Einstein's field equations under conditions of D dimensions geometrized completely

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Aim: The problem of the geometrization of the Einstein field equations under conditions of D dimensions is reviewed again.
Methods: The usual tensor calculus rules were used.
Results: The stress-energy-tensor $\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ has been geometrized under conditions of D dimensions. The Ricci tensor $R_{\mu\nu}$ has been expressed completely in terms of the metric tensor $g_{\mu\nu}$ under conditions of D dimensions too. Based on the geometrized Einstein field equations under conditions of D dimensions, a mathematical formalism how to calculate the exact value the cosmological constant Λ under conditions of D dimensions has been derived.
Conclusion: The Einstein field equations under conditions of D dimensions open the door to a logically consistent unified field theory.
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1 Introduction

In point of fact, Einstein's geometrization of gravity (see Newton, 1687), the gravitational field (see Barukčić, 2011; Einstein, 1916) and the various proposals for a unified field theory (see Einstein, 1925; Weyl, 1918), "**a generalization of the theory of the gravitational field**"(see Einstein, 1950), were not yet able to produce the desired scientific success. In particular, the stress-energy momentum tensor of the electromagnetic field and Einstein's stress-energy momentum tensor of matter of the general (see Einstein, 1916; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity (GTR) are the weak spots of this theory because these fields are thus far devoid (see Goenner, 2004) of any geometrical significance. However, in order to complete the geometrization of the general theory of relativity (see Einstein, 1905) as started but not completed by Einstein himself, it is necessary to geometrize the electromagnetic field and the stress energy tensor of matter too. It is necessary to expressly clarify that the dominance of geometry in physics, of the metric tensor $g_{\mu\nu}$ and today's understanding of the gravitational field as something like the manifestation of space - time curvature has closed our scientific horizon a little bit and prevents us to much from the possibility of handing over to future (scientific) generations the description of the gravitational field (see Barukčić, 2011) by other mathematical tools (see Barukčić, 2016a, 2016c) than geometry. Generally speaking, although both, 'geometrization " and 'unification 'are not incompatible as such, both need not to be (mathematically) conceptually identical either. The deep hope and believe that a complete geometrization of the Einstein's gravitational field equations even if a delicate and fragile plant might end up at **a unified field theory** in the sense of Einstein or Weyl's and Eddington's classical field theory in which all fundamental interactions are described by objects of space-time geometry might comfort and console our tortured scientific soul.

2 Material and methods

The first confrontation between GTR and experiment occurred on May 29, 1919 by a momentous expedition at Sobral in northern Brazil (lead by Crommelin), and on the island of Príncipe off the coast of West Africa (lead by Eddington). Astronomical observations made by a special British team (Crommelin and Eddington) during the total solar eclipse occurred on May 29, 1919 provided the first empirical test of the validity of Einstein's general theory of relativity as discussed by the Royal Society of London and the Royal Astronomical Society announced at their joint meeting on the sixth of November 1919 (see Dyson et al., 1920; Thmoson, 1919). A deeper knowledge of the foundations of nature and physics as such should be able to reduce the shadows of doubts (Barukčić, 2016a).

Definitions

Definition 2.1 (Anti tensor). *Let $a_{\mu\nu}$ denote a co-variant (lower index) second-rank tensor. Let $b_{\mu\nu}$ denote another co-variant second-rank et cetera. Let $E_{\mu\nu}$ denote the sum of these co-variant second-rank tensors. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + ... \equiv E_{\mu\nu}$ be given. A co-variant second-rank anti tensor (Barukčić, 2020) of a tensor $a_{\mu\nu}$ denoted in general as $\underline{a}_{\mu\nu}$ is defined*

$$\begin{aligned} \underline{a}_{\mu\nu} &\equiv E_{\mu\nu} - a_{\mu\nu} \\ &\equiv b_{\mu\nu} + ... \end{aligned} \tag{1}$$

Let $a^{\mu\nu}$ denote a contra-variant (upper index) second-rank tensor. Let $b^{\mu\nu}$ denote another contra-variant (upper index) second-rank et cetera. Let $E^{\mu\nu}$ denote the sum of these *contra-variant (upper index) second-rank tensors. Let the relationship $a^{\mu\nu} + b^{\mu\nu} + ... \equiv E^{\mu\nu}$ be given. A co-variant second-rank anti tensor of a tensor $a^{\mu\nu}$ denoted in general as $\underline{a}^{\mu\nu}$ is defined*

$$\begin{aligned} \underline{a}^{\mu\nu} &\equiv E^{\mu\nu} - a^{\mu\nu} \\ &\equiv b^{\mu\nu} + ... \end{aligned} \tag{2}$$

Let $a_{\mu}{}^{\nu}$ denote a mixed second-rank tensor. Let $b_{\mu}{}^{\nu}$ denote another mixed second-rank et cetera. Let $E_{\mu}{}^{\nu}$ denote the sum of these mixed second-rank tensors. Let the relationship $a_{\mu}{}^{\nu} + b_{\mu}{}^{\nu} + ... \equiv E_{\mu}{}^{\nu}$ be given. A mixed second-rank anti tensor of a tensor $a_{\mu}{}^{\nu}$ denoted in general as $\underline{a}_{\mu}{}^{\nu}$ is defined

$$\begin{aligned} \underline{a}_{\mu}{}^{\nu} &\equiv E_{\mu}{}^{\nu} - a_{\mu}{}^{\nu} \\ &\equiv b_{\mu}{}^{\nu} + ... \end{aligned} \tag{3}$$

Symmetric tensors of rank 2 may represent many physical properties objective reality. A co-variant second-rank tensor $a_{\mu\nu}$ is symmetric if

$$a_{\mu\nu} \equiv a_{\nu\mu} \tag{4}$$

However, there are circumstances, where a tensor is anti-symmetric. A co-variant second-rank tensor $a_{\mu\nu}$ is anti-symmetric if

$$a_{\mu\nu} \equiv -a_{\nu\mu} \tag{5}$$

Thus far, there are circumstances were an anti-tensor is identical with an anti-symmetrical tensor.

$$a_{\mu\nu} \equiv E_{\mu\nu} - b_{\mu\nu} + ... \equiv E_{\mu\nu} - \underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{6}$$

Under conditions where $E_{\mu\nu} = 0$, an anti-tensor is identical with an anti-symmetrical tensor or it is

$$-\underline{a}_{\mu\nu} \equiv -a_{\nu\mu} \tag{7}$$

However, an anti-tensor is not identical with an anti-symmetrical tensor as such.

