Absolute accuracy analysis and improvement of a hybrid 6-DOF medical robot

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Abstract

Purpose – The purpose of this paper is to describe a calibration method developed to improve the accuracy of a six degrees-of-freedom medical robot. The proposed calibration approach aims to enhance the robot’s accuracy in a specific target workspace. A comparison of five observability indices is also done in order to choose the most appropriate calibration robot configurations.

Design/methodology/approach – The calibration method is based on the forward kinematic approach, which uses a nonlinear optimization model. The used experimental data are 84 end-effector positions, which are measured using a laser tracker. The calibration configurations are chosen through an observability analysis, while the validation after calibration is carried out in 336 positions within the target workspace.

Findings – Simulations allowed finding the most appropriate observability index for choosing the optimal calibration configurations. They also showed the ability of our calibration model to identify most of the considered robot’s parameters, despite measurement errors. Experimental tests confirmed the simulation findings and showed that the robot’s mean position error is reduced from 3.992 mm before calibration to 0.387 mm after, and the maximum error is reduced from 5.957 mm to 0.851 mm.

Originality/value – This paper presents a calibration method which makes it possible to accurately identify the kinematic errors for a novel medical robot. Also, this paper presents a comparison between the five observability indices proposed in literature. The proposed method might be applied to any industrial or medical robot similar to the robot studied in this paper.

Keywords Robot calibration, Kinematic calibration, Robot accuracy, Manipulator accuracy, Kinematic parameters, Observability.

Paper type Research paper.
1 Introduction

Parallel robots are often considered more precise than serial robots and have a greater load-to-weight ratio because their structure is stiffer without being bulkier (Joubair et al. 2013 a). The poor accuracy of serial robots is mainly attributed to error accumulation but serial robots offer larger workspaces. Consequently, the so-called hybrid serial-parallel manipulators are considered as a compromise solution. They offer a stiff structure and relatively large workspace. Therefore, greater repeatability is obtained but, to ensure good accuracy, the robot must be calibrated.

Two categories of robot calibration are used: kinematic and non-kinematic calibration. The first method is based on the identification of and compensation for the errors related to the robot's kinematic parameters (e.g. link lengths, active joint offsets, as well as the base and the tool frame locations). Non-kinematic calibration considers additional parameters related to other sources of errors, such as thermal and stiffness effects. For more details about the different categories of robot calibration, interested readers are referred to (Roth et al., 1987; Elatta et al., 2004; Joubair, 2012 a). In this paper only kinematic calibration is considered and is referred to here simply as robot calibration.

This paper describes a full kinematic calibration of a 6-DOF hybrid robot which is dedicated to medical applications (echography). Few research studies have been dedicated to enhancing the accuracy of medical robots. Mavroidis et al. (1998) performed a positioning error analysis on a 6-DOF manipulator, considering the kinematic and non-kinematic errors of the robot’s parameters. The robot studied was designed for the accurate positioning of patients under treatment using particle beams, and its repeatability and accuracy were evaluated at ±0.15 mm and ±5 mm, respectively, and no calibration was carried out to improve this robot's accuracy. Janvier et al. (2008) evaluated the performance of a medical robotic 3D-ultrasound imaging system, dedicated to lower limb blood vessels. The evaluated robot’s repeatability was around 0.20 mm, while the robot’s accuracy was improved to 0.57 mm and 1.6 mm, for the mean and maximum values respectively. Szep et al. (2009) analyzed, by simulation, the accuracy of a 2-DOF (RPRPR) medical parallel robot, with respect to the kinematic errors. In (Song et al., 2009), the positioning accuracy of a medical robotic system designed for spine surgery was evaluated and improved to 1.1418 mm and 0.8878 mm for the maximum and the mean values respectively. The robot’s accuracy was improved by a kinematic calibration. More recently, Torres et al., (2011) presented the calibration of an ultrasound acquisition system, in which the probe is attached to a 6-DOF serial robot. The probing of a human femur shows the capability of the system to measure the corresponding dimensions with an error of 6.0553 mm.

In all the works presented above, the obtained accuracy (i.e. the maximum position error) after calibration is higher than 1 mm, and in some works, only simulations are performed. In our work, a calibration framework is developed and applied to significantly improve the accuracy of an ultrasound imaging medical robot. This robot is expected to be remotely-controlled by a doctor or any specialized operator. For this reason, good accuracy and therefore a calibration of the robot is required. Our calibration approach is based on reducing the residual of the end-effector position errors by using forward kinematic equations (forward calibration method). The calibration configurations are optimally chosen through an observability analysis.

The observability analysis determines the most appropriate poses inside the robot workspace that provide the best identification of the robot’s parameter errors. This analysis is based on the so-called observability indices, which allow the quantifying of the efficiency of the parameters' identification process. Five observability indices (O₁, O₂, O₃, O₄ and O₅) are presented in the literature (Sun and Hollerbach, 2008; Borm and Meng, 1989; Nahvi and Hollerbach, 1996; Driels and Pathre, 1990). Comparisons of the efficiency of these indices were achieved in (Sun and Hollerbach, 2008; Joubair et al., 2013 b; Joubair et al., 2013 c), and most conclusions showed
that the most appropriate index for the kinematic calibration is $O_1$. A verification of this claim and a detailed explanation of this index are presented in the section *Observability analysis.*

The end-effector position measurements are usually carried out by means of 3D measurement devices, such as a probing CMM (Joubair *et al.*, 2013; Joubair *et al.*, 2012), an optical CMM (Nubiola *et al.*, 2013), a probing articulated arm (Joubair *et al.*, 2012), or a laser tracker (Joubair *et al.*, 2013; Xueyou and Shenghua, 2007). Some 1D measurement instruments are also used: a ballbar (Kim, 2005; Huang *et al.*, 2006; Nubiola and Bonev, 2014) or a laser interferometer (Alici and Shirinzadeh, 2003). In our measurement process we use a laser tracker having an uncertainty of ±40 µm.

