INT. COMM. HEAT MASS TRANSFER 0735-1933/85 $3.00 + .00
Vol. 12, pp. 3-22, 1985

HEAT TRANSFER IN AUTOMOBILE RADIATORS OF THE TUBULAR TYPE

BY

F. W. DITTUS AND L. M. K. BOELTER

CONTENTS

INTRODUCTION

Heat to be dissipated from water-cooled internal combustion engines is usually transferred to the atmosphere by means of devices commonly called radiators. The medium conveying heat to the radiator is generally water, the medium conveying heat away is air.

In this article it is intended to discuss the fundamentals involved in the transfer of heat from water to the atmosphere in the simplest type of tubular radiator. No attempt will be made to discuss the effect of the rate of heat transfer when using fins, honeycomb section, or any type other than the plain tube.

The unit of measure of heat transfer in heat exchange equipment is the "Overall Transfer Factor," which is the heat transferred per unit area of heat transmitting surface per unit time per unit of temperature difference between the hot and cold fluids.

A_m = Mean area of heat transfer section based on mean tube diameter (sq. ft.)

A_a = Area of heat transfer section on air side (sq. ft.)

A_w = Area of heat transfer section on water side (sq. ft.)

R = Ratio of outer tube surface (air) to surface of tube at mean diameter per unit length of tube

R' = Ratio of inner tube surface (water) to surface of tube at mean diameter per unit length of tube

$R = \frac{2D}{D+d}$

$R' = \frac{2d}{D+d}$

D = Outside diameter of tube (inches)

d = Inside diameter of tube (inches)

The value of U_t for a curved separating wall is:

$$U_t = \frac{4k}{(D+d)\ ln\ D/d} \tag{6}$$

Substituting equation (6) for the term U_t and also substituting in equation (5) the equivalent values of R and R', the latter becomes:

$$\frac{1}{U_o} = \left(\frac{D+d}{2}\right)\left(\frac{1}{U_a D} + \frac{ln\ D/d}{2k} + \frac{1}{U_w D}\right) \tag{7}$$

Film Transfer Factor on the Liquid Side of a Radiator

Types of fluid flow through tubes

On the liquid side of a radiator heat is carried from the warm water to the colder tube wall by two methods:

(1) convection
(2) conduction

In the region of turbulent flow, most of the heat is transferred from the liquid to the tube wall by forced convection. Because of the low thermal conductivity of fluids, very little heat is transferred from the center of the stream to the tube wall by conduction. In forced circulation systems the fluid flow through the radiator is turbulent unless the tubes are of very small diameter.

In the viscous flow region practically all of the heat is transferred from the interior of the stream to the tube wall by conduction.

The rate of heat flow from the water to the air is retarded by

(*a*) film resistance on the water side of the tube surface,
(*b*) thermal resistance of tube,
(*c*) film resistance on the air side of the tube.

If we denote the three resistances mentioned above by R_w, R_t, and R_a, respectively, we may write the following equation:

$$R_o = R_w + R_t + R_a \tag{1}$$

where R_o = overall or total heat flow resistance.

Ordinarily, however, the term employed is not thermal resistance but thermal conductance, which is the reciprocal of resistance. Denoting thermal conductance by U we may then write:

$$\frac{1}{U_o} = \frac{1}{U_w} + \frac{1}{U_t} + \frac{1}{U_a} \tag{2}$$

where

U_o = Overall transfer factor (BTU/sq.ft./°F./hr.)
U_w = Film transfer factor on water side (BTU/sq.ft./°F./hr.)
U_a = Film transfer factor on air side (BTU/sq.ft./°F./hr.)
U_t = Thermal conductance of separating wall (BTU/sq.ft./°F./hr.)

The value of U_t can be readily calculated by the use of the following equation:

$$U_t = \frac{k}{t} \tag{3}$$

where

t = Thickness of separating wall (ft.)
k = Thermal conductivity of separating wall material (BTU/sq.ft./hr./°F./ft.)

Equation (2) holds when heat is transferred through a body with parallel heat-transmitting surfaces. In the case of heat flow through curved surfaces, for example, tube walls, a correction should be made for the fact that the outer surface per unit length of tube is greater than the inner surface for the same length of tube. Equation (2) then becomes:

$$\frac{1}{U_o A_m} = \frac{1}{U_a A_a} + \frac{1}{U_t A_m} + \frac{1}{U_w A_w} \tag{4}$$

or, referred to the mean diameter of the tube, equation (4) becomes:

$$\frac{1}{U_o} = \frac{1}{U_a R} + \frac{1}{U_t} + \frac{1}{U_w R'} \tag{5}$$

where

The type of fluid flow existing within a tube may be determined by calculating "Reynolds criterion," which is defined as follows:

$$C_r = \frac{vd}{u/s} = \frac{Vd}{0.624z} \tag{8}$$

where

C_r = Reynolds criterion
C_r' = Reynolds critical number (see following paragraph)
v = mean linear velocity of fluid (ft./sec.)
V = mean mass velocity (lbs./sq. ft./sec.)
d = inside diameter of tube (inches)
u = absolute viscosity of fluid at mean stream temperature (poises)
z = absolute viscosity of fluid at mean stream temperature (centipoises)
s = density of fluid (numerically equal to the specific gravity of fluid referred to water at 60° F.) (gram./cc.)

If, upon substitution of the proper values in the above equation, the numerical result (C_r) is greater than 40, the flow is turbulent. If, on the other hand, the result is less than 25, the flow is non-turbulent or viscous. In the event that a ratio (C_r') having a value between the just mentioned numbers is obtained, the flow may be either turbulent or viscous depending to a great extent upon the entrance and exit conditions of the installation in question and the roughness of the tube surface.

Film transfer factors for turbulent flow—

Most of the experimental work done on heat transfer covers the turbulent region for fluid flow inside of tubes. McAdams and Frost (1922) correlated all the published data and proposed the following equation for heat transfer existing at turbulent flow:

$$\frac{Ud}{k} = B_1\left(\frac{vd}{u/s}\right)^{0.796} \tag{9}$$

which is a simplified form of the following equation proposed by Nusselt (1910):

$$\frac{Ud}{k} = f_1\left(\frac{vd}{u/s}\right) \cdot f_2\left(\frac{cu}{k}\right) \tag{10}$$

where B_1 = constant
c = specific heat of fluid (BTU/lb./°F.)
k = thermal conductivity of fluid (BTU/sq. ft./hr./°F./ft.), and all other terms as mentioned above.

McAdams and Frost eliminated the third term of equation (10) because the correlated data fell along the same straight line when plotted on logarithmic paper according to equation (9). Most of the data plotted were results of heat transfer tests conducted with water flowing through tubes.