Definition 2.2 (Einstein's field equations). *Let $R_{\mu\nu}$ denote the Ricci tensor (Ricci & Levi-Civita, 1900) of 'Einstein's general theory of relativity'(Einstein, 1916), a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. Let R denote the Ricci scalar, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a Riemannian manifold. Ricci scalar curvature is the contraction of the Ricci tensor and is written as R without subscripts or arguments. Let Λ denote the Einstein's cosmological constant. Let $\underline{\Lambda}$ denote the "**anti cosmological constant**"(Barukčić, 2015). Let $g_{\mu\nu}$ metric tensor of Einstein's general theory of relativity. Let $G_{\mu\nu}$ denote Einstein's curvature tensor. Let $\underline{G}_{\mu\nu}$ denote the "**anti tensor**"(Barukčić, 2016c) of Einstein's curvature tensor. Let $E_{\mu\nu}$ denote stress-energy tensor of energy. Let $\underline{E}_{\mu\nu}$ denote tensor of non-energy, the anti-tensor of the stress-energy tensor of energy. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field. Let c denote the speed of the light in vacuum, let γ denote Newton's gravitational "**constant**"(Barukčić, 2014, 2015, 2016b, 2016c). Let π denote the number pi. Einstein's field equation, published by Albert Einstein (Einstein, 1915) for the first time in 1915, and finally 1916 (Einstein, 1916) but later with the "**cosmological constant**"(Einstein, 1935; Einstein & de Sitter, 1932; Einstein, 1917) term are determined as*

$$\begin{aligned} R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv E_{\mu\nu} \end{aligned} \tag{8}$$

However, the above left-hand side of the Einstein field equations represents only one part (Ricci curvature) of the geometric structure (Weyl curvature).

Definition 2.3 (Laue's scalar T). *Max von Laue (1879-1960) proposed the meanwhile so called Laue scalar (Laue, 1911) (criticised by Einstein (Einstein & Grossmann, 1913)) as the contraction of the the stress–energy momentum tensor $T_{\mu\nu}$ denoted as T and written without subscripts or arguments. Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, it is*

$$T \equiv g^{\mu\nu} \times T_{\mu\nu} \tag{9}$$

Taken Einstein seriously, $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter"(see Einstein, 1923, p. 88). In other words, "Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense."(see Einstein, 1923, p. 93)

Definition 2.4 (The entity E). *In general, we define the entity E as*

$$\begin{aligned} E &\equiv \left(\frac{8 \times \pi \times \gamma}{c^4 \times D}\right) \times T \\ &\equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \end{aligned} \tag{10}$$

*where D is the space-time dimension, where c denote the speed of the light in vacuum, γ denote Newton's gravitational "**constant**"(Barukčić, 2014, 2015, 2016b, 2016c), π is the number pi and T denote Laue's scalar.*

Definition 2.5 (**The tensor of energy and momentum $a_{\mu\nu}$ + $b_{\mu\nu}$**). *The tensor of stress-energy-momentum denoted as $E_{\mu\nu}$ is determined in detail as follows.*

$$\begin{aligned} E_{\mu\nu} &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \\ &\equiv G_{\mu\nu} + \left(\Lambda \times g_{\mu\nu}\right) \\ &\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\ &\equiv a_{\mu\nu} + b_{\mu\nu} \\ &\equiv E \times g_{\mu\nu} \end{aligned} \tag{11}$$

In our understanding, the stress-energy tensor of the electromagnetic field ($b_{\mu\nu}$) is equivalent to the portion of the stress-energy tensor of matter / energy ($E_{\mu\nu}$)due to the electromagnetic field where where $T_{\mu\nu}$ "denotes the co-variant energy tensor of matter"(see Einstein, 1923, p. 88). In other words, there is no third tensor between the stress-energy tensor of the electromagnetic field ($b_{\mu\nu}$) and the tensor of ordinary matter or matter in the narrower sense ($a_{\mu\nu}$), **a third tensor is not given, tertium non datur**! In other words, as outlined view lines before: "*Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense.* "(see Einstein, 1923, p. 93)

Electromagnetic field $b_{\mu\nu}$

Ordinary matter $a_{\mu\nu}$

Figure. Energy tensor as identity of ordinary matter and electromagnetic field.

Vranceanu (see Vranceanu, 1936) is elaborating on the same issue too. In point of fact, the energy tensor T_{kl} is treated by Vranceanu as the sum of two tensors one of which is due to the electromagnetic field ($b_{\mu\nu}$).

"On peut aussi supposer que le tenseur d'énergie T_{kl} soit la somme de deux tenseurs dont un dû au champ électromagnétique ..."(see Vranceanu, 1936)

Translated into English: '*One can also assume that the energy tensor T_{kl} be the sum of two tensors one of which is due to the electromagnetic field.*'In this context, it is necessary to make a distinction between the relationship between ordinary matter and electromagnetic field and matter and gravitational field. Matter and ordinary matter are not completely the same.

Definition 2.6 (**The tensor of non-energy**). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of non-energy or the anti tensor of the stress energy tensor is defined/derived/determined as follows:*

$$\begin{aligned} \underline{E}_{\mu\nu} &\equiv R_{\mu\nu} - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \\ &\equiv \left(\left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu}\right) \\ &\equiv c_{\mu\nu} + d_{\mu\nu} \end{aligned} \tag{12}$$

Definition 2.7 (**The anti Einstein's curvature tensor or the tensor or non-curvature**). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of non-curvature is defined/derived/determined as follows:*

$$\begin{aligned} \underline{G}_{\mu\nu} &\equiv R_{\mu\nu} - G_{\mu\nu} \\ &\equiv R_{\mu\nu} - \left(R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right)\right) \\ &\equiv \left(\frac{R}{2}\right) \times g_{\mu\nu} \\ &\equiv b_{\mu\nu} + d_{\mu\nu} \end{aligned} \tag{13}$$

Definition 2.8 (**The tensor $d_{\mu\nu}$ (neither curvature nor momentum)**). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of neither curvature nor momentum is defined/derived/determined as follows:*

$$\begin{aligned} d_{\mu\nu} &\equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - b_{\mu\nu} \\ &\equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) - c_{\mu\nu} \end{aligned} \tag{14}$$

There may exist circumstances where this tensor indicates pure vacuum, the space devoid of any matter.