This paper is organized as follows: A description of the kinematic calibration model is given in the next section, followed by a presentation of the parameter identification approach and an observability analysis, as well as an explanation of the identifiable parameters. The next sections present a simulation study followed by an experimental calibration, including an analysis of the results. Finally, conclusions are drawn in the last section.

## 2 Robot Calibration Model

### 2.1 The robot description and the main reference frames

The MedRUE robot (Medical Robot for vascular Ultrasound Examination) shown in Figure 1 is a 6-DOF hybrid serial-parallel robot with five active revolute joints and one linear guide. It is composed of two five-bar mechanisms (Figure 2) symmetrically assembled on the robot base, which in turn is fixed to a linear guide actuated through a servomotor $M_1$. We note that the two five-bar mechanisms are considered to be perfectly parallel to each other and perpendicular to the $x_B$, which is the $x$ axis of the base reference frame ($F_B$). Each five-bar mechanism $i$ ($i=1…2$) has five links: the distance $d_i$ between the anchor points of the two proximal links, and the four mobile links having $L_{ij} (j=1…4)$ as lengths. The five links connect five revolute joints ($A_i, B_i, C_i, D_i, E_i$), among which only two ($A_i$ and $C_i$) are actuated by using rotary motors $M_{i×2}$ and $M_{i×2+1}$, which involve two angles, $q_{i×2}$ and $q_{i×2+1}$, respectively. A total of five angles of active joints are considered ($q_2, …, q_6$), while the variable attributed to the linear motion is denoted $q_1$.

![Fig. 1. The MedRUE robot with the tool part.](image)
To close the kinematic chain, joints $E_1$ and $E_2$ are linked through the probe support (Figure 1), which has a universal joint at each extremity $G_i$. The $x$ coordinate of $G_i$ with respect to the base frame is denoted by $d_{i_1}$. When the robot is at its home (zero) position, the vector $\mathbf{r}_{E_1E_2}$ is supposed to be parallel to $\mathbf{r}_{G_1G_2}$ and $||\mathbf{r}_{G_1G_2}||$ is equal to $d_{i_1} + d_{i_2}$; otherwise, it will be the difference between these two distances. This difference is compensated by a passive prismatic joint located between $G_2$ and the probe support. Finally, the origin of the last reference frame $F_6$ is located midway between $G_1$ and $G_2$: its $x$ axis ($x_5$) is defined to align with $\mathbf{r}_{G_1G_2}$ and $z_6$ is pointing toward the probe center. The probe is actuated—having a rotation around $x_6$—by a small rotary actuator ($M_6$) attached to the link, having $L_{i_4}$ as its length. In our calibration process, the probe is replaced with a specially designed part on which a measurement target is attached, and the corresponding center is considered to be the origin of the tool reference frame ($F_{\text{tool}}$). We also use a world reference frame ($F_{\text{world}}$), which is associated with the robot’s work-cell.

$F_{\text{world}}$ (with axes $x_w$, $y_w$ and $z_w$) is chosen to be on the robot’s work table and to have approximately the same orientation as $F_B$, which is located on the robot’s base at $O_B$. As shown in Figure 1, the axis $x_B$ is aligned with the axis of the linear guide, and $z_B$ is normal to the plane defined by the platform of the robot’s base. The translation $\mathbf{T}_B = [x_B, y_B, z_B]^T$ of $F_B$ with respect to $F_{\text{world}}$, and the orientation $(\phi_{bx}, \phi_{by}, \phi_{bz})$, described in XYZ fixed Euler angles, are expected to be identified by the calibration process.

Knowing that the end-effector’s orientation is not used in our calibration process, therefore, only the translation $\mathbf{T}_{\text{tool}} = [x, y, z]^T$ of $F_{\text{tool}}$ with respect to the robot’s last reference frame $F_6$ is identified. The origin of $F_{\text{tool}}$ is described to be the center of the end-effector’s probe (replaced in our experiments by a measurement target), and its orientation is the same as that of $F_6$.

Fig. 2. One five-bar mechanism of the MedRUE robot.
2.2 Kinematic model of the robot

Given the vector \( \psi = [q_1, q_2, \ldots, q_6]^T \) of the active joint variables, the end-effector’s pose with respect to the world frame is represented by using homogeneous matrices as follows:

\[
A_{\text{world}}(\psi) = \begin{bmatrix} R_{\text{world}} & T_{\text{world}} \\ 0 & 1 \end{bmatrix} = A_B^B A_6^B A_{\text{tool}}^6,
\]

where,

\[
A_B^B = \begin{bmatrix} R_B & T_B \\ 0 & 1 \end{bmatrix},
\]

\[
A_6^B = \begin{bmatrix} 1 & 0 & 0 & x_i \\ 0 & 1 & 0 & y_i \\ 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix},
\]

with \( A_a^b \) denoting the homogeneous matrix representing a frame \( a \) with respect to a frame \( b \). The calculation of \( A_6^B \) is presented in the following paragraphs.

The forward kinematic equations of MedRUE are obtained by solving the direct kinematics of the two five-bar mechanisms (Figure 2); i.e., identify the position of each \( E_i \) with respect to \( F_B \), and then find the coordinates of \( O_{\text{tool}} \) (the origin of the tool reference frame) with respect to \( F_B \). Finally, the \( O_{\text{tool}} \) coordinates are transformed so that they are with respect to \( F_{\text{world}} \). Hence, the position of the last joint \( E_i \) of each five-bar mechanism is calculated with respect to its own reference frame \( F_i \) having \( O_i \) as origin, and then the obtained positions are transformed so that they are with respect to \( F_B \). The origin \( O_i \) of each five-bar mechanism is defined to be midway on \( d_i \); \( x_i \) is aligned with \( r_{CA} \) and \( z_i \) is parallel to \( x_B \).