Equation (9) was later modified by McAdams and Frost (1924) to include a correction for the increased heat transfer rate due to turbulence at the entrance of the tube. This modified equation is as follows:

$$\frac{Ud}{k}=B_2\left(1+\frac{N}{r}\right)\cdot f\left(\frac{vd}{u'/s}\right) \tag{11}$$

where

B_2 = constant
N = empirical number
r = ratio of tube length to diameter $=\frac{l}{d}$
u' = viscosity of fluid at film temperature (poises)

Upon considering the results of a number of experiments the equation proposed by the last mentioned authors was:

$$\frac{Ud}{k}=B_2\left(1+\frac{50}{r}\right)\left(\frac{vd}{u'/s}\right)^{0.80} \tag{12}$$

As mentioned above, in most of the experiments performed the fluid used was water and the heat was generally flowing from the tube to the liquid, i.e., heating the liquid.

Morris and Whitman (1928) conducted a series of experiments in which oils having a wide range of viscosities were used. In addition to this they studied the heat transfer rates for cooling as well as heating of the liquid flowing through the tube. The result of the investigation showed that film transfer factors may be expressed by the following equation:

$$\frac{Ud}{k}=f\left(\frac{Vd}{z}\right)\cdot\left(\frac{cz}{k}\right)^{0.37} \tag{13}$$

which is of the same form as the Nusselt equation previously mentioned, except that mass velocity (lbs./sq. ft./sec.) is used instead of linear velocity and absolute viscosity expressed in centi-poises instead of poises. The two just mentioned variables are denoted by "V" and "z" respectively. Figure 1 shows the experimental data of these investigators plotted according to equation (13). It will be noted that there are two separate groups of points, one for heating liquids and another for cooling liquids. As pointed out by Morris and Whitman, the film transfer factor for cooling a liquid is about 75 per cent of that for heating a liquid when the comparison is made at the same flow conditions.

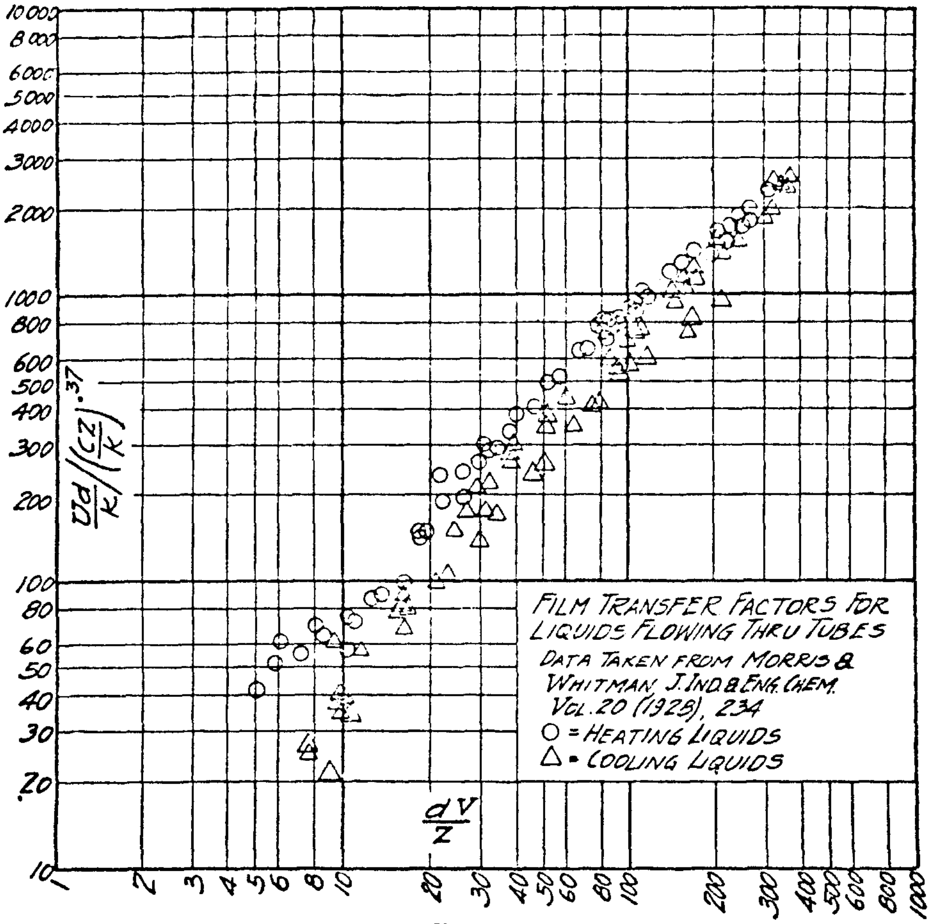

Fig. 1

This variation is no doubt due to the fact that the physical properties of the fluid particles conveying and conducting heat are different for the two conditions, even though the mean fluid temperatures are the same. Perhaps a better procedure would be to plot the film transfer factors as a function of the various thermal properties of the fluid at the film temperature instead of the mean stream temperature.

The curves obtained when using the physical properties of the fluid at the tube temperature instead of at the mean stream temperature are in no better agreement than those shown by Morris and Whitman, nor is there a better agreement when the physical properties are taken at a mean temperature between the tube wall and mean stream temperatures. In every case a separate curve was obtained for heating and cooling, in some cases the cooling curve lying above and in some cases lying below the heating curve, depending entirely upon the temperature used to determine the physical properties of the liquid.