Definition 2.9 (**The tensor $c_{\mu\nu}$**). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor of non-momentum and curvature is defined/derived/determined as follows:*

$$c_{\mu\nu} \equiv b_{\mu\nu} - \left(\Lambda \times g_{\mu\nu}\right) \tag{15}$$

Definition 2.10 (**The tensor $b_{\mu\nu}$**). *The co-variant stress-energy tensor of the electromagnetic field, in this context* *denoted by $b_{\mu\nu}$, is of order two and its components can be displayed by a 4 × 4 matrix too. Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the tensor $b_{\mu\nu}$ denotes the stress-energy tensor of the electromagnetic field (Hughston & Tod, 1990, p. 38) expressed more compactly and in a coordinate-independent is*

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu c}\times F_{\nu d}\times g^{cd}\right)-\left(\frac{1}{4}\times g_{\mu\nu}\times F_{de}\times F^{de}\right)\right)\right) \quad (16)$$

where F_{de} is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 2.11 (**The stress-energy tensor of ordinary matter $\mathbf{a}_{\mu\nu}$**). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the stress-energy tensor of ordinary matter $a_{\mu\nu}$ is defined/derived/determined as follows:*

$$\begin{aligned} a_{\mu\nu} &\equiv \left(\left(\frac{4\times 2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right)-b_{\mu\nu}\\ &\equiv G_{\mu\nu}+\left(\Lambda\times g_{\mu\nu}\right)-b_{\mu\nu}\\ &\equiv R_{\mu\nu}-\left(R\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)+d_{\mu\nu}\end{aligned} \quad (17)$$

or

$$\begin{aligned} a_{\mu\nu} \equiv\ & R_{\mu\nu}-\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)+\left(\Lambda\times g_{\mu\nu}\right)-\\ &\left(\frac{1}{4\times\pi}\times\left(\left(F_{\mu c}\times F_{\nu d}\times g^{cd}\right)-\left(\frac{1}{4}\times g_{\mu\nu}\times F_{de}\times F^{de}\right)\right)\right)\end{aligned} \quad (18)$$

Definition 2.12 (**The Ricci tensor $\mathbf{R}_{\mu\nu}$**). *Let $R_{\mu\nu}$ denote the Ricci tensor (Ricci & Levi-Civita, 1900) of 'Einstein's general theory of relativity'(Einstein, 1916), a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) able to measure of the degree to which a certain geometry of a given metric differs from that of ordinary Euclidean space. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field.*

$$\begin{aligned} R_{\mu\nu} &\equiv \underbrace{\left(\left(\frac{4\times 2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}\right)}_{a_{\mu\nu}+b_{\mu\nu}}+\underbrace{\left(\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right)-\left(\Lambda\times g_{\mu\nu}\right)\right)}_{c_{\mu\nu}+d_{\mu\nu}}\\ &\equiv \left(a_{\mu\nu}+b_{\mu\nu}\right)+\left(c_{\mu\nu}+d_{\mu\nu}\right)\\ &\equiv \left(a_{\mu\nu}+c_{\mu\nu}\right)+\left(b_{\mu\nu}+d_{\mu\nu}\right)\\ &\equiv \left(a_{\mu\nu}\right)+\left(+b_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}\right)\\ &\equiv \left(b_{\mu\nu}\right)+\left(+a_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}\right)\\ &\equiv \left(c_{\mu\nu}\right)+\left(+a_{\mu\nu}+b_{\mu\nu}+d_{\mu\nu}\right)\\ &\equiv \left(d_{\mu\nu}\right)+\left(+a_{\mu\nu}+b_{\mu\nu}+c_{\mu\nu}\right)\\ &\equiv a_{\mu\nu}+b_{\mu\nu}+c_{\mu\nu}+d_{\mu\nu}\\ &\equiv S\times g_{\mu\nu}\\ &\equiv \left(\frac{R}{D}\right)\times g_{\mu\nu}\end{aligned} \quad (19)$$

Remark 2.1. *In general relativity, it is common to present the Riemann and Ricci tensors by the Christoffel symbols. However, Christoffel symbols are given through the metric tensor itself. Therefore, giving the Ricci tensor while using the metric tensor explicitly, is theoretically possible.* ***Equation (19)*** *provide us with one way to present the Ricci tensor in terms of the metric tensor directly as $R_{\mu\nu} \equiv S\times g_{\mu\nu} \equiv \left(\frac{R}{D}\right)\times g_{\mu\nu}$.*

Definition 2.13 (**The Ricci scalar R**). *Under conditions of Einstein's general (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric is determined at each point in space-time by lamda Λ and anti-lamda (Barukčić, 2015) $\underline{\Lambda}$ as*

$$R \equiv g^{\mu\nu}\times R_{\mu\nu} \equiv (\Lambda)+(\underline{\Lambda}) \equiv D\times S \quad (20)$$

where D is the number of space-time dimension and $S \equiv \left(\frac{R}{D}\right)$ (lemma 3.1, equation 53). A Ricci scalar curvature R which is positive at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In contrast to this, a Ricci scalar curvature R which is negative at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general it is

$$R\times g_{\mu\nu} \equiv \left(\Lambda\times g_{\mu\nu}\right)+\left(\underline{\Lambda}\times g_{\mu\nu}\right) \quad (21)$$

The cosmological constant can also be written algebraically as part of the stress–energy tensor, a second order tensor as the source of gravity (energy density).

Table 1 provides an overview of the general definition of the relationships between the four basic (Barukčić, 2016a, 2016c) fields of nature under conditions of the general theory of relativity.

		Curvature YES	Curvature NO	
Momentum	YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$E_{\mu\nu}$
	NO	$c_{\mu\nu}$	$d_{\mu\nu}$	$\underline{E}_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$R_{\mu\nu}$

Table 1: Einstein field equations and the four basic fields of nature

Table 2 provides a more detailed overview of the definitions of the four basic (Barukčić, 2016a, 2016c) fields of nature.