The coordinates of \( B_i \) and \( D_i \) are expressed with respect to the local frame \( F_i \) as follows:

\[
r_{B_i} = r_{O_{A_i}} + \begin{bmatrix} L_i \cos(q_{i-2} + \delta q_{i-2}) \\ L_i \sin(q_{i-2} + \delta q_{i-2}) \end{bmatrix},
\]

\[
r_{D_i} = r_{O_{C_i}} + \begin{bmatrix} L_3 \cos(q_{i+2+1} + \delta q_{i+2+1}) \\ L_3 \sin(q_{i+2+1} + \delta q_{i+2+1}) \end{bmatrix},
\]

where \( L_i \) and \( L_3 \) are the lengths of the four swinging links as shown in Figure 2, and \( \delta q_i \) is the offset of active joint \( i \). While, the vector \( r_{O_{A_i}} \) is calculated as follows:

\[
r_{O_{A_i}} = \begin{bmatrix} \sqrt{(C_{1y} - A_{1z})^2 + (C_{1z} - A_{1z})^2} \\ 2 \end{bmatrix}^T,
\]

and

\[
r_{O_{C_i}} = -r_{O_{A_i}}.
\]
The coordinates of $E_i$ with respect to a frame $F_i$ are obtained as follows:

$$\mathbf{r}_{O,E_i} = \mathbf{r}_{O,D_i} + \mathbf{r}_{D,S_i} + \mathbf{r}_{S,E_i},$$  \hspace{1cm} (8)$$

where

$$\mathbf{r}_{D,S_i} = \frac{\mathbf{r}_{D,B}}{2} \left( \frac{L_{44}^2 - L_{52}^2}{\|\mathbf{r}_{D,B}\|^2} + 1 \right),$$  \hspace{1cm} (9)$$

$$\mathbf{r}_{S,E_i} = \sqrt{L_{44}^2 - \|\mathbf{r}_{D,S_i}\|^2} \left( \frac{\pi}{2} \right) \mathbf{r}_{D,B},$$  \hspace{1cm} (10)$$

and $\mathbf{R}_z$ is the rotation matrix around the $z$ axis, and $\mathbf{r}_{D,B}$ is the unit vector along $\mathbf{r}_{D,B}$.

The coordinates of $E_i$ (Figures 1 and 2) with respect to a frame $F_B$ are obtained as follows

$$\mathbf{r}_{O,E_i} = A_i^B \mathbf{r}_{O,E_i},$$  \hspace{1cm} (11)$$

where

$$A_i^B = \begin{bmatrix}
0 & 0 & 1 & (-1)^i d_{u_1} + q_i + \delta q_i \\
-\sin(\theta_i) & \cos(\theta_i) & 0 & \frac{A_i + C_i}{2} \\
-\cos(\theta_i) & -\sin(\theta_i) & 0 & \frac{A_i + C_i}{2} \\
0 & 0 & 0 & 1
\end{bmatrix},$$  \hspace{1cm} (12)$$

and $\theta_i$, which is the angle between $\mathbf{r}_{A_i}$ and the normal of the $x_By_B$ plane, is calculated as follows:

$$\theta_i = \operatorname{atan2}(y_{u_1} - y_{u_1}, z_{u_1} - z_{u_1}).$$  \hspace{1cm} (13)$$

The orientation of $F_6$ with respect to $F_B$ is obtained from the corresponding rotation matrix $\mathbf{R}_6^B = [\mathbf{R}_y(\gamma) \mathbf{R}_y(\beta) \mathbf{R}_z(\alpha)]$, where $\gamma$, $\alpha$ and $\beta$ are the fixed XYZ Euler angles. The angle $\gamma$ can be calculated directly as $\gamma = q_{DE_1} + q_6$, while $\alpha$ and $\beta$ are obtained from the first column of the rotation matrix, which in turn corresponds to the unit vector $\mathbf{r}_{G,G_2} = [u_x \quad u_y \quad u_z]^T$. As a result:

$$\alpha = \sin^{-1}(u_x \cos \gamma + u_z \sin \gamma),$$  \hspace{1cm} (14)$$

$$\beta = \arctan2(u_x \sin \gamma - u_z \cos \gamma, \sqrt{(\cos \alpha)^2 - (u_x \sin \gamma - u_z \cos \gamma)^2}),$$  \hspace{1cm} (15)$$

The translation of $F_6$ with respect to $F_B$ is calculated as follows:

$$\mathbf{T}_6^B = O_B E_1 + \begin{bmatrix} d_{31} - d_{41} \\ 0 \\ 0 \end{bmatrix} + \mathbf{R}(\gamma, \beta, \alpha) \begin{bmatrix} d_{41} + d_5 \\ 0 \\ 0 \end{bmatrix},$$  \hspace{1cm} (16)$$
Finally, the pose of the last joint with respect to $F_B$ is expressed as follows:

$$A^B_6 = \begin{bmatrix} R^B_p & T^B_p \\ 0 & 1 \end{bmatrix}.$$  \hfill (17)

To summarize, the considered robot’s parameters are:

- The lengths of the ten links of the two five-bar mechanisms: $L_{11}, L_{12}, L_{13}, L_{14}, L_{21}, L_{22}, L_{23}$ and $L_{24}$.
- The $y$ and $z$ coordinates of the anchor points of the two proximal links of the five-bar mechanisms: $A_{1y}, C_{1y}, A_{2y}, A_{2z}, C_{2y}$, and $C_{2z}$.
- The offsets of the six active joints: $\delta q_1, \delta q_2, \delta q_3, \delta q_4, \delta q_5, \delta q_6$.
- The offset parameters for the tool part: $d_{31}, d_{32}, d_{41}, d_{42}, d_{5}$.
- The parameters defining the base with respect to $F_{\text{world}}$: $x_b, y_b, z_b, \phi_b, \phi_{bx}$ and $\phi_{bz}$.
- The position of the tool frame with respect to the last frame: $x_t, y_t$, and $z_t$.