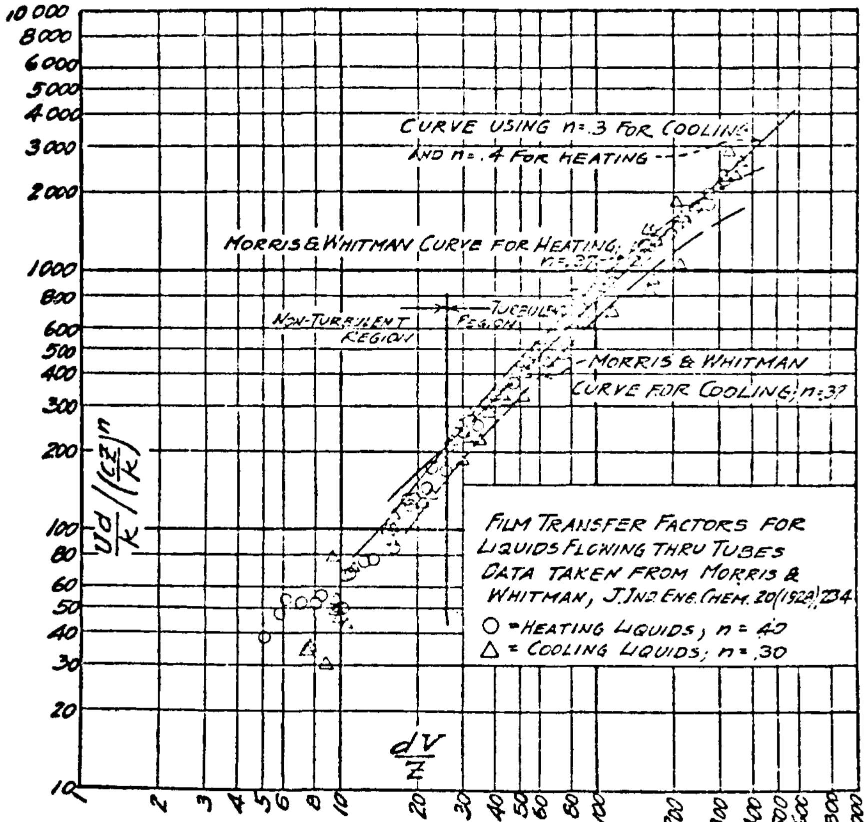

Fig. 2

In order to obtain a common curve for heating and cooling, it is suggested to use two different exponents in the term $\left(\frac{cz}{k}\right)^n$ for each process. Figure 2 shows the plotted results calculated from Morris and Whitman's published data, using n equals 0.4 and 0.3 respectively for heating and cooling a liquid flowing to a tube. Unfortunately, no other data are available to test the use of two different exponents for heating and cooling.

The fluids used by Morris and Whitman in their experiments were water and oils covering a considerable range of viscosities. Neither these authors nor McAdams and Frost showed any experimental values for gases flowing through tubes.

In order to determine whether or not the Morris and Whitman curve also applies to gases, the published results of a number of investigators using gases in their heat transfer experiments were analyzed and plotted according to the following equations:

$$\left(\frac{Ud}{k}\right)=f\left(\frac{Vd}{z}\right)\cdot\left(\frac{cz}{k}\right)^n \tag{14}$$

where

$n=0.3$ for cooling

$n=0.4$ for heating

all other variables as previously defined.

The curves thus obtained for gases are shown in figure 3 together with others published by McAdams and Frost for liquids. The curves shown for gas flow cover a range of tube diameters from ½ inch to about 6 inches and a temperature range from 60° F. to 1400° F. The mass velocities varied from 0.2 to 6.6 lbs. per sq. ft. per second. The pressure within the pipe varied from 1.5 to 190 pounds per sq. inch absolute. Considering this wide variation in operating conditions, the agreement of the curves is remarkable, although it is somewhat difficult to draw a mean curve.

The mean curve shown on figure 3 may be expressed by the following equation:

$$\frac{Ud}{k}=19.5\left(\frac{dV}{z}\right)^{.08}\left(\frac{cz}{k}\right)^n \tag{15}$$

where n is the 0.3 and 0.4 for cooling and heating, respectively, as mentioned above.

The entrance correction factor $\left(1+\frac{50}{r}\right)$ proposed by McAdams and Frost was omitted by Morris and Whitman in the equation proposed by the latter. The reason for the omission is that not sufficient data are available definitely to determine the end correction factor. For the same reason the factor is also omitted from equation (15).

Film transfer factor for viscous or non-turbulent flow—

Very little data have been published on heat transfer for viscous flow. McAdams (1925) published a curve having a slope of about 1/10 when the variable $\left(\frac{Ud}{k}\right)\Big/\left(1+\frac{50}{r}\right)$ is plotted against $\left(\frac{Vd}{z}\right)$ or, in other words,

$$\frac{Ud}{k}=B_3\left(\frac{Vd}{z}\right)^{1/10}\cdot\left(1+\frac{50}{r}\right) \tag{16}$$

No experimental data were shown to support this equation.

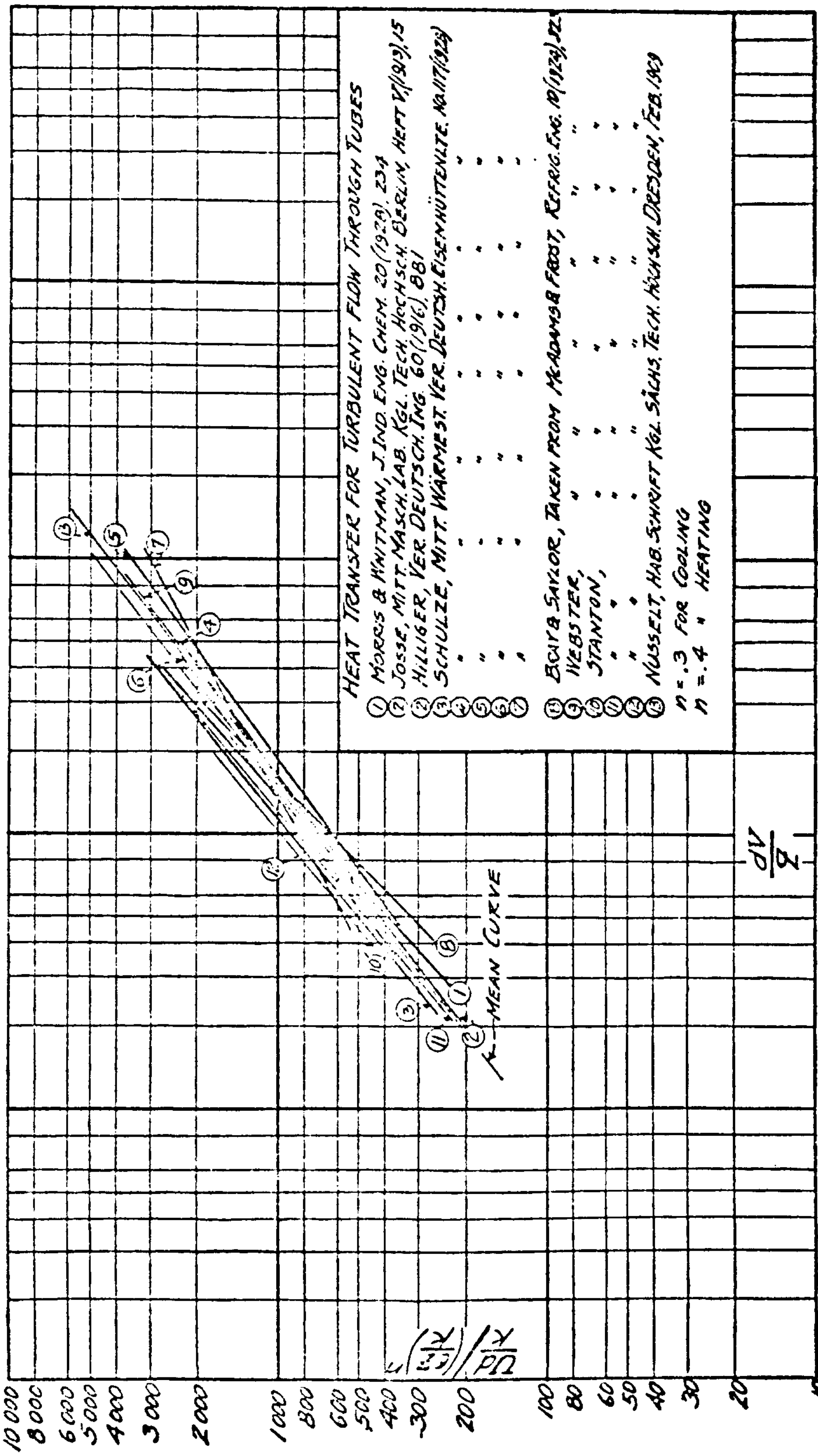
HEAT TRANSFER FOR TURBULENT FLOW THROUGH TUBES
MORRIS & WHITMAN, J. IND. ENG. CHEM. 20 (1928), 234
JOSSE, MITT. MASCH. LAB. KGL. TECH. HOCHSCH. BERLIN, HEFT V (1909), 15
HILLIGER, VER. DEUTSCH. ING. 60 (1916), 881
SCHULZE, MITT. WÄRMEST. VER. DEUTSH. EISENHÜTTENLTE. No. 117 (1928)
BOAY & SAYLOR, TAKEN FROM McADAMS & FROST, REFRIG. ENG. 10 (1924), 32
WEBSTER,
STANTON,
NUSSELT, HAB. SCHRIFT KGL. SÄCHS. TECH. HOCHSCH. DRESDEN, FEB. 1909
n = .3 FOR COOLING
n = .4 " HEATING
MEAN CURVE