	Curvature YES	Curvature NO	
Momentum YES	$a_{\mu\nu}$	$b_{\mu\nu}$	$\frac{8\times\pi\times\gamma}{c^4}\times T_{\mu\nu}$
NO	$(b_{\mu\nu} - \Lambda\times g_{\mu\nu})$	$(\frac{R}{2}\times g_{\mu\nu} - b_{\mu\nu})$	$(\frac{R}{2}\times g_{\mu\nu} - \Lambda\times g_{\mu\nu})$
	$G_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$R_{\mu\nu}$

Table 2: Einstein field equations and the four basic fields of nature

Remark 2.2. *There are manifolds where the stress–energy tensor arise entirely from an electromagnetic field with the consequence that the only source for the gravitational field is the field energy (and momentum) of the electromagnetic field. Such manifolds are characterized by the condition that $a_{\mu\nu}$ = 0 (electrovacuum solutions). However, among the many well-known exact solutions in general relativity is the lambdavacuum solution too, which is distinct from electrovacuum and vacuum solutions. In general, $T_{\mu\nu}$ contains **all** forms of energy, momentum et cetera which includes any matter present but if there is some electromagnetic radiation then it to must be included in $T_{\mu\nu}$. However, regions of space-time devoid of any matter but also of radiative energy and momentum were $T_{\mu\nu}$ = 0 (i. e. in a vacuum region) might exist. Under conditions were* $\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu} \equiv 0$ *the Einstein field equations becomes*

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + \left(\Lambda\times g_{\mu\nu}\right) \equiv \left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu} \equiv 0 \quad (22)$$

or

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + \left(\Lambda\times g_{\mu\nu}\right) \equiv 0 \quad (23)$$

Considering Ricci-flat manifolds, manifolds with a vanishing Ricci tensor, $R_{\mu\nu}$ = 0, an equivalent formulation of the relationship above follows as

$$0 - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + \left(\Lambda\times g_{\mu\nu}\right) \equiv 0 \quad (24)$$

Equation 24 simplifies as

$$\left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) \equiv \left(\Lambda\times g_{\mu\nu}\right) \quad (25)$$

Manipulating equation 25 it is

$$\left(\frac{R}{2}\right) \equiv (\Lambda) \quad (26)$$

or

$$R \equiv 2\times(\Lambda) \equiv (\Lambda) + (\Lambda) \quad (27)$$

Rearranging equation 27, it is

$$R - (\Lambda) \equiv (\Lambda) \quad (28)$$

Under conditions were $T_{\mu\nu}$ = 0 (i. e. in a vacuum region) and $R_{\mu\nu}$ = 0 (Ricci-flat manifolds), equation 28 simplifies (see theorem 3.8, equation 129) as

$$\underline{\Lambda} \equiv \Lambda \quad (29)$$

Under conditions where the Ricci scalar itself is equal to R = 0, equation 28 changes to

$$-\Lambda \equiv +\Lambda \quad (30)$$

and the state of pure symmetry, a possible state nature before the beginning of this world, is given. In other words, a nonzero cosmological constant can be positive (as in de Sitter space, named after Willem de Sitter (1872–1934) (see de Sitter, 1917a, 1917b)) or negative (as in anti-de Sitter space). Thus far, has this world developed out of the state of pure symmetry (equation 30) where

(anti-de Sitter space) = (de Sitter space)

is one among the many far-reaching questions which might follow from equation 30. Nonetheless, equation 28 changes under conditions of manifolds where $T_{\mu\nu}$= 0 and $R_{\mu\nu}$= 0. In general, manifolds where $T_{\mu\nu}$= 0 and $R_{\mu\nu}$= 0 are determined according to the definition 2.13, equation 20, by the equation

$$\underline{\Lambda} \equiv \Lambda \quad (31)$$

Rearranging equation 23 it is

$$G_{\mu\nu} + \left(\Lambda\times g_{\mu\nu}\right) \equiv 0 \quad (32)$$

or

$$G_{\mu\nu} + \equiv -\left(\Lambda \times g_{\mu\nu}\right) \tag{33}$$

An equivalent formulation of the exact lambdavacuum solutions in general relativity in terms of the Ricci tensor is given by

$$R_{\mu\nu} \equiv \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\Lambda \times g_{\mu\nu}\right) \tag{34}$$

Table 3 provides an overview of Einstein field equations and the lambdavacuum solution (Barukčić, 2016a, 2016c).

	Curvature YES	Curvature NO	
Momentum YES	0	0	0
NO	$(-\Lambda\times g_{\mu\nu})$	$(\frac{R}{2}\times g_{\mu\nu})$	$(\frac{R}{2}\times g_{\mu\nu} - \Lambda\times g_{\mu\nu})$
	$G_{\mu\nu}$	$\frac{R}{2}\times g_{\mu\nu}$	$R_{\mu\nu}$

Table 3: Einstein field equations and the lambdavacuum solution

Definition 2.14 (The metric tensor $g_{\mu\nu}$ and the inverse metric tensor $g^{\mu\nu}$). *Einstein described (local) stress-energy and momentum by a tensor* $\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}$. *In the same respect, the (local) space-time curvature has been described by Einstein as* $R_{\mu\nu} - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + \left(\Lambda\times g_{\mu\nu}\right)$. *Finally, Einstein has been of the opinion that there are conditions where (local) stress-energy and momentum and (local) space-time curvature are related to each other. In general, Einstein field equations relate (local) space-time curvature with (local) energy and momentum by the equation*

$$\underbrace{R_{\mu\nu} - \left(\left(\frac{R}{2}\right)\times g_{\mu\nu}\right) + \left(\Lambda\times g_{\mu\nu}\right)}_{(local)\ space-time\ curvature} \equiv \underbrace{\left(\frac{4\times 2\times\pi\times\gamma}{c^4}\right)\times T_{\mu\nu}}_{(local)\ energy\ and\ momentum} \tag{35}$$

The expression on the left side of Einstein field equations represents the curvature of space-time as determined by the metric while the expression on the right side of Einstein field equations represents the matter–energy content of space-time. Mathematically, it is necessary to consider circumstances that it is possible to take the trace with respect to the metric of both sides of the Einstein field equations. Therefore, we define in general

$$g_{\mu\nu} \times g^{\mu\nu} \equiv D \tag{36}$$

where D might denote the number of space-time dimensions. In point of fact, Einstein field equations (Einstein, 1916, 1935; Einstein & de Sitter, 1932; Einstein, 1915, 1917) were initially formulated by Einstein himself in the context of a four-dimensional theory even though Einstein field equations need not to break down under conditions of D space-time dimensions (see Stephani, 2003). Nonetheless, based on Einstein's statement (Einstein, 1916, p. 796), one gets

$$g_{\mu\nu} \times g^{\mu\nu} \equiv D \equiv +4 \tag{37}$$

or

$$\frac{1}{g_{\mu\nu}\times g^{\mu\nu}} \equiv \frac{1}{4} \tag{38}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other.

Einstein's point of view is that

> "... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential"
> (Einstein, 1923, p. 88)

The inverse metric tensor $g^{\mu\nu}$ is of the same size as the metric tensor $g_{\mu\nu}$. Thus far, whatever $g_{\mu\nu}$ does, $g^{\mu\nu}$ undoes and their product is the identity.