Eight parameters — among all 36 considered parameters — are redundant, which means that we need to reduce the number of the identifiable parameters to 28 (see section Identifiable parameters).

### 3 Parameter Identification

To identify the values of the robot’s parameters, we decided to use forward kinematic calibration, which is based on minimizing the errors of the end-effector poses. However, only positions are considered in our identification process, since they are easier to measure. To achieve efficient parameter identification, excluding the non-identifiable parameters from the identification process is appropriate. Also, the efficiency of this identification might be considerably improved through suitable selection of the calibration configurations. More details about the identification process and the most appropriate calibration configurations are given in subsequent sections.

#### 3.1 Identifiable parameters

As mentioned previously, our robot has 36 kinematic parameters. Some of these parameters are not identifiable, mainly because of their redundancy with other parameters. Therefore, the redundant parameters should be excluded from the identification process. To determine the redundant parameters, we use the identification Jacobian matrix $J$, which is composed of the derivative of the calibration poses (positions only in our case) with respect to the parameters expected to be identified (i.e. the 36 kinematic parameters):

$$J = [J_1 \ldots J_n]^T$$  \hfill (18)

where

$$J_i = \begin{bmatrix} \frac{\partial x_i(\Psi, P_{\text{nom}})}{\partial p_1} & \cdots & \frac{\partial x_i(\Psi, P_{\text{nom}})}{\partial p_n} \\ \frac{\partial y_i(\Psi, P_{\text{nom}})}{\partial p_1} & \cdots & \frac{\partial y_i(\Psi, P_{\text{nom}})}{\partial p_n} \\ \frac{\partial z_i(\Psi, P_{\text{nom}})}{\partial p_1} & \cdots & \frac{\partial z_i(\Psi, P_{\text{nom}})}{\partial p_n} \end{bmatrix}.$$  \hfill (19)
and $\Psi_i = [q_{i1}, q_{i2}, \ldots, q_{i6}]^T$ is the vector of the joint variables for the calibration configuration $i (i=1\ldots m)$, and $n$ is the total number of the kinematic parameters ($n = 36$).

The number of the independent (i.e. identifiable) parameters corresponds to the rank $r_J$ of $J$. The rank of our Jacobian matrix is 28, which means that eight parameters are not identifiable. To determine these eight parameters, we use an iterative algorithm, in which each iteration allows us to find the parameter that, if eliminated from $J$, leads to the maximum reduction of the condition number while retaining the rank $r_J$ at its initial value (i.e. $r_J = 28$). Considering that the number of columns of $J$ is denoted by $m$, the used column elimination algorithm is explained below:

While $r_J < m$, do:

For $i = 1$ to $m$

- Remove the column $i$ from $J$, and calculate both the new rank $r^*_J$ and the new condition number $c^*_J$ for the new Jacobian matrix.

- If $r^*_J = r_J$: Save $i$, $r^*_J$ and $c^*_J$ as a line in a $(m \times 3)$ result matrix $M$.

End For

- Execute an ascending sort on $M$, based on the third column (i.e. $c^*_J$). The sorted matrix is denoted by $M^*$. If the separated elimination of some parameters gives the same $c^*_J$, the priority in the sort process – for these parameters – is given randomly.

- Extract the column number $i$ from the first line of $M^*$ (The line having the lowest $r^*_J$) and remove the corresponding column from $J$. The obtained Jacobian matrix is denoted by $J^*$ and its number of columns is $m^* = m - 1$.

- Replace $J$ by $J^*$.

- Replace $m$ by $m^*$.

End while

After executing the above algorithm, which is programmed in Matlab, the found non-identifiable parameters are: $A_{21}$, $A_{22}$, $d_{31}$, $d_{32}$, $d_{41}$, $d_5$, $x_b$, $z_b$. Consequently, the corresponding columns of these parameters are removed from matrix $J$, and the number of identifiable parameters becomes $n = 28$, which correspond to the rank of $J$.

### 3.2 Target workspace and identification process

Our robot will be calibrated in a specific area of its workspace. This volume is the workspace covering the equivalent of the patient’s leg. Figure 3 shows the target workspace, which has been defined as a half-cylinder, 300 mm in diameter and 700 mm in length. Therefore, the aim of the robot calibration is to improve its accuracy inside this target workspace.
Fig. 3. The robot’s world, base and tool reference frames, with the target workspace as well as calibration positions and trajectory.

The parameters are identified by using the least-square minimization method, which minimizes the residual of the end-effector positions:

\[
\text{minimize} \sum_{i=1}^{m} \left( (x_{\text{est},i} - x_{\text{meas},i})^2 + (y_{\text{est},i} - y_{\text{meas},i})^2 + (z_{\text{est},i} - z_{\text{meas},i})^2 \right).
\]

(20)

**X**_{\text{est},i} is the vector of the estimated coordinates of the calibration configuration \(i\), calculated by using the corresponding set of joint values \(\psi_i = [q_{i1}, q_{i2}, \cdots, q_{i6}]^T\) and the parameter vector \(\mathbf{p}\). Vector \(\mathbf{p}\) is initialized with the parameters’ nominal values and is updated at each iteration of the identification algorithm. Thus, **X**_{\text{est},i} is expressed as follows:

\[
\mathbf{X}_{\text{est},i}(\psi_i, \mathbf{p}) = [x_{\text{est},i}, y_{\text{est},i}, z_{\text{est},i}]^T.
\]

(21)

Finally, **X**_{\text{meas},i} represents the measured coordinates of the position \(i\). It is expressed as the coordinates on the end-effector’s position with respect to the world reference frame.