Fig. 3

Some experiments were conducted by Dittus at the University of California (1929) on the heat transfer from tubes to liquids in viscous motion. It was found upon plotting the experimental results according to the equation $\frac{Ud}{k}=f\left(\frac{Vd}{z}\right)$ that the latter equation did not hold, but separate curves were obtained, each of which had a slope of roughly 1/12, for each diameter tube tested. It was apparent that some additional variables should be included in the above equation in order to obtain a satisfactory curve. When the experimental data were plotted according to Nusselt's equation (10), the results again did not fall along the same curve. Upon including the additional variables, specific heat and temperature difference, the following equation was derived by the application of the principle of dimensional homogeneity:

$$\frac{Ud}{k}=f\left[\left(\frac{vd}{u/s}\right),\left(\frac{dvsc}{k}\right),\left(\frac{k\Delta}{dv^3s}\right)\right] \tag{17}$$

where Δ = logarithmic mean temperature difference between tube and liquid, and all other terms are the same as mentioned before.

By combining the variables in equation (17) in a particular manner, it was possible to derive a new group of variables, which caused all experimental points to fall along the same curve when plotted accordingly.

The final equation proposed is as follows:

$$U=20\left(\frac{k\Delta}{d}\right)^{1/3}\cdot s^{2/3}\cdot c\cdot\left(\frac{vd}{u/s}\right)^{1/12} \tag{18}$$

Further information on heat transfer in the viscous or non-turbulent flow region is available in an article published by Thomas and Wadlow (1929) on the heat transfer performance of tubular radiators for automobiles. These investigators determined the overall transfer from water to air by maintaining a constant water velocity and varying the air velocity in one set of experiments and reversing the procedure in another set of experiments. These tests covered both the turbulent and the non-turbulent or viscous flow on the water side of the radiator tubes. Because of the method employed of varying only one of either the air or water velocity, it is possible to calculate the film transfer factors from the overall transfer factors. The results are shown in figure 4.

It will be noted upon reviewing figure 4 that results calculated from Thomas and Wadlow's data do not agree with the tests of Dittus. The reason for this may be that in the latter tests every effort was

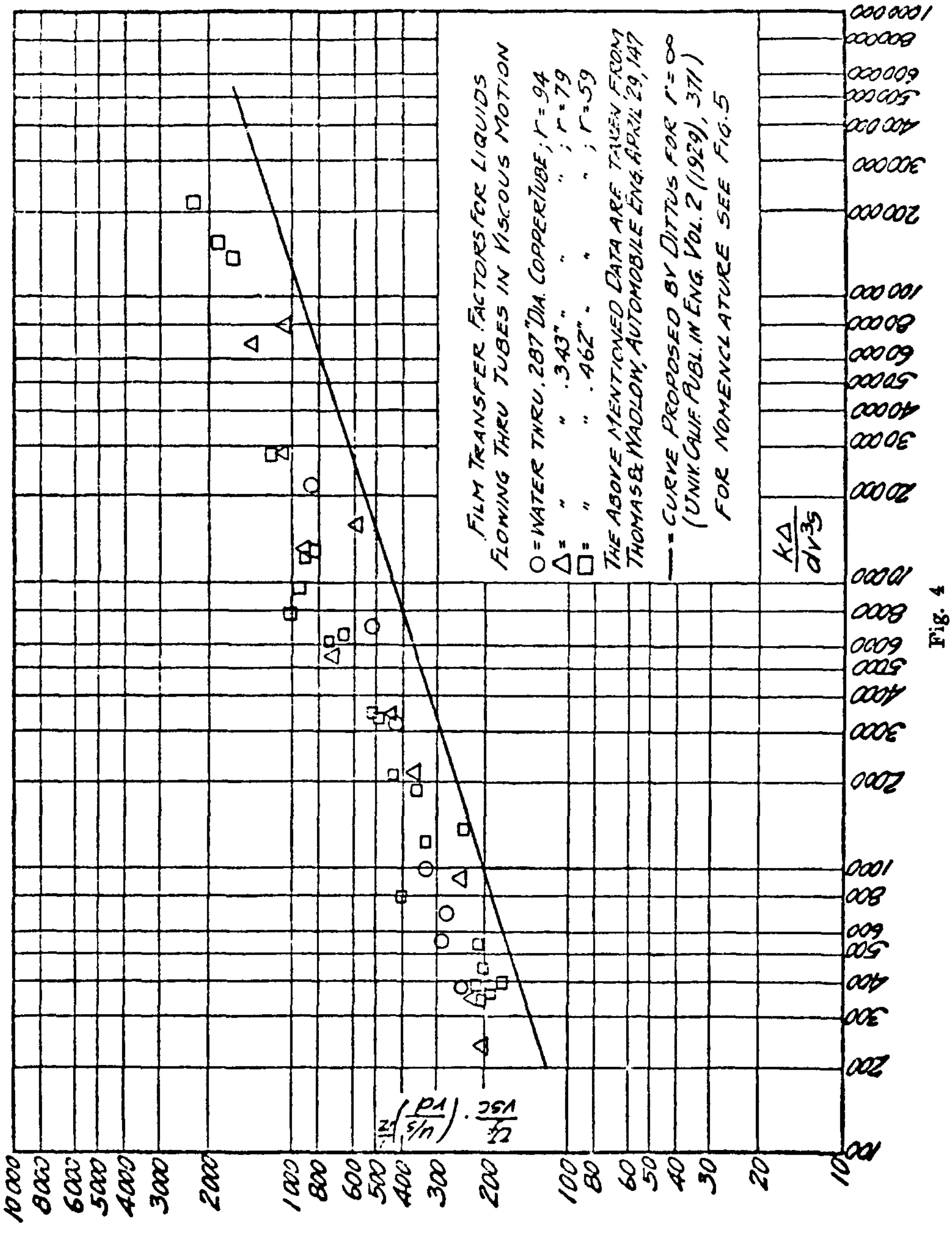
FILM TRANSFER FACTORS FOR LIQUIDS
FLOWING THRU TUBES IN VISCOUS MOTION
○ = WATER THRU .287" DIA. COPPER TUBE; r = 94
△ = " " .343" " ; r = 79
□ = " " .462" " ; r = 59
THE ABOVE MENTIONED DATA ARE TAKEN FROM
THOMAS & WADLOW, AUTOMOBILE ENG. APRIL '29, 147
— = CURVE PROPOSED BY DITTUS FOR r = ∞
(UNIV. CALIF. PUBL. IN ENG. VOL. 2 (1929), 371)
FOR NOMENCLATURE SEE FIG. 5

