3/2}S}e^{-\frac{x_i^2}{RL}} \quad (17)$$

where, $L = \frac{\alpha\Omega_i(1-\rho^2)}{1-3\rho^2}$, $R = \frac{L+2\sigma_{w_i}^2}{L}$, and $S = \frac{\Omega_i(1-\rho^2)^2}{\alpha}$. As $t_i{=}|x_i|^2$ and channel with unit power ($E[{h_i}^2] = 2\sigma_i^2 = 1$), using methods of transformation of variables and plugging $\Omega_i = 1$ in eq.(17) we get,

$$P(t_i|H_1) = \frac{1}{2\sqrt{t_i}}[P_{x_i|H_1}(\sqrt{t_i}) + P_{x_i|H_1}(-\sqrt{t_i})] = \frac{2\sigma_{w_i}}{RS\sqrt{2\pi}} t_i^{-\frac{1}{2}} e^{-\frac{t_i}{2\sigma_{w_i}^2}} + \frac{1}{R^{3/2}S}e^{-\frac{t_i}{RL}} erf\left(\frac{\sqrt{t_i}}{\sigma_{w_i}\sqrt{2R}}\right) \quad (18)$$

The CDF of t_i under H_1 and therefore P_{d_i} is obtained by integrating eq.(18) as follows,

$$
\begin{aligned}
P_{d_i} &= 1 - F(t_i|H_1) = 1 - \int_{t_i<\tau_i} P(t_i|H_1)\, dt_i = 1 - \\
&\frac{\sqrt{2}\sigma_{w_i}}{RS\sqrt{\pi}}\int_0^{\tau_i} t_i^{-\frac{1}{2}} e^{-\frac{t_i}{2\sigma_{w_i}^2}} dt_i + \frac{1}{R^{3/2}S}\int_0^{\tau_i} e^{-\frac{t_i}{RL}} erf\left(\frac{\sqrt{t_i}}{\sigma_{w_i}\sqrt{2R}}\right) dt_i \\
&= 1 - \frac{L}{S}\left\{erf\left(\frac{\sqrt{\tau_i}}{\sigma_{w_i}\sqrt{2}}\right) - \frac{1}{\sqrt{R}} e^{-\frac{\tau_i}{RL}} erf\left(\frac{\sqrt{\tau_i}}{\sigma_{w_i}\sqrt{2R}}\right)\right\} \\
&= 1 - \frac{(1-2\rho^2)^2}{(1-\rho^2)(1-3\rho^2)}\left\{\frac{\gamma(N,\frac{\tau_i}{2\sigma_{w_i}^2})}{\Gamma(N)} - \frac{1}{\sqrt{R}} e^{-\frac{\tau_i}{RL}} \frac{\gamma(N,\frac{\tau_i}{2R\sigma_{w_i}^2})}{\Gamma(N)}\right\}
\end{aligned} \tag{19}
$$

As, t_k follows eq.(18) under H_1 with $2N$ degrees of freedom and $\mathrm{erf}(x) = \frac{\gamma(\frac{1}{2},x^2)}{\Gamma(\frac{1}{2})}$ as [[35], 6.5.16]. Finally, required likelihood ratio test (LRT) statistic becomes,

$$
L(y_1^K) = \frac{P(y_1^K|H_1)}{P(y_1^K|H_0)} = \prod_{i=1}^{K} \frac{P(y_i|u_i=0)(1-P_{d_i}) + P(y_i|u_i=1)P_{d_i}}{P(y_i|u_i=0)(1-P_{f_i}) + P(y_i|u_i=1)P_{f_i}} \tag{20}
$$

Let us consider that each SU-FC channel is independent Rayleigh fading with AWGN i.e. $y_i = r_i u_i + n_i$, where $n_i \sim CN(0, \sigma_{n_i}^2)$ and r_i is Rayleigh distributed with unit power ($E[{r_i}^2] = 1$). Therefore $P(y_i|u_i = 0) = \frac{1}{\sqrt{2\pi\sigma_{n_i}^2}} e^{-\frac{{y_i}^2}{2\sigma_{n_i}^2}}$ and $P(y_i|u_i=1)$ follows eq.(17) by replacing x_i with y_i, h_i with r_i and $\sigma_{w_i}^2$ with $\sigma_{n_i}^2$. It is obvious that instantaneous CSI is not required to compute P_{d_i} (eq.(19)) and P_{f_i} (eq.(15)) , only channel statistics are required. Corresponding LRT statistic is obtained by putting those values, P_{d_i} and P_{f_i} in eq.(20). Therefore $L(y_1^K)$ depends only on channels statistics $\sigma_{w_i}^2$, $\sigma_{n_i}^2$ and ρ. If all SU's have same local threshold and experience the same channel condition, then $\sigma_i^2 = \sigma^2$, $\sigma_{w_i}^2 = \sigma_w^2$, $\sigma_{n_i}^2 = \sigma_n^2$, and we can write $P_{d_i} = P_d$, $P_{f_i} = P_f$. We are unable to provide the intermediate steps of all calculations in this paper due to the page limit.