The calibration model (equation 20) is solved by using the function \textit{lsqnonlin} from the Matlab optimization toolbox.
3.3 Observability analysis

As explained in the previous section, 28 parameters are expected to be identified in our calibration process. Theoretically, only 28 equations are needed to achieve this parameter identification. These equations can be obtained by using 10 calibration configurations, which give 30 equations, since each configuration implies three equations (for x, y and z). However, because of the measurement noise, an over-constraining of the identification system is necessary. Knowing that collecting the data for a specific position takes only a few seconds, we choose to use 84 calibration configurations.

After deciding on the number of calibration configurations, an optimal selection of these is desirable in order to achieve efficient parameter identification. Usually, selecting calibration configurations which are uniformly distributed inside the robot workspace allows good parameter identification. However, to get even better results, an optimization process — called an observability analysis — can be used to select the best configurations for the identification process. This analysis allows us to find the most suitable robot configurations in which the parameter errors are precisely identifiable.

The observability analysis uses the so-called observability indices. These indices are calculated by using the singular values $\sigma_i \ (i = 1...n)$ obtained from the diagonal matrix $\Sigma$ of the singular value decomposition (SVD) of the sensitivity matrix, which is also called the identification Jacobian matrix $J$. For $n$ independent parameters, $\Sigma$ is presented as follows:

$$
\Sigma = \begin{bmatrix}
\sigma_1 & 0 & \cdots & 0 \\
0 & \sigma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_n \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0
\end{bmatrix}
$$

(22)

Five observability indices have been presented in the literature ($O_1\ldots O_5$). According to (Sun and Hollerbach, 2008; Joubair et al., 2013 b; Joubair et al., 2013 c), the index $O_1$ seems to be the most appropriate index for the kinematic calibration. Furthermore, this claim is verified in our simulation study, and the obtained results have been experimentally confirmed. The index $O_1$ is calculated as follows:

$$
O_1 = \frac{(\sigma_1\sigma_2...\sigma_n)^{\frac{1}{n}}}{\sqrt{m}},
$$

(23)

where $m$ is the number of calibration configurations, and $\sigma_1 \ldots \sigma_n$ are the singular values of the identification matrix $J$ for the $n = 28$ identifiable parameters.

To select the appropriate calibration configurations, we use an add-subtract algorithm (Joubair et al., 2013 b; Joubair et al., 2013 c) based on the DETMAX method, which was originally presented in (Mitchell, 1974). A set of $N = 420$ configurations, uniformly distributed in the robot’s Cartesian workspace (Figure 3), is selected. The 420 configurations are considered as candidates from which the $M = 84$ calibration configurations will be chosen. The initial set of calibration configurations, denoted by $\omega_0$, is uniformly selected among the $N$ candidates and is updated at each iteration of the algorithm. The update of $\omega$ is achieved by the so-called addition and subtraction steps. The addition consists of adding the configuration among the remaining
\(N-M\) leading to the highest value of the used observability index \(O_i\), and the subtraction allows us to eliminate the configuration having the minimum contribution to the enhancement of \(O_i\). These two steps (i.e. addition and subtraction) are executed successively until the added configuration is itself removed in the following subtraction step.

The pose selection algorithm is separately executed for each of the five observability indices in order to compare their effectiveness, by examining the obtained accuracy after calibration for each index. The results of this comparison are presented in the next section and in the experimental tests. For illustration purposes, the improvement of the value of the most appropriate index \(O_1\) with respect to the successive iterations is shown in Figure 4.

![Graph](image)

**Fig. 4.** Evolution of the observability index \(O_1\) with respect to the add-subtract algorithm iterations.

## 4 Simulation study

The aim of this study is to evaluate the efficiency of our calibration process. Hence, the impact of the measurement noise on the parameter identification is evaluated, and the effectiveness of the chosen calibration configurations is also verified. Furthermore, the calibration results from a set of calibration configurations uniformly distributed inside the robot workspace are compared with the calibration configurations obtained by an observability analysis which involves the five indices presented in the literature.

For simulation purposes, the actual parameters’ values are simulated by introducing randomly generated errors into the ranges ±2 mm and ±1.5° for the distances and angles, respectively. These errors lead to poor robot accuracy that will be improved through the calibration process, by identifying the actual values of the robot’s parameters as closely as possible, despite the presence of measurement noise. The measurement errors are generated according to a normal distribution with a range of ±120 μm, which is an exaggeration of the uncertainty of our measurement instrument (i.e. a laser tracker with an uncertainty of ±40 μm). A total of 420 positions uniformly-distributed inside the robot’s workspace are used (see Figure 3). Of these, 84 positions are dedicated to the parameter identification, and the 336 remaining positions are used in the validation process.
The parameters’ values are identified as explained in the previous section (Identification process). However, the measured positions are simulated by substituting in the forward kinematic equations vector \( \mathbf{p}_{\text{act}} \) of the actual parameters’ values (Table 1) and vector 
\[
\Psi_i = [q_{i_1}, q_{i_2}, \cdots, q_{i_6}]^T
\]
corresponding to each calibration configuration. Also, as shown in equation 24, a vector \( \mathbf{E}_i \) of the simulated measurement noise is added to the corresponding position \( i \):
\[
\mathbf{X}_{\text{meas},i}(\Psi_i, \mathbf{p}_{\text{act}}) = [x_{\text{meas},i}, y_{\text{meas},i}, z_{\text{meas},i}]^T + \mathbf{E}_i,
\]  \hspace{1cm} (24)

The parameters’ nominal and actual values are presented in Table 1, as well as the identified values with and without measurement errors.