Fig. 4

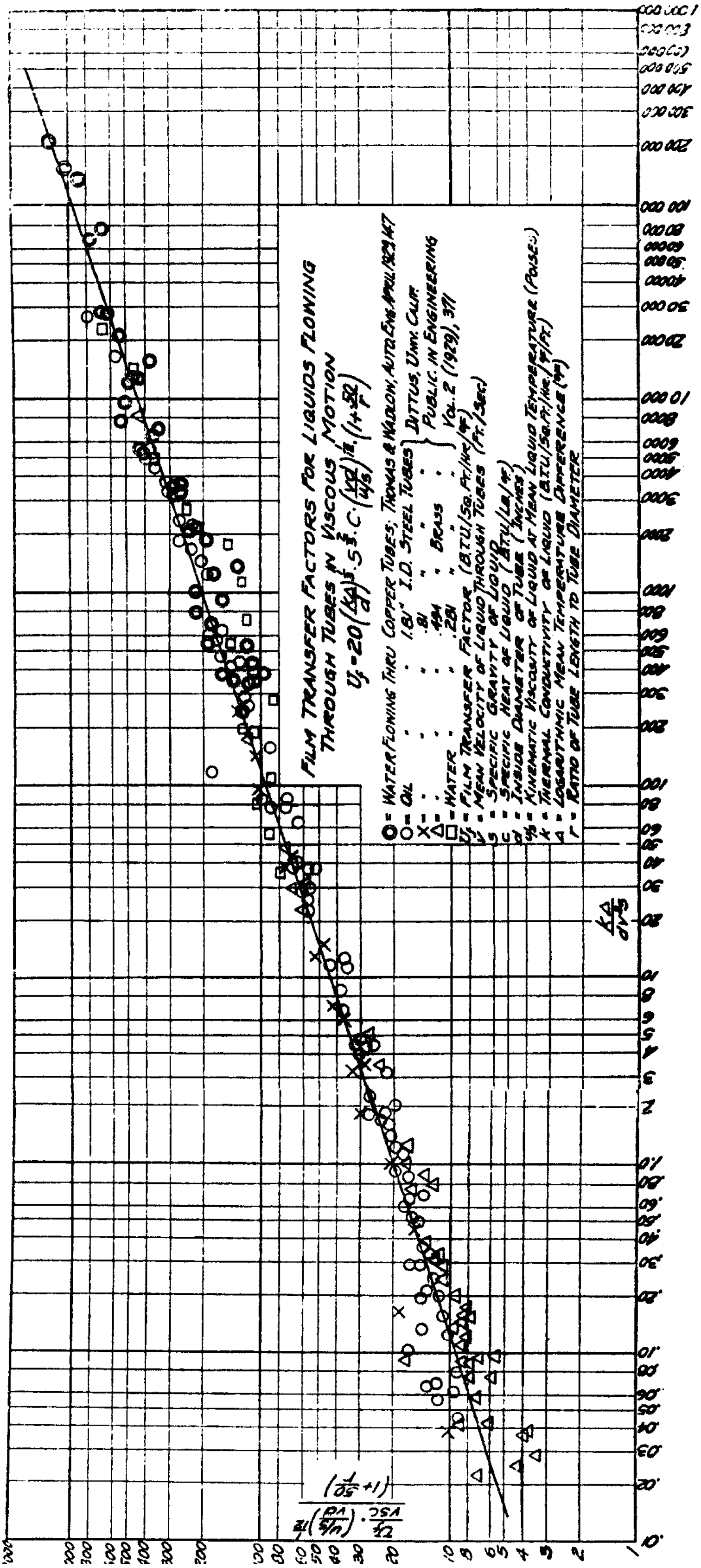
FILM TRANSFER FACTORS FOR LIQUIDS FLOWING
THROUGH TUBES IN VISCOUS MOTION
= WATER FLOWING THRU COPPER TUBES; THOMAS & WALDON, AUTO ENG. APRIL 1929
= OIL 1.81" I.D. STEEL TUBES
= BRASS
= WATER
DITTUS, UNIV. CALIF.
PUBLIC. IN ENGINEERING
VOL. 2 (1929), 371
= FILM TRANSFER FACTOR (B.T.U./SQ.FT./HR./°F.)
= MEAN VELOCITY OF LIQUID THROUGH TUBES (FT./SEC.)
= SPECIFIC GRAVITY OF LIQUID
= SPECIFIC HEAT OF LIQUID (B.T.U./LB./°F.)
= INSIDE DIAMETER OF TUBE (INCHES)
= KINEMATIC VISCOSITY OF LIQUID AT MEAN LIQUID TEMPERATURE (POISES)
= THERMAL CONDUCTIVITY OF LIQUID (B.T.U./SQ.FT./HR./°F./FT.)
= LOGARITHMIC MEAN TEMPERATURE DIFFERENCE (°F.)
= RATIO OF TUBE LENGTH TO TUBE DIAMETER

Fig. 5

made to eliminate turbulence due to entrance and exit disturbances, while in the former tests no such precaution was observed. For this reason, turbulence existed at the entrance to the radiator tubes, which may have resulted in a higher overall transfer factor.

It will be remembered that McAdams and Frost suggested an end correction factor for the case of turbulent flow.

Upon applying the same correction factors $\left(1+\frac{50}{r}\right)$ to the results calculated from Thomas' and Wadlow's data and plotting the values thus corrected, it was found that the plotted points coincided remarkably well with those reported by Dittus (1929), as is shown in figure 5.