Definition 2.15 (Index raising). *For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices (Kay, 1988) raises each index. In simple words, it is*

$$F^{\binom{1\ 3}{\mu\ c}} \equiv g^{\binom{1\ 2}{\mu\ \nu}} \times g^{\binom{3\ 4}{c\ d}} \times F_{\binom{\nu\ d}{2\ 4}} \tag{39}$$

or more professionally

$$F^{\mu\, c} \equiv g^{\mu\nu} \times g^{cd} \times F_{\nu\, d} \tag{40}$$

2.1 Axioms

2.2 Axioms in general

Axioms (Hilbert, 1917) and rules which are chosen carefully can be of use to avoid logical inconsistency and equally preventing science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms (Easwaran, 2008) too.

2.2.1 Axiom I. Lex identitatis

To say that +1 is identical to +1 is to say that both are the same.

AXIOM 1. LEX IDENTITATIS.

$$+1 \equiv +1 \tag{41}$$

2.2.2 Axiom II. Lex contradictionis

AXIOM 2. LEX CONTRADICTIONIS.

$$+0 \equiv +1 \tag{42}$$

2.2.3 Axiom III. Lex negationis

AXIOM 3. LEX NEGATIONIS.

$$\neg(0) \times (+0) \equiv (+1) \tag{43}$$

where ¬ denotes the (natural/logical) process of negation.

3 Results

Theorem 3.1 (The relationship between S and the number of dimensions of space-time D)**.** *Einstein Field Equations are defined in space-time dimensions (see Málek, 2012, p. 31) other than 3+1 too.*
In general, S is given by

$$S \equiv \left(\frac{R}{D}\right) \tag{44}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{45}$$

is true, **then** the conclusion

$$S \equiv \left(\frac{R}{D}\right) \tag{46}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{47}$$

is true. Multiplying this premise by Ricci tensor $R_{\mu\nu}$ it is

$$R_{\mu\nu} \equiv R_{\mu\nu} \tag{48}$$

Our theoretical assumption is that the Ricci tensor $R_{\mu\nu}$ can be given completely through the metric tensor $g_{\mu\nu}$ in an easy and straightforward way. In general, the Ricci tensor $R_{\mu\nu}$ is completely determined by S and the metric tensor $g_{\mu\nu}$ as

$$R_{\mu\nu} \equiv S \times g_{\mu\nu} \tag{49}$$

while, at this stage, we don't know the exact value of S. Rearranging it is,

$$R_{\mu\nu} \times g^{\mu\nu} \equiv S \times g_{\mu\nu} \times g^{\mu\nu} \tag{50}$$

or in accordance to definition 2.13

$$R \equiv S \times g_{\mu\nu} \times g^{\mu\nu} \tag{51}$$

According to definition 2.14 (definition 2.14, equation 36) it is

$$R \equiv S \times D \tag{52}$$

The entity S is depending on the number of space-time dimensions D and follows as

$$S \equiv \left(\frac{R}{D}\right) \tag{53}$$

In other words, our conclusion is true.

***Quod erat demonstrandum*.**

Remark 3.1. *The complete geometrization of Einstein field equations as provided by Ilija Barukčić (see Barukčić, 2020) has been derived under conditions where the number of space-time dimensions D is equal to D = 4.*

Theorem 3.2 (The Ricci tensor $R_{\mu\nu}$ given through the metric tensor $g_{\mu\nu}$ under conditions of D space-time dimensions.)**.** *In general relativity, it is common to present the Riemann and Ricci tensors using the Christoffel symbols while Christoffel symbols are given through the metric. Mathematical formulas giving the Ricci tensor $R_{\mu\nu}$ under conditions of D space-time dimensions (see Málek, 2012, p. 31) while using explicitly the metric tensor $g_{\mu\nu}$ are missing.*
In general, the Ricci tensor $R_{\mu\nu}$ is given by the metric tensor $g_{\mu\nu}$ as

$$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \equiv S \times g_{\mu\nu} \tag{54}$$

where D is the number of space-time dimensions.

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{55}$$

is true, **then** the conclusion

$$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \tag{56}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{57}$$

is true. Multiplying this premise by the Ricci scalar R it is

$$R \equiv R \tag{58}$$

According to lemma 3.1, equation 52, the equation before (equation 58) is equivalent with

$$S \times D \equiv R \tag{59}$$

Rearranging, we obtain

$$S \equiv \frac{R}{D} \tag{60}$$

Multiplying by the metric tensor $g_{\mu\nu}$, it is

$$S \times g_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \tag{61}$$

According to lemma 3.1, equation 49 it is $R_{\mu\nu} \equiv S \times g_{\mu\nu}$ The Ricci tensor $R_{\mu\nu}$ expressed directly in terms of the metric tensor $g_{\mu\nu}$ under conditions of D space-time dimensions follows as

$$R_{\mu\nu} \equiv \frac{R}{D} \times g_{\mu\nu} \tag{62}$$

In other words, our conclusion is true.