IV. Simulation Results

In this section simulation results are presented to evaluate the proposed cooperative sensing scheme. Some of the relevant simulation parameters are given below:

Number of SUs (K) is considered as 10 and number of sensing samples (N) is 5. $\sigma_w^2 = -10dB$ for PU-SU channels; Assume Rayleigh fading with unit power i.e. $\sigma_i^2 = 0.5$; $\rho = 0.8$ as correlation coefficient between two adjacent PU-SU channels; $\sigma_n^2 = -10dB$ when SU-FC channels are AWGN or Rayleigh fading. We have also considered normalized signal power throughout our simulation. It is considered that all the SUs are experiencing same channel conditions. The receiver operating characteristic (ROC) curve is defined by probability of detection (P_d) vs probability of false alarm (P_f). We are generating correlated Rayleigh RVs of equal correlation case using the method described in [38].

Fig.4 characterizes the ROC curve of the cooperative spectrum sensing at FC for correlated PU-SU channels with frequency non-selective fading and SU-FC as independent Rayleigh fading. Average SNR is 10dB i.e. $\sigma_w^2 = \sigma_n^2 = -10dB$. The dashed curve indicates the analytical ROC curve obtained from eq.(17) and eq.(19) and the solid curve represents the Monte Carlo simulation of the same. It shows that, theoretical and Monte Carlo simulation of ROC for correlated PU-SU and independent SU-FC channels are matching.

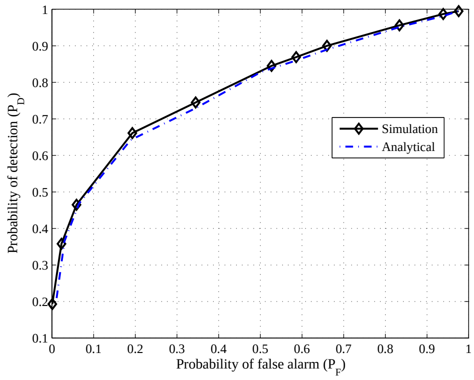

Fig. 4. ROC curves (analytical vs. simulation) of correlated Rayleigh faded PU-SU channels with independent SU-FC channels. $K = 10$, $N = 5$.

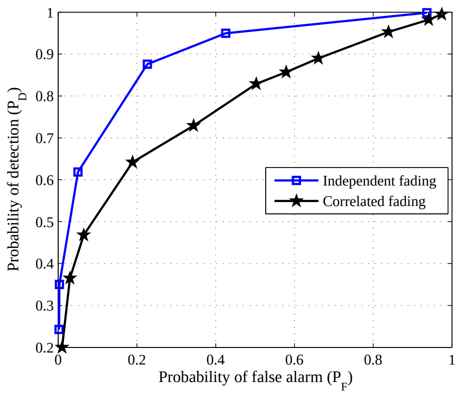

Fig. 5. Comparison of ROC curves for correlated and independent Rayleigh faded PU-SU channels with independent SU-FC channels. $K = 10$, $N = 5$.

Fig.5 represents the comparison of ROC curves between correlated and independent PU-SU Rayleigh fading channels with independent Rayleigh faded SU-FC channels. Top curve represents independent and lower one represents correlated scenario. It shows that, ROC curve for independent PU-SU channels performs better than correlated case. Therefore we may conclude that ROC performance increases as the effect of correlation decreases.

V. Conclusion

The problem of centralized cooperative spectrum sensing over correlated Rayleigh fading channels in CRN is addressed in this paper. We have presented LRT based new fusion rule which requires only statistical characteristics of the wireless channels between PU-SU-FC. The effect of correlated and independent Rayleigh fading over sensing and reporting channels are considered respectively. We have derived corresponding closed form solutions for local probabilities of detection (P_d) and false alarm (P_f). We have also derived a new trivariate Rayleigh pdf (eq.(8)) considering identical correlation between adjacent channels using Miller's [9] approach.

Factor graphs provide a natural graphical description of the factorization of a global function into product of local functions. Analysis based on SPA over normal factor graphs has been used to achieve near-optimal detection of cooperative spectrum sensing in CR environment. It is obvious that considerable amount of gain in computational complexity may be achieved and the model is suitable for complex situation.