The equation now proposed for heat transfer existing at viscous flow is as follows:

$$U=20\left(\frac{k\Delta}{d}\right)^{1/3}\cdot s^{2/3}\cdot c\left(\cdot\ \frac{vd}{u/s}\right)^{1/12}\cdot\left(1+\frac{50}{r}\right) \tag{19}$$

As mentioned above, in the tests conducted by Dittus provisions were made to eliminate all disturbance due to entrance conditions. For that particular case the term r then becomes infinite and equation (19) will be the same as equation (18).

It is true that the introduction of the end correction term is based on rather meager data, but until more data are available, it is advisable to employ the McAdams and Frost end correction factor as shown in equation (19).

Film Transfer Factor with Flow Transverse to Tubes

Single row of tubes—

Much work has been done on heat transfer from cylinders to fluids and vice versa with flow at right angles to the cylinder. Some experiments have been conducted with heated wires moving at a fixed velocity through liquids or air in order to obtain a relation of heat transmission and relative velocity and thus calibrate the device for velocity measurements. Some work has also been done with air flow transverse to tubes. The most recent data of the latter type of tests are those published by Reiher (1925) in which hot air was cooled by flowing transverse to cold tubes. Reiher proposed the following equation:

$$\frac{UD}{k_m}=f\left(\frac{v_M D}{u_m/s_m}\right) \tag{20}$$

which may also be expressed as follows:

$$\frac{UD}{k_m}=f\left(\frac{V_M D}{z_m}\right) \tag{21}$$

where

v_M = average velocity between two adjacent tubes (ft./sec.)

V_M = average mass velocity between two adjacent tubes (lbs./sq. ft./sec.)

D = outside diameter of tubes (inches)

u_m = absolute viscosity of fluid at mean of tube and fluid temperature (poises)

z_m = absolute viscosity of fluid at mean of tube and fluid temperature (centi-poises)

k_m = thermal conductivity of fluid at mean of fluid and tube temperatures (BTU/sq. ft./hr./°F./ft.)

c_m = specific heat of fluid at mean of fluid and tube temperature (BTU/lb./°F.).

s_m = density of fluid at mean of fluid and tube temperature (gr./cc.)

and all other terms are as mentioned before.

The term V_M may be calculated as follows:

$$V_M=V_o\cdot\frac{b}{b-\frac{\pi D}{4}} \tag{22}$$

$$\text{also}\quad V_o=V_{max}\cdot\frac{b-D}{b} \tag{23}$$

where

V_o = mass velocity of gas just before entering tube bundle (lbs./sq. ft./sec.)

V_{max} = maximum mass velocity of gas between two adjacent tubes (lbs./sq. ft./sec.)

b = center to center spacing of tubes (inches)

D = outside diameter of tubes (inches)

Using the term V_M instead of either V_o or V_{max} eliminates the use of another group of variables involving the ratio of tube spacing to tube diameter. Reiher found that upon plotting all his experimental data in that manner, all points fell along the same straight line.

The data of several experimenters are plotted in that manner in figure 6. It will be noted that points obtained with air flowing at right angles to the cylinder fall along the same curve, while the points obtained from experiments using oil and water instead of air formed separate curves depending upon the liquid used.

Expressing each curve according to the following equation:

$$\frac{UD}{k_m}=B_3\left(\frac{V_M D}{z_m}\right)^n \tag{24}$$

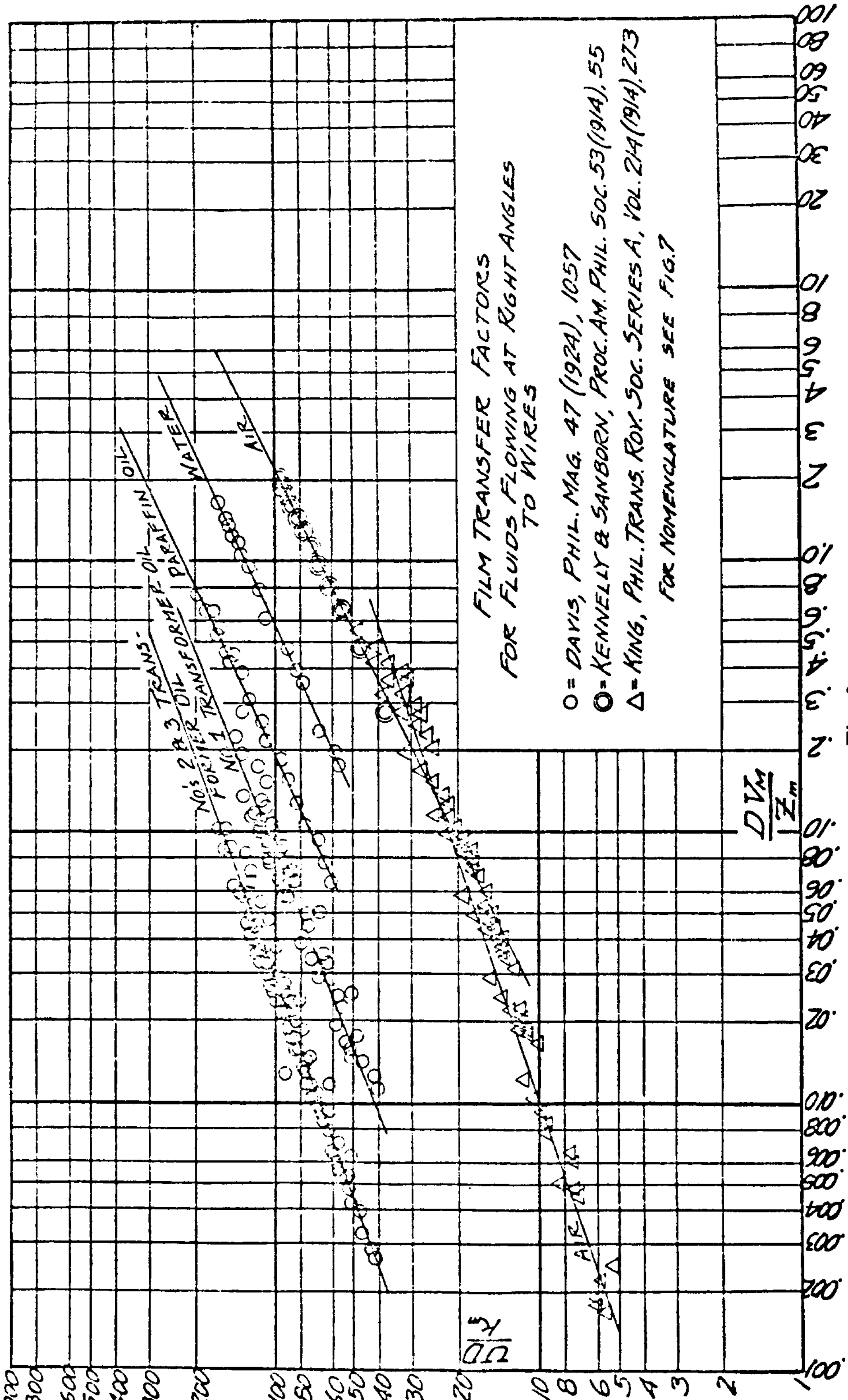
FILM TRANSFER FACTORS
FOR FLUIDS FLOWING AT RIGHT ANGLES
TO WIRES
O = DAVIS, PHIL. MAG. 47 (1924), 1057
⊙ = KENNELLY & SANBORN, PROC. AM. PHIL. SOC. 53 (1914), 55
Δ = KING, PHIL. TRANS. ROY. SOC. SERIES A, VOL. 214 (1914), 273
FOR NOMENCLATURE SEE FIG. 7
OIL
TRANS-
TRANSFORMER-OIL
PARAFFIN
No's 2 & 3 TRANSFORMER OIL
No. 1 TRANSFORMER OIL
WATER
AIR
DVm/Zm
UD/k