***Quod erat demonstrandum*.**

Remark 3.2. *Einstein's general theory of relativity does not in any way privilege a particular space-time geometry. In this context, the string theory is a theoretical framework in which particles of particle physics are replaced by strings, a kind of one-dimensional objects and is treated more or less as not manifestly background independent. Under conditions of D = 1 space-time dimensions (see theorem 3.2, equation 62) it is* $R_{\mu\nu} \equiv \frac{R}{1} \times g_{\mu\nu} \equiv R \times g_{\mu\nu}$ *while the background independent Einstein field equations changes to* $\left(\frac{R}{1} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times 1}\right) \times g_{\mu\nu}$ *or to* $+\left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$*. Under conditions of D = 2 space-time dimensions (see theorem 3.2, equation 62) it is necessary to consider the possibility that* $R_{\mu\nu} \equiv \frac{R}{2} \times g_{\mu\nu}$ *while the Einstein field equations changes to* $\left(\frac{R}{2} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times 2}\right) \times g_{\mu\nu}$ *or to* $+\left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}$*. In particular, under conditions of D = 3 space-time dimensions (see theorem 3.2, equation 62) we obtain* $R_{\mu\nu} \equiv \frac{R}{3} \times g_{\mu\nu}$ *while the Einstein field equations changes to* $\left(\frac{R}{3} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times 3}\right) \times g_{\mu\nu}$*. Under conditions of original general relativty (D = 4 space-time dimensions), it is (see theorem 3.2, equation 62)* $R_{\mu\nu} \equiv \frac{R}{4} \times g_{\mu\nu}$ *while the Einstein field equations changes to* $\left(\frac{R}{4} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times 4}\right) \times g_{\mu\nu}$*. The Kaluza–Klein theory (KK theory), a historical precursor to string theory, is a kind of a classical unified field theory of gravitation and electromagnetism built around fifth dimension. Under conditions of D = 5 space-time dimensions (KK theory), it is (see theorem 3.2, equation 62)* $R_{\mu\nu} \equiv \frac{R}{5} \times g_{\mu\nu}$ *while the Einstein field equations changes to* $\left(\frac{R}{5} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times 5}\right) \times g_{\mu\nu}$*. In string theory, space-time is ten-dimensional (nine spatial dimensions, and one time dimension) and it is* $R_{\mu\nu} \equiv \frac{R}{10} \times g_{\mu\nu}$ *while the Einstein field equations changes to* $\left(\frac{R}{10} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times 10}\right) \times g_{\mu\nu}$*. In the year 1995, Edward Witten (see Witten, 1995) suggested that the five consistent versions of superstring theory (type I, type IIA, type IIB, and two versions of heterotic string theory) were just special limiting cases of an eleven-dimensional theory called M-theory. In M-theory space-time is eleven-dimensional (ten spatial dimensions, and one time dimension). Under conditions of M-theory (D = 11 space-time dimensions), it is (see theorem 3.2, equation 62)* $R_{\mu\nu} \equiv \frac{R}{11} \times g_{\mu\nu}$ *while the Einstein field equations changes to* $\left(\frac{R}{11} \times g_{\mu\nu}\right) - \left(\frac{R}{2} \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times 11}\right) \times g_{\mu\nu}$*. For now there is no end in sight on the number of space-time dimensions D and the theories associated with the same.*

Theorem 3.3 (The relationship between E and the number of dimensions of space-time D)**.** *Einstein field Equations in other space-time dimensions (see Málek, 2012, p. 31) than 3+1 need not lead to insurmountable contradictions.*
In general, it is

$$E \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{63}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{64}$$

is true, **then** the conclusion

$$E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{65}$$

is also true, the absence of any technical errors presupposed. The premise (i. e axiom)

$$(+1) = (+1) \tag{66}$$

is true. Multiplying this premise by the stress-energy momentum tensor it is

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{67}$$

We do expect that the stress-energy momentum tensor can be geometrized completely as

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv E \times g_{\mu\nu} \tag{68}$$

Rearranging it is,

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \equiv E \times g_{\mu\nu} \times g^{\mu\nu} \tag{69}$$

According to definition of Laue's scalar (definition 2.3) it is

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T \equiv E \times g_{\mu\nu} \times g^{\mu\nu} \tag{70}$$

According to definition 2.14 (definition 2.14, equation 36) it is

$$\left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4}\right) \equiv E \times D \tag{71}$$

The entity E is depending on the number of space-time dimensions D and follows as

$$E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{72}$$

In other words, our conclusion is true.

Quod erat demonstrandum.

Theorem 3.4 (The stress-energy-momentum tensor of matter under conditions of D space-time dimensions)**.** *The starting point of Einstein's theory of general relativity is that gravity as such is a property of space-time geometry. Consequently, Einstein published a geometric theory of gravitation (Einstein, 1916) while Einstein's initial hope to construct a purely geometric theory of gravitation in which even the sources of gravitation themselves would be of geometric origin has still not been fulfilled. Einstein's field equations have a source term, the stress-energy tensor of matter, radiation and vacuum et cetera, which is of order two and is still devoid of any geometry and free of any geometrical significance. In general, the completely geometrical form of the stress-energy momentum tensor of Einstein's theory of general relativity under conditions of D space-time dimensions is given by*

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{73}$$

Proof by modus ponens. **If** the premise

$$\underbrace{+1 = +1}_{(Premise)} \tag{74}$$

is true, **then** the conclusion

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{75}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) = (+1) \tag{76}$$

is true. Multiplying this premise by Einstein's stress-energy-momentum tensor of matter $\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu}$ it is

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{77}$$

Again, it is possible to express the stress-energy-momentum tensor of matter completely in terms of the metric tensor $g_{\mu\nu}$. We obtain

$$E_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv E \times g_{\mu\nu} \tag{78}$$

According to theorem 3.3, equation 72, equation 78 can be simplified. The stress-energy-momentum tensor of matter under conditions of D space-time dimensions is determined by the equation

$$E_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{79}$$

where D is the number of space-time dimensions.

Quod erat demonstrandum.

Remark 3.3. *Theorem 3.4, equation 79 demands that*

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{80}$$

This equation leads straightforward to the need that

$$T_{\mu\nu} \equiv \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{81}$$

Lemma 3.1. *It is*

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{82}$$

Simplifying equation before, the most simple geometrical form of the pure stress–energy momentum tensor $T_{\mu\nu}$ under conditions of D dimensions is determined by the equation

$$T_{\mu\nu} \equiv \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{83}$$

Quod erat demonstrandum.

In more detail, under conditions of D = 4 dimensions the the pure stress–energy momentum tensor $T_{\mu\nu}$ is determined by the metric, enriched only by view constants and a scalar as

$$\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{84}$$

However, describing the fundamental stress–energy momentum tensor $T_{\mu\nu}$, the source term of the gravitational field in Einstein's general theory of relativity, as an inherent geometrical structure, as being determined and dependent on the metric field $g_{\mu\nu}$ is associated with several and far reaching consequences. The properties of energy, momentum, mass, stress et cetera need no longer to be seen as intrinsic properties of matter. Theoretically, the properties which material systems posses could be determined in virtue of their relation to space-time structures too. The question could arise whether the energy tensor $T_{\mu\nu}$ at the end could be in different aspects less fundamental than the metric field $g_{\mu\nu}$ itself. Is and why is matter more fundamental (Lehmkuhl, 2011; Lehmkuhl, 2014) than space-time? In contrast to such a position, is the assumption justified that **without** the space-time structure encoded in the metric **no** energy tensor? To bring it to the point, can space-time (and its geometric structure) exist without matter and if yes, what kind of existence could this be? Einstein's starting point was to derive space-time structure from the properties of material systems. In contrast to this position, theorem 3.4 allow us to see that, on the contrary, the energy tensor depend on the metric field and is completely determined by the metric field. Consequently, the matter fields themselves are derivable from the structure of space-time or the very definition of an energy tensor is determined by space-time structures too. Thus far, the question is not answered definitely, which came first, either space-time structure or energy tensor.So it is reasonable to ask, is the energy-momentum tensor of matter only dependent on the structure of space-time or even determined by the structure of space-time or both or none? In other words, granddaddies **either chicken or the egg** dilemma is asking for an innovative and a comprehensive solution and may end up in an Anti-Machian theory. However, this leads us at this point too far afield.