### Table 1. Results of the simulated parameter identification

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal values</th>
<th>Actual parameter values</th>
<th>Identified parameter values, without measurement errors</th>
<th>Identified parameter values, with measurement errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{1y} ) [mm]</td>
<td>-233.000</td>
<td>-232.034</td>
<td>-232.034</td>
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<td>-82.595</td>
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</tr>
<tr>
<td>( C_{1z} ) [mm]</td>
<td>438.000</td>
<td>438.250</td>
<td>438.250</td>
<td>438.110</td>
</tr>
<tr>
<td>( A_{2y} ) [mm]*</td>
<td>-233.000</td>
<td>-233.000</td>
<td>-233.000</td>
<td>-233.000</td>
</tr>
<tr>
<td>( A_{2z} ) [mm]*</td>
<td>178.000</td>
<td>178.000</td>
<td>178.000</td>
<td>178.000</td>
</tr>
<tr>
<td>( C_{2y} ) [mm]</td>
<td>-83.000</td>
<td>-82.156</td>
<td>-82.156</td>
<td>-82.602</td>
</tr>
<tr>
<td>( C_{2z} ) [mm]</td>
<td>438.000</td>
<td>436.199</td>
<td>436.199</td>
<td>436.574</td>
</tr>
<tr>
<td>( L_{11} ) [mm]</td>
<td>400.000</td>
<td>400.930</td>
<td>400.930</td>
<td>400.924</td>
</tr>
<tr>
<td>( L_{12} ) [mm]</td>
<td>520.000</td>
<td>520.221</td>
<td>520.221</td>
<td>519.831</td>
</tr>
<tr>
<td>( L_{13} ) [mm]</td>
<td>400.000</td>
<td>399.285</td>
<td>399.285</td>
<td>399.301</td>
</tr>
<tr>
<td>( L_{14} ) [mm]</td>
<td>520.000</td>
<td>520.332</td>
<td>520.332</td>
<td>519.842</td>
</tr>
<tr>
<td>( L_{21} ) [mm]</td>
<td>400.000</td>
<td>399.799</td>
<td>399.799</td>
<td>399.692</td>
</tr>
<tr>
<td>( L_{22} ) [mm]</td>
<td>520.000</td>
<td>520.664</td>
<td>520.664</td>
<td>520.835</td>
</tr>
<tr>
<td>( L_{23} ) [mm]</td>
<td>400.000</td>
<td>399.959</td>
<td>399.959</td>
<td>399.896</td>
</tr>
<tr>
<td>( L_{24} ) [mm]</td>
<td>520.000</td>
<td>520.235</td>
<td>520.235</td>
<td>520.223</td>
</tr>
<tr>
<td>( d_{31} ) [mm]*</td>
<td>166.800</td>
<td>166.800</td>
<td>166.800</td>
<td>166.800</td>
</tr>
<tr>
<td>( d_{32} ) [mm]*</td>
<td>166.800</td>
<td>166.800</td>
<td>166.800</td>
<td>166.800</td>
</tr>
<tr>
<td>( d_{41} ) [mm]*</td>
<td>41.500</td>
<td>41.500</td>
<td>41.500</td>
<td>41.500</td>
</tr>
<tr>
<td>( d_{42} ) [mm]</td>
<td>41.500</td>
<td>41.952</td>
<td>41.952</td>
<td>41.994</td>
</tr>
<tr>
<td>( d_{5} ) [mm]*</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( \delta q_1 ) [mm]</td>
<td>-0.657</td>
<td>0.102</td>
<td>0.102</td>
<td>0.060</td>
</tr>
</tbody>
</table>
After the parameters have been identified, the position errors (i.e. the absolute position accuracy) are assessed in the robot’s target workspace. As mentioned earlier, 420 positions were considered in our calibration process. Only 84 positions were used in the parameter identification, while the remaining 336 positions were dedicated to validation. The position errors are calculated for each validation position, by calculating the difference between the actual (i.e. real position) and the desired position. An actual position is calculated by substituting the corresponding joint values and the actual parameter values in equation 21, while a desired position is obtained by using the identified parameters’ values.

The parameter identification and the accuracy assessment are repeated both for the five sets of calibration configurations obtained by separately using the five observability indices ($O_1$, $O_2$, $O_3$, $O_4$ and $O_5$), and for the 84 calibration configurations uniformly-distributed in the robot’s target workspace. The composed $xyz$ position errors, before and after calibration, are summarized in Table 2, which shows that the index $O_1$ is the best observability index for our robot’s calibration. Figure 5 illustrates the histograms representing the distribution of the $xyz$ composed errors before and after calibration, by using the observability index $O_1$.

**Table 2.** Position errors, before and after calibration, with different sets of calibration configurations.

<table>
<thead>
<tr>
<th>Position errors [mm]</th>
<th>Before calibration</th>
<th>After calibration by using calibration configurations based on:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$O_1$</td>
</tr>
<tr>
<td>Mean</td>
<td>11.474</td>
<td><strong>0.014</strong></td>
</tr>
<tr>
<td>Max</td>
<td>20.168</td>
<td><strong>0.034</strong></td>
</tr>
<tr>
<td>STD</td>
<td>4.253</td>
<td><strong>0.006</strong></td>
</tr>
</tbody>
</table>
Fig. 5. Histograms of position errors in the target workspace: (a) before calibration; and (b) after calibration using the observability index $O_1$.