Fig. 6

it was found that the constant B_3 could be plotted as a function of the term $\left(\frac{c_m z_m}{k_m}\right)$ and varied as the one-third power of the last-mentioned group of variables.

Because of this fact all data were replotted as shown in figure 7 according to the following equation:

$$\frac{UD}{k_m}=f\left(\frac{V_M D}{z_m}\right)^n\left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{25}$$

Upon reviewing figure 7 it will be noted that all points, regardless of whether tests were conducted with water, oil, or air, fall along the same curve. It will furthermore be noted that for the region where $\frac{V_M D}{z_m}$ is less than 0.25 the line has a slope of 0.4 while for the region above that the line has a slope of about 0.56. Apparently there is a change of film conditions at the intersection of the two curves. The equations for heat transfer with fluid flow at right angles to cylinders may then be written as follows:

(*a*) for $\frac{V_M D}{z_m}<0.25$

$$\frac{UD}{k_m}=83\left(\frac{V_M D}{z_m}\right)^{0.4}\left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{26}$$

(*b*) for $\frac{V_M D}{z_m}>0.25$

$$\frac{UD}{k_m}=105\left(\frac{V_M D}{z_m}\right)^{0.56}\cdot\left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{27}$$

Several rows of tubes—

A series of tests for the determination of heat transfer when several rows of tubes are involved were made by Reiher. In all these experiments the heat transfer took place from tubes to air; no data are available from experiments in which liquids were used. Furthermore, all of Reiher's results are for the region where $\frac{V_M D}{z_m}$ is greater than 0.25 so that no data are available at all for several rows of tubes in the region where the just mentioned criterion is less than 0.25.

Until experiments prove otherwise, it may be assumed that Reiher's equation applying to air flow only may be modified to include liquid flow by introducing the term $\left(\frac{c_m z_m}{k_m}\right)^{1/3}$ as was done in the case of the equations for single tubes. The modified equations applying to several rows of tubes for the region where $\left(\frac{V_M D}{z_m}\right)$ is greater than 0.25 are as follows:

(1) Staggered rows of tubes:

$$2 \text{ rows, } \frac{UD}{k_m} = 53\left(\frac{V_{max.}D}{z_m}\right)^{0.69} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{28}$$

$$3 \text{ rows, } \frac{UD}{k_m} = 60\left(\frac{V_{max.}D}{z_m}\right)^{0.69} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{29}$$

$$4 \text{ rows, } \frac{UD}{k_m} = 65\left(\frac{V_{max.}D}{z_m}\right)^{0.69} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{30}$$

$$5 \text{ rows, } \frac{UD}{k_m} = 69\left(\frac{V_{max.}D}{z_m}\right)^{0.69} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{31}$$

(2) Rows of tubes directly behind each other:

$$2 \text{ rows, } \frac{UD}{k_m} = 55\left(\frac{V_{max.}D}{z_m}\right)^{0.654} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{32}$$

$$3 \text{ rows, } \frac{UD}{k_m} = 57\left(\frac{V_{max.}D}{z_m}\right)^{0.654} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{33}$$

$$4 \text{ rows, } \frac{UD}{k_m} = 58\left(\frac{V_{max.}D}{z_m}\right)^{0.654} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{34}$$

$$5 \text{ rows, } \frac{UD}{k_m} = 59\left(\frac{V_{max.}D}{z_m}\right)^{0.654} \cdot \left(\frac{c_m z_m}{k_m}\right)^{1/3} \tag{35}$$

It should be noted that in the two last mentioned groups of equations the term $V_{max.}$ instead of V_{Mean} is used. Just why this was done is not explained in Reiher's paper; unless he assumes that in heat exchange equipment, when several rows of tubes are used, the bulk of the heat transfer takes place at the point of maximum velocity. Reiher also states that the above equations are valid only when the tubes are fairly close together.

DISCUSSION AND CONCLUSIONS

Analysis of published data on heat transfer in tubular radiators indicates that a large part of the total resistance to heat flow is due to the relatively low film transfer factors on the air side of the tubes. For this reason, any attempt to increase the overall transfer factor in radiators should begin by improving the film transfer factor on the air side of the tube. Some improvements may also be made by decreasing the film resistance on the liquid side of the tubes, although the total gain will be slight unless the air film transfer factor is increased materially at the same time. The film transfer factor on the liquid side of a tubular radiator may be calculated readily with the aid of the equations (15) and (19) according as the flow is turbulent or non-turbulent respectively.

The critical point at which the flow within a tube changes from non-turbulent to turbulent may be determined by application of Reynolds criterion listed as equation (8).

It appears that there is also a critical region for flow transverse to tubes at which the flow changes from non-turbulent or viscous to turbulent; this is clearly shown in figure 7. The change of flow from non-turbulent to turbulent appears to take place at the point where $\left(\frac{V_m D}{z_m}\right)$ is equal to 0.25.

For the non-turbulent or viscous region of flow transverse to tubes the film transfer factor for single rows of tubes is expressed by equation (26) while for the turbulent region equation (27) applies.

The film transfer factor for turbulent flow transverse to several successive rows of tubes may be calculated with the aid of the modified Reiher's equations, listed in this paper as numbers 28 to 35 inclusive.