Theorem 3.5 (Completely geometrized Einstein's field equations under conditions of D space-time dimensions)**.** *Both admirers and sceptics agree that Einstein field equations are not completely geometrized. However, a complete geometrization of Einstein field equations may go hand-in-hand with a unification of all known interactions. Therefore, we derive now a completely geometrical form of Einstein field equations (Einstein, 1916; Einstein & de Sitter, 1932; Einstein, 1917), the mathematical foundation of the generalised theory of gravitation(Einstein, 1950) under conditions of D dimensions (Moulin, 2017) as*

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \quad (85)$$

Proof by modus ponens. **If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \quad (86)$$

is true, **then** the following conclusion

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \quad (87)$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \quad (88)$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$(+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv (+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \quad (89)$$

or

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu} \quad (90)$$

Einstein worked out the relationship between curvature and momentum as presented by the definition 2.2 as

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \quad (91)$$

Taking the trace with respect to the metric of both sides of the Einstein field equations one gets

$$R_{\mu\nu} \times g^{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu} \times g^{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \quad (92)$$

Equation 92 simplifies as

$$R - \left(\left(\frac{R}{2}\right) \times D\right) + (\Lambda \times D) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T \quad (93)$$

Dividing equation 93 by D, the number of space-time dimensions, it is

$$\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \quad (94)$$

In point of fact, due to theorem 3.1, equation 53, it is $S \equiv \left(\frac{R}{D}\right)$. Substituting this relationship into the equation 94, we obtain

$$S - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \quad (95)$$

According to theorem 3.3, equation 72, it is $E \equiv \left(\frac{4 \times 2 \times \pi \times \gamma \times T}{c^4 \times D}\right)$ and equation 95 changes to

$$S - \left(\frac{R}{2}\right) + (\Lambda) \equiv E \quad (96)$$

The general geometrical form of Einstein field equation under conditions of D dimensions is obtained by multiplying equation 96 by the metric tensor $g_{\mu\nu}$ as

$$\left(S \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv E \times g_{\mu\nu} \quad (97)$$

or in more detail as

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \quad (98)$$

To bring it again to the point, our conclusion is true.

Quod erat demonstrandum.

Remark 3.4. *Einstein's general theory of relativity gave a very big boost to physics. However, there are still many and deepest problems in modern physics that remain to be solved. How does space-time develop as such? Does space-time develop from lower and more simple to higher and more complex space-time dimensions? How will space-time develop in the future? Meanwhile, various other theories entered the 'scientific market'. It is known that in M-theory (see Witten, 1998, p. 1129) space-time is 11-dimensional, while in in super-string theory (de Haro et al., 2013) it is 10-dimensional, and in bosonic string theory (de Haro et al., 2013), it is 26-dimensional. The general geometrical form of Einstein field equation under conditions of D dimensions is obtained as* $\left(S \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv E \times g_{\mu\nu}$ *and should be able to cope with these theoretical* *challenges. However, one starting point of string theory is the idea that the point-like particles of particle physics can also be described as one-dimensional objects called strings. Under conditions of one-dimension, Einsteins field equation becomes* $\left(R \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T \times g_{\mu\nu}$*. String theory as an area of current research in theoretical physics seeks to unite the current theory of very small objects (quantum mechanics) with the theory of very large objects (general relativity). Under conditions of one-dimension, it should be possible to study the nature of strings completely by the equation* $\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T \times g_{\mu\nu}$.

Theorem 3.6 (Einstein's field equations under conditions of D=2 space-time dimensions)**.** *The Einstein field equations (Einstein, 1916; Einstein & de Sitter, 1932; Einstein, 1917, 1950) simplifies under conditions of two space-time dimensions as*

$$(\Lambda) \times g_{\mu\nu} \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{99}$$

Proof by modus ponens. **If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{100}$$

is true, **then** the following conclusion

$$(\Lambda) \times g_{\mu\nu} \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{101}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{102}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$+1 \times \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \equiv +1 \times \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{103}$$

or

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{104}$$

According to theorem 3.5 (equation 98), equation 104 changes to

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu} \tag{105}$$

which is the general form of the Einstein's field equations under conditions of D dimensions. Under conditions of D=2 space-time conditions, the Einstein's field equations becomes

$$\left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{2}\right) \times g_{\mu\nu} \tag{106}$$

or

$$0 + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{2}\right) \times g_{\mu\nu} \tag{107}$$

To bring it again to the point, our conclusion that

$$(\Lambda) \times g_{\mu\nu} \equiv \left(\frac{4 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{108}$$

is true.

Quod erat demonstrandum.

Remark 3.5. *Under conditions of D=2 space-time dimensions equation 108 determines Einstein's cosmological constant as* $\Lambda \equiv \left(\frac{4 \times \pi \times \gamma}{c^4}\right) \times T$ *or something as* $\Lambda \approx T$.

Theorem 3.7 (Einstein's cosmological constant Λ)**.** *An even more severe violation of our trust into physics is created by the cosmological constant Λ, which specifies as the overall vacuum energy density. Depending on the specific assumptions made, the physical value (Weinberg, 1987) of the cosmological constant Λ is found to be very contradictory. Now, we can calculate the value of the cosmological constant Λ very precisely. In general, the value of the cosmological constant Λ is given by*

$$\Lambda \equiv \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) + \left(\frac{R}{2}\right) - \left(\frac{R}{D}\right) \tag{109}$$

Proof by modus ponens. **If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{110}$$

is true, **then** the following conclusion

$$\Lambda \equiv \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) + \left(\frac{R}{2}\right) - \left(\frac{R}{D}\right) \tag{111}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{112}$$

is true. Multiplying this premise by Einstein's stress-energy tensor of general relativity, we obtain

$$(+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv (+1) \times \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{113}$$

or

$$\left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu} \tag{114}$$

According to Einstein (definition 2.2), equation 114 changes to

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \tag{115}$$

Taking the trace with respect to the metric of both sides of the Einstein field equations one gets

$$R_{\mu\nu} \times g^{\mu\nu} - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu} \times g^{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu} \times g^{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \times g^{\mu\nu} \tag{116}$$

According to definition 2.14 (definition 2.14, equation 36), equation 116 simplifies as

$$R - \left(\left(\frac{R}{2}\right) \times D\right) + (\Lambda \times D) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T \tag{117}$$

Dividing equation 117 by D, the number of space-time dimensions, it is

$$\frac{R}{D} - \left(\frac{R}{2}\right) + (\Lambda) \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \tag{118}$$

Rearranging equation 118 yields **the exact value of Einstein's cosmological constant Λ under D dimensions** as

$$\Lambda \equiv \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) + \left(\frac{R}{2}\right) - \left(\frac{R}{D}\right) \tag{119}$$

In other words, our conclusion is true. ***Quod erat demonstrandum***.