In the present work, we have focused on correlated PU-SU channels. More complex channel conditions, e.g. both PU-SU and SU-FC channels as correlated, can be focussed as future work. Development of fusion rules for soft decisions may also be interesting future works.

APPENDIX A
MESSAGE PASSING ON FACTOR GRAPH (FIG.3)

$$
\begin{aligned}
&(1)M_{y_i\to P(y_i|u_i)} = 1 \text{ and } (7)M_{s\to P(t_i|h_i,s)} = 1\\
&(2a)M_{P(r_i)\to r_i} = M_{r_i\to P(y_i|u_i,r_i)} = P(r_i)\\
&(2b)M_{P(h_{i-1},h_{i+1})\to(h_{i-1},h_{i+1})}\\
&= M_{(h_{i-1},h_{i+1})\to P(h_i|h_{i-1},h_{i+1})} = P(h_{i-1},h_{i+1})\\
&(3)M_{P(y_i|u_i,r_i)\to u_i} = M_{u_i\to P(u_i|t_i)}\\
&= \int P(y_i|u_i,r_i)P(r_i)\ dr_i = P(y_i|u_i)\\
&(4)M_{P(u_i|t_i)\to t_i} = M_{t_i\to P(t_i|h,s)}\\
&= P(y_i|u_i=0)I(t_i<\tau_i) + P(y_i|u_i=1)I(t_i>\tau_i)\\
&(5)M_{P(h_i|h_{i-1},h_{i+1})\to h_i} = M_{h_i\to P(t_i|h_i,s)}\\
&= \int P(h_i|h_{i-1},h_{i+1})P(h_{i-1},h_{i+1})dh_{i-1}dh_{i+1} = P(h_i)\\
&(6)M_{P(t_i|h_i,s)\to s} = \int_{t_i}\int_{h_i} P(t_i|h_i,s)\times(5)\times(4)\ dh_i\ dt_i\\
&= P(h_i)\left[P(y_i|0)\int_0^{\tau_i} P(t_i|s)dt_i + P(y_i|1)\int_{\tau_i}^{\infty} P(t_i|s)dt_i\right] \qquad (21)
\end{aligned}
$$

REFERENCES

[1] V. Aalo and R. Viswanathou, "On distributed detection with correlated sensors: two examples," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 25, no. 3, pp. 414–421, 1989.

[2] E. Drakopoulos and C. Lee, "Optimum multisensor fusion of correlated local decisions," *Aerospace and Electronic Systems, IEEE Transactions on*, vol. 27, no. 4, pp. 593–606, 1991.

[3] M. Kam, Q. Zhu, and W. Gray, "Optimal data fusion of correlated local decisions in multiple sensor detection systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 3, pp. 916–920, 1992.

[4] A. Ghasemi and E. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *New Frontiers in Dynamic Spectrum Access Networks, 2005. DySPAN 2005. 2005 First IEEE International Symposium on*, 2005, pp. 131–136.

[5] S. Mishra, A. Sahai, and R. Brodersen, "Cooperative sensing among cognitive radios," in *Communications, 2006. ICC '06. IEEE International Conference on*, vol. 4, 2006, pp. 1658–1663.

[6] J. Unnikrishnan and V. V. Veeravalli, "Cooperative sensing for primary detection in cognitive radio," *IEEE J. Sel. Topics Signal Process.*, vol. 2, no. 1, pp. 18–27, Feb. 2008.

[7] A. Cacciapuoti, I. Akyildiz, and L. Paura, "Correlation-aware user selection for cooperative spectrum sensing in cognitive radio ad hoc networks," *IEEE J. Sel. Areas Com.*, vol. 30, no. 2, pp. 297–306, 2012.

[8] D. Bera, I. Chakrabarti, P. Ray, and S. Pathak, "Factor graph based cooperative spectrum sensing in CR network," in *VTC Spring*, June 2013.

[9] L. Blumenson and K. Miller, "Properties of generalized rayleigh distributions," *Ann. Math. Stat.*, pp. 903–910, 1963.

[10] FCC, "Facilitating opportunities for flexible, efficient, and reliable spectrum use employing cognitive radio technologies," ET Docket No 03-108, December 2003, www.fcc.gov.

[11] J. Mitola and G. Maguire, "Cognitive radio: making software radios more personal," *Persn Com, IEEE*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[12] D. Bera, S. Pathak, and I. Chakrabarti, "A normal factor graph approach for co-operative spectrum sensing in cognitive radio," in *Communications (NCC), 2012 National Conference on*, Feb. 2012, pp. 1–5.

[13] J. Fuhl, J.-P. Rossi, and E. Bonek, "High-resolution 3-d direction-of-arrival determination for urban mobile radio," *Antennas and Propagation, IEEE Transactions on*, vol. 45, no. 4, pp. 672–682, 1997.