In summary, we can conclude that our calibration model is able to effectively identify the kinematic parameter errors and, therefore, to significantly enhance the robot’s accuracy, regardless of the measurement errors. Also, our simulation has demonstrated that the observability index $O_1$ yields the best improvement of the robot’s accuracy. However, we notice that the indices $O_2$ and $O_3$ also give results which are close to the accuracy obtained by $O_1$. Therefore, more investigations were carried out in the experimental analysis to shed more light on this matter. These will be discussed in the next section.
5 Actual calibration and validation

As mentioned earlier, MedRUE is dedicated to the ultrasound imaging of lower limb blood vessels. Therefore the target workspace has been precisely selected to replicate the area in which the patient’s leg is reached by the ultrasound probe (Figure 3).

The measurement instrument used in our calibration is a laser tracker, which has an uncertainty of ±40µm. This uncertainty is reduced in our data acquisition process by taking the measurement for each position five times, and the average of each of these five measurements is considered to be the measured value of the corresponding position. The target used for the position measurement is a 1.5 inch SMR placed on a magnetic nest, which in turn is attached to the robot’s end-effector (Figure 6). Finally, to identify the robot’s world frame with respect to the laser tracker frame, three additional nests are permanently attached to the robot’s frame.

![Fig. 6. The calibration setup.](image)

The position measurements were carried out at the same 420 configurations used in our simulation study. A total of 84 positions were used in the parameters’ identification process, which employed the minimization method presented in equation 20. The identified parameters’ values are presented in Table 3, where the non-identifiable parameters are denoted by ‘*’.
Table 3. Identified values of the robot’s parameters by using the observability index $O_1$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Nominal values</th>
<th>Identified values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{1y}$ [mm]</td>
<td>−233</td>
<td>−224.895</td>
</tr>
<tr>
<td>$A_{1z}$ [mm]</td>
<td>178</td>
<td>172.092</td>
</tr>
<tr>
<td>$C_{1y}$ [mm]</td>
<td>−83</td>
<td>−75.641</td>
</tr>
<tr>
<td>$C_{1z}$ [mm]</td>
<td>438</td>
<td>433.902</td>
</tr>
<tr>
<td>$A_{2y}$ [mm]*</td>
<td>−233</td>
<td>−233.000</td>
</tr>
<tr>
<td>$A_{2z}$ [mm]*</td>
<td>178</td>
<td>178.000</td>
</tr>
<tr>
<td>$C_{2y}$ [mm]</td>
<td>−83</td>
<td>−79.920</td>
</tr>
<tr>
<td>$C_{2z}$ [mm]</td>
<td>438</td>
<td>438.724</td>
</tr>
<tr>
<td>$L_{11}$ [mm]</td>
<td>400</td>
<td>400.342</td>
</tr>
<tr>
<td>$L_{12}$ [mm]</td>
<td>520</td>
<td>516.704</td>
</tr>
<tr>
<td>$L_{13}$ [mm]</td>
<td>400</td>
<td>401.443</td>
</tr>
<tr>
<td>$L_{14}$ [mm]</td>
<td>520</td>
<td>520.762</td>
</tr>
<tr>
<td>$L_{21}$ [mm]</td>
<td>400</td>
<td>401.732</td>
</tr>
<tr>
<td>$L_{22}$ [mm]</td>
<td>520</td>
<td>524.236</td>
</tr>
<tr>
<td>$L_{23}$ [mm]</td>
<td>400</td>
<td>401.282</td>
</tr>
<tr>
<td>$L_{24}$ [mm]</td>
<td>520</td>
<td>525.544</td>
</tr>
<tr>
<td>$d_{31}$ [mm]*</td>
<td>166.800</td>
<td>166.800</td>
</tr>
<tr>
<td>$d_{32}$ [mm]*</td>
<td>166.800</td>
<td>166.800</td>
</tr>
<tr>
<td>$d_{41}$ [mm]*</td>
<td>41.5</td>
<td>41.500</td>
</tr>
<tr>
<td>$d_{42}$ [mm]</td>
<td>41.5</td>
<td>39.195</td>
</tr>
<tr>
<td>$d_5$ [mm]*</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\delta q_1$ [mm]</td>
<td>0</td>
<td>8.521</td>
</tr>
<tr>
<td>$\delta q_2$ [º]</td>
<td>0</td>
<td>−0.464</td>
</tr>
<tr>
<td>$\delta q_3$ [º]</td>
<td>0</td>
<td>−0.351</td>
</tr>
<tr>
<td>$\delta q_4$ [º]</td>
<td>0</td>
<td>−0.433</td>
</tr>
<tr>
<td>$\delta q_5$ [º]</td>
<td>0</td>
<td>−0.313</td>
</tr>
<tr>
<td>$\delta q_6$ [º]</td>
<td>0</td>
<td>5.727</td>
</tr>
<tr>
<td>$x_b$ [mm]*</td>
<td>−109</td>
<td>−109</td>
</tr>
<tr>
<td>$y_b$ [mm]</td>
<td>−139</td>
<td>−134.354</td>
</tr>
<tr>
<td>$z_b$ [mm]*</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>$\phi_{bx}$ [º]</td>
<td>0</td>
<td>−0.568</td>
</tr>
<tr>
<td>$\phi_{by}$ [º]</td>
<td>0</td>
<td>−0.115</td>
</tr>
<tr>
<td>$\phi_{bz}$ [º]</td>
<td>0</td>
<td>0.274</td>
</tr>
<tr>
<td>$x_t$ [mm]</td>
<td>3</td>
<td>−7.402</td>
</tr>
<tr>
<td>$y_t$ [mm]</td>
<td>−55</td>
<td>−45.224</td>
</tr>
<tr>
<td>$z_t$ [mm]</td>
<td>138</td>
<td>140.046</td>
</tr>
</tbody>
</table>
The accuracy after calibration was evaluated in 336 positions (i.e. the remaining positions among the 420 initially measured). The obtained results for the robot’s accuracy (i.e. the xyz composed errors of the end-effector’s position with respect to $F_{\text{world}}$) before and after calibration are presented in Figure 7 and summarized in Table 4, while the corresponding distributions are presented as histograms in Figure 8. In order to confirm the effectiveness of the observability index $O_1$, the parameter identification was carried out separately, by using six calibration sets composed of 84 configurations each: those obtained by using the five observability indices and a set of configurations uniformly-distributed inside the target workspace. The results of this comparison are presented in Table 4. We note that the robot’s position errors were calculated in the same fashion as presented in the simulation study, the only difference being that the actual position values were measured by the laser tracker.