Remark 3.6. *The most important outcome of theorem 3.7, equation 119 is the discovery that the exact value of* ***Einstein's cosmological constant Λ depends on D, the number of space-time dimensions and probably vice versa****. It is evident to all of us that theorem 3.7, equation 119 provides crystal clear and objectively provable facts that Einstein's cosmological constant Λ cannot be treated as a constant. Additionally, especially appropriate measurements and experiments may proof as true that theorem 3.7 induces more than only reasonable doubts with respect to the constancy of Einstein's cosmological constant Λ. Furthermore, one striking consequence of theorem 3.7, equation 119 is the fact that as soon as Λ is known, it is possible to calculate the number of space-time dimensions D of a manifold (theorem 3.7, equation 119).*

Theorem 3.8 (Anti cosmological constant $\underline{\Lambda}$). *The value of the anti-cosmological constant $\underline{\Lambda}$ can be calculated very precisely. In general, the value of the anti-cosmological constant $\underline{\Lambda}$ (Einstein, 1916; Einstein & de Sitter, 1932; Einstein, 1917) is given by*

$$\underline{\Lambda} \equiv \left(\frac{R}{D}\right) + \left(\frac{R}{2}\right) - \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) \tag{120}$$

Proof by modus ponens. **If** the premise of modus ponens

$$\underbrace{+1 = +1}_{(Premise)} \tag{121}$$

is true, **then** the following conclusion

$$\underline{\Lambda} \equiv \left(\frac{R}{D}\right) + \left(\frac{R}{2}\right) - \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) \tag{122}$$

is also true, again the absence of any technical errors presupposed. The premise

$$+1 \equiv +1 \tag{123}$$

is true. Multiplying this premise by Ricci scalar (see definition 2.12), we obtain

$$(+1) \times (R) \equiv (+1) \times (R) \tag{124}$$

or

$$R \equiv R \tag{125}$$

Adding Λ and subtracting Λ, the cosmological constant, it is

$$R - \Lambda + \Lambda \equiv R - \Lambda + \Lambda \tag{126}$$

or

$$R - \Lambda + \Lambda \equiv R + 0 \tag{127}$$

According to our definition 2.12 it is

$$\underline{\Lambda} + \Lambda \equiv R \tag{128}$$

and therefore

$$\underline{\Lambda} \equiv R - \Lambda \tag{129}$$

The exact value of the cosmological constant Λ under conditions of D space-time dimensions was calculated by theorem 3.7, equation 119 as $\Lambda \equiv \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) + \left(\frac{R}{2}\right) - \left(\frac{R}{D}\right)$. **The exact value of the anti cosmological constant** $\underline{\Lambda}$ can be calculated as

$$\underline{\Lambda} \equiv R - \left(\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) + \left(\frac{R}{2}\right) - \left(\frac{R}{D}\right)\right) \tag{130}$$

or as

$$\begin{aligned} \underline{\Lambda} &\equiv \left(\frac{R}{D}\right) + \left(\frac{2 \times R}{2}\right) - \left(\frac{R}{2}\right) - \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) \\ &\equiv \left(\frac{R}{D}\right) + \left(\frac{R}{2}\right) - \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \frac{T}{D}\right) \end{aligned} \tag{131}$$

with the consequence that our conclusion is true.

Quod erat demonstrandum.

4 Discussion

Having overthrown the Newtonian gravitational theory, the general theory of relativity did not enable general relativity's geometrization of gravitation to non-gravitational interactions, in particular, to electromagnetism. Among Weyl and Eddington and other, Einstein was one of the first to use explicitly the term "**unified field theory**"in the title (Einstein, 1925) of a publication in 1925. In the following, Einstein himself published more than thirty technical papers on unification of all physical interactions. However, Einstein's unified field theory program was in vain (Barukčić, 2016a, 2016c).

Further lack of clarity to geometrize all fundamental interactions and to provide a completely geometrized (Einstein, 1950) theory of relativity stemmed from the cosmological constant Λ, the energy density of space, or vacuum energy, and the uncertainties associated with the same. Although some physicists including Einstein himself initially opposed the cosmological constant Λ, today, there is some experimental evidence (Perlmutter et al. (Perlmutter et al., 1999) *Supernova Cosmology Project* and Riess et al. (Riess et al., 1998) *High-Z Supernova Search Team*) that the expansion of the universe is accelerating, implying the possibility of a positive nonzero value for the cosmological constant Λ. Considered Einstein's insight (see Einstein, 1916, p. 796) that $g_{\mu\nu} \times g^{\mu\nu} \equiv D = +4$ (definition 2.14) it was possible to geometrize the Einstein field equations under conditions of D dimensions. The Einstein field equations under conditions of D dimensions (see theorem 3.5, equation 98) might be seen as fully compliant with the relationship

$$\left(\left(\frac{R}{D}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times g_{\mu\nu}\right) + \left(\Lambda \times g_{\mu\nu}\right) \equiv \left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times \left(\frac{T}{D}\right) \times g_{\mu\nu}$$

Encouraged by this result, it was possible to calculate **the exact value of the cosmological constant Λ under conditions where** $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ (definition 2.14). However, an answer to the question whether the condition $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ is generally given may predominantly be found elsewhere. Under conditions where $g_{\mu\nu} \times g^{\mu\nu} \equiv D$ where D is the number of space-time dimensions, we are able to calculate the exact value of the cosmological constant Λ very precisely as

$$\Lambda \equiv \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right) + \left(\frac{R}{2}\right) - S$$

and much more than this. For the reasons set out above, the inevitable conclusion is that even the value of the anti cosmological constant $\underline{\Lambda}$ follows as

$$\underline{\Lambda} \equiv S + \left(\frac{R}{2}\right) - \left(\frac{8 \times \pi \times \gamma \times T}{c^4 \times D}\right).$$

5 Conclusion

In combination with other already published (Barukčić, 2016a, 2016c) papers, Einstein's general theory of relativity is completely geometrized. The theoretical value of the cosmological constant Λ and the value of the anti cosmological constant $\underline{\Lambda}$ was calculated very precisely.

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