[14] R. Viswanathan and P. Varshney, "Distributed detection with multiple sensors: Part-i. fundamentals," *Proceedings of the IEEE*, vol. 85, no. 1, pp. 54–63, Jan. 1997.

[15] J. Tsitsiklis, "Problems in decentralized decision making and computation," Ph.D. dissertation, Dept. Elec. Eng. Comput. Sci., Massachusetts Inst. Technology (M.I.T.), Boston, MA, 1984.

[16] A. Sahai, N. Hoven, and R. Tandra, "Some fundamental limits on cognitive radio," in *Communications, Control and Computing, in Proc. Allerton Conference on*, Oct. 2004.

[17] H. L. V. Trees, *Detection, Estimation, and Modulation Theory*. John Wiley and Sons. Inc., 1968, vol. 1.

[18] V. Aalo and R. Viswanathan, "Asymptotic performance of a distributed detection system in correlated gaussian noise," *IEEE Trans. Signal Process.*, vol. 40, no. 1, pp. 211–213, Jan. 1992.

[19] Z. Chair and P. Varshney, "Optimal data fusion in multiple sensor detection systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-22, no. 1, pp. 98–101, Jan. 1986.

[20] B. Chen, R. Jiang, T. Kasetkasem, and P. Varshney, "Channel aware decision fusion in wireless sensor networks," *IEEE Trans. Signal Process.*, vol. 52, no. 12, pp. 3454–3458, Dec. 2004.

[21] Y. An and Z. Gao, "Spatial correlation analysis between the secondary users in distributed cognitive radio networks," in *Computer Science and Network Technology (ICCSNT), 2011 International Conference on*, vol. 4, 2011, pp. 2574–2577.

[22] A. Abdi and M. Kaveh, "A space-time correlation model for multielement antenna systems in mobile fading channels," *Selected Areas in Communications, IEEE Journal on*, vol. 20, no. 3, pp. 550–560, 2002.

[23] C. M. Bishop, *Pattern Recognition and Machine Learning*. Springer.

[24] H. Wymeersch, *Iterative Receiver Design*. Cambr. Univ. Press, 2007.

[25] F. Kschischang, B. Frey, and H.-A. Loeliger, "Factor graphs and the sum-product algorithm," *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 498–519, Feb. 2001.

[26] J. M. Hammersley and P. Clifford, "Markov fields on finite graphs and lattices," *Unpublished*, 1971.

[27] F. Kschischang and B. Frey, "Iterative decoding of compound codes by probability propagation in graphical models," *IEEE J. Sel. Areas Com.*, vol. 16, no. 2, pp. 219–230, Feb. 1998.

[28] M. K. Simon and M. S. Alouini, *Digital Communication over Fading Channels*, 2nd ed. New York: Wiley, 2005.

[29] G. Karagiannidis, D. Zogas, and S. Kotsopoulos, "On the multivariate nakagami-m distribution with exponential correlation," *IEEE Trans. Commun.*, vol. 51, no. 8, pp. 1240–1244, Aug. 2003.

[30] P. J. Hagedorn, M. Smith, P. J. Bones, R. P. Millane, and D. Pairman, "A trivariate Chi-square distribution derived from the complex Wishart distribution," *Multivariate Anal., Journal on*, vol. 97, no. 3, pp. 655–674, March 2006.

[31] P. Dharmawansa, N. Rajatheva, and C. Tellambura, "Infinite series representations of the trivariate and quadrivariate nakagami-m distributions," *IEEE Trans. Wireless Commun.*, vol. 6, no. 12, pp. 4320–4328, 2007.

[32] R. Mallik, "On multivariate rayleigh and exponential distributions," *IEEE Trans. Inf. Theory*, vol. 49, no. 6, pp. 1499–1515, june 2003.

[33] S. Zarrin and T. J. Lim, "Belief propagation on factor graphs for cooperative spectrum sensing in cognitive radio," in *DySPAN 2008. 3rd IEEE Symposium on*, Oct. 2008, pp. 1–9.

[34] P. Bello, "Characterization of randomly time-variant linear channels," *Communications Systems, IEEE Transactions on*, vol. 11, no. 4, pp. 360–393, 1963.

[35] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions*, 10th ed. New York: Dover Publications, 1972.

[36] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series*, 3rd ed., ser. Special Functions. New York: Gordon and Breach Science Publishers, 1992, vol. 2.

[37] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Prentice Hall, 1998, vol. 2.

[38] N. Beaulieu and K. Hemachandra, "Novel simple representations for gaussian class multivariate distributions with generalized correlation," *IEEE Trans. Inf. Theory*, vol. 57, no. 12, pp. 8072–8083, 2011.