![Fig. 7. Position errors, before and after calibration.](image)

**Table 4.** Position $xyz$ composed errors, before and after calibration, obtained by using the six sets of calibration configurations.

<table>
<thead>
<tr>
<th>Position errors</th>
<th>Before calibration</th>
<th>After calibration by using calibration configurations based on:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$O_1$</td>
</tr>
<tr>
<td>Mean [mm]</td>
<td>3.992</td>
<td>0.387</td>
</tr>
<tr>
<td>Max [mm]</td>
<td>5.957</td>
<td>0.851</td>
</tr>
<tr>
<td>STD [mm]</td>
<td>0.639</td>
<td>0.170</td>
</tr>
</tbody>
</table>
The results summarized in Table 4 confirm that $O_1$ is the most appropriate index for calibrating our robot; it gave the best accuracy after calibration. For more details about the accuracy before and after calibration, Table 5 shows the position errors according to each coordinate separately.
Table 5. Position errors, according to \(x\), \(y\) and \(z\) coordinates, by using the observability index \(O_1\).

<table>
<thead>
<tr>
<th>Position errors</th>
<th>Before calibration</th>
<th>After calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(x)</td>
<td>(y)</td>
</tr>
<tr>
<td>Mean [mm]</td>
<td>3.159</td>
<td>1.645</td>
</tr>
<tr>
<td>Max [mm]</td>
<td>4.860</td>
<td>4.319</td>
</tr>
<tr>
<td>STD [mm]</td>
<td>0.407</td>
<td>1.044</td>
</tr>
</tbody>
</table>

By analyzing Table 5, we notice that the \(x\) coordinate makes the highest contribution to the robot’s position errors in both before and after calibration assessments. The fact that this high contribution remains even after calibration leads us to attribute these position errors to non-modelled parameter errors. Knowing that no load was applied to the robot’s end-effector, during our tests, and the temperature was controlled around 24°C in our laboratory, the only remaining and significant source of error is that of backlash. Also, the movement along the \(x\) axis is mainly achieved by the linear motor, which allows the displacement of the whole structure of the robot according to the \(x\) coordinate. As a conclusion, we can attribute the high errors on the \(x\) axis mainly to the backlash, and partially to the lead errors of the linear guide, which are specified by the manufacturer to be 50 \(\mu\)m/300 mm. The lead errors provided by the manufacturer were initially considered in the robot controller; however, an accurate assessment of these errors might only marginally enhance the accuracy along the \(x\) axis, since the displacement along this axis is only 700 mm. The compensation for the lead errors requires a more accurate measuring instrument than a laser tracker (ideally a laser interferometer).

The backlash impact cannot be compensated by the kinematic calibration. To reduce backlash errors, an improvement of the gearbox quality of the robot’s actuators is suggested (mainly the linear guide). Also, we emphasise that it is possible to model and compensate the backlash errors; it forms part of the so-called non-kinematic calibration, which is not considered in this paper.

We recall that in our calibration model, we neglected some kinematic errors, such as the angle between the links of the two five bar mechanisms and the \(y_Bz_B\) plane (i.e. the links were supposed to be perfectly perpendicular to \(x_B\)). By considering these errors, the accuracy after calibration might be better but the kinematic model would be too complicated, which in turn would significantly increase the delays of the robot’s controller calculations (e.g. trajectory generation). Besides, the obtained accuracy after calibration is sufficient for the considered application.

6 Conclusions

In this paper, we have presented a full kinematic calibration of a novel medical robot which is dedicated to the 3D ultrasound imaging of lower limb blood vessels. The identified parameters are the classical kinematic parameters (i.e. distances and link lengths), the active joint offsets, the parameters defining the tool reference frame with respect to the last joint, and the robot’s base reference frame with respect to the world frame. A simulation study showed that the used calibration model, despite measurement errors, was effective in identifying 28 among the 36 considered parameters, since eight parameters were redundant. The calibration configurations were chosen through an observability analysis, which demonstrated that the index \(O_1\) was the most appropriate observability index, allowing the best accuracy improvement of the robot. The experimental tests confirmed the effectiveness of our calibration process, by improving the robot’s mean positioning errors from 3.992 mm to 0.387 mm and the maximum positioning errors from 5.957 mm to 0.851 mm. The accuracy after calibration (i.e. 0.851 mm) is sufficient for the requirements of the application to which the calibrated robot is dedicated. Furthermore, we note...
that the improvement in accuracy was limited by the robot’s repeatability, which in turn was evaluated to be around 0.1 mm. The main reason for the repeatability errors was attributed to the backlash in the robot’s linear joint.

7  Acknowledgments

We thank the Canada Research Chairs program, the Canadian Foundation for Innovation and NSERC for funding this work.

8  References