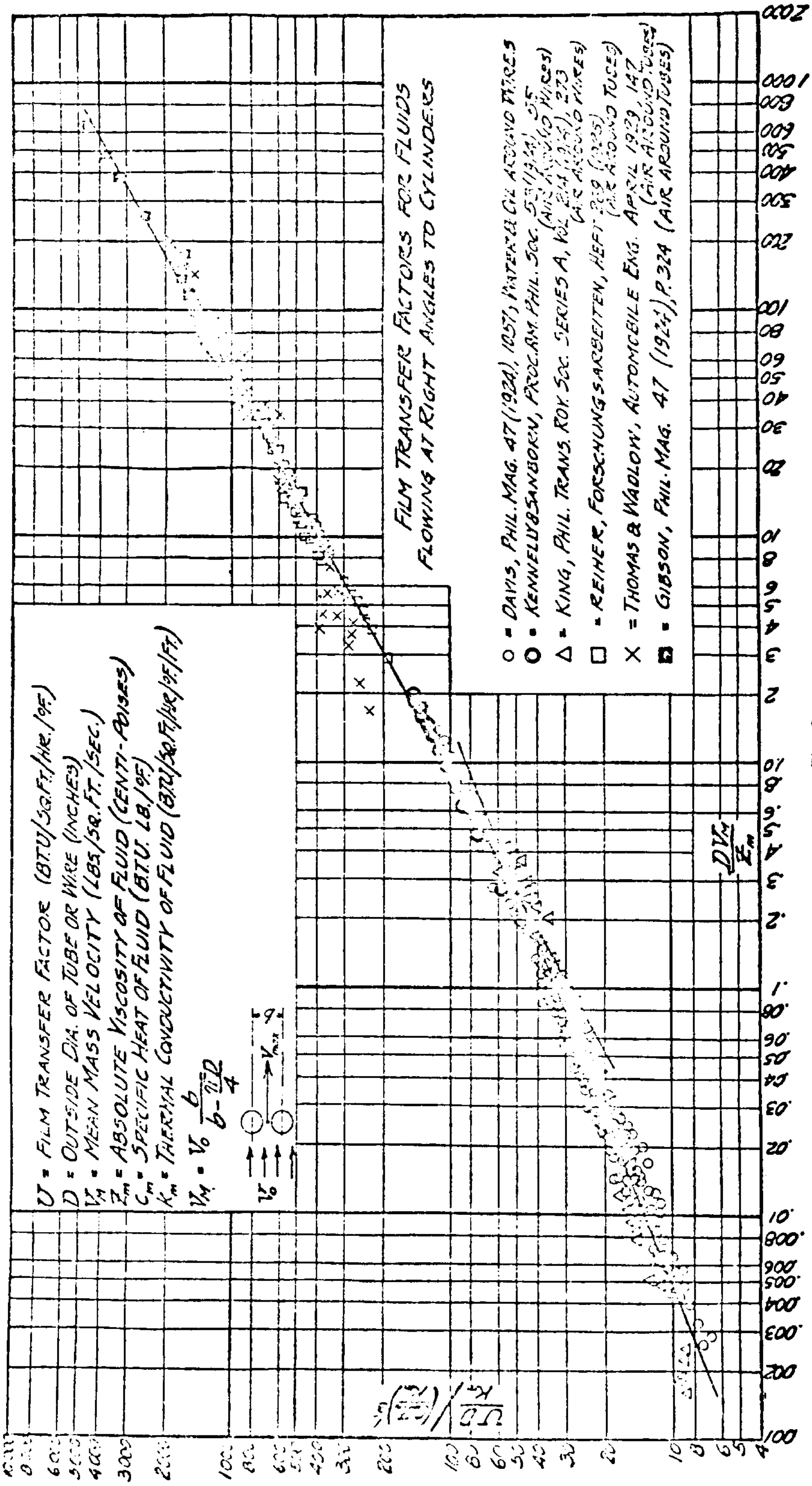
FILM TRANSFER FACTORS FOR FLUIDS
FLOWING AT RIGHT ANGLES TO CYLINDERS
○ = DAVIS, PHIL. MAG. 47 (1924), 1057, (WATER & OIL AROUND WIRES)
○ = KENNELLY & SANBORN, PROC. AM. PHIL. SOC. (AIR AROUND WIRES)
△ = KING, PHIL. TRANS. ROY. SOC. SERIES A, (AIR AROUND WIRES)
□ = REIHER, FORSCHUNGSARBEITEN, HEFT (AIR AROUND TUBES)
× = THOMAS & WADLOW, AUTOMOBILE ENG. APRIL 1929, 147 (AIR AROUND TUBES)
■ = GIBSON, PHIL. MAG. 47 (1924), P. 324 (AIR AROUND TUBES)
U = FILM TRANSFER FACTOR (BTU/SQ.FT./HR./°F.)
D = OUTSIDE DIA. OF TUBE OR WIRE (INCHES)
V_M = MEAN MASS VELOCITY (LBS/SQ.FT./SEC.)
Z_m = ABSOLUTE VISCOSITY OF FLUID (CENTI-POISES)
C_m = SPECIFIC HEAT OF FLUID (B.T.U./LB./°F.)
K_m = THERMAL CONDUCTIVITY OF FLUID (BTU/SQ.FT./HR/°F/FT.)
$V_M = V_0 \frac{b}{b - \frac{\pi D}{4}}$
$\frac{DV_M}{Z_m}$

Fig. 7

BIBLIOGRAPHY

DAVIS, A. H.
1924. Convective Cooling of Wires in Streams of Viscous Liquids. Philos. Mag., 47 : 1057.

DITTUS, F. W.
1929. Heat Transfer from Tubes to Liquids in Viscous Motion. Univ. Calif. Publ. Engin., 2 : 371.

GIBSON, A.H.
1924. Heat Dissipation from the Surfaces of Pipes and Cylinders in an Air Current. Philos. Mag., 47 : 324.

HILLIGER, B.
1916. Untersuchungen über die Wirkung von Einlagekörpern in den Rauchröhren von Lokomobilkesseln. Zeitschr. des Vereines deutscher Ingenieure, 60 : 899.

JOSSE, E.
1913. Wärmetechnische Grundlagen beim Kondensations-Vorgange. Mitteilungen aus dem Maschinen-Laboratorium der Kgl. Techn. Hochschule zu Berlin, V. Heft : 15.

KENNELLY, A. E., and SANBORN, H. S.
1914. The Influence of Atmospheric Pressure upon the Forced Thermal Convection from Small Electrically Heated Platinum Wires. Proc. Am. Phil. Soc., 53 : 55.

KING, L. V.
1914. On the Convection of Heat from Small Cylinders in a Stream of Fluid: Determination of the Convection Constants of Small Platinum Wires with Applications to Hot Wire Anemometry. Phil. Trans. Royal Society, Series A, 214 : 373.

MCADAMS, W. H.
1925. The Flow of Liquids. Publications Mass. Inst. Tech., Serial No. 12: 5, 6.

MCADAMS, W. H., and FROST, T. H.
1922. Heat Transfer by Conduction and Convection. Industrial and Engineering Chemistry, 14 : 1101.
1924. Heat Transfer for Water Flowing inside Pipes. Refrigerating Engineering, 10 : 23.

MORRIS, F. H., and WHITMAN, W. G.
1928. Heat Transfer for Oils and Water in Pipes. Industrial and Engineering Chemistry, 20 : 234.

NUSSELT, W.
1909. Der Wärmeübergang in Rohrleitungen, Zur Habilitation an der Kgl. Sächs. Technischen Hochschule zu Dresden.
1910. Der Wärmeübergang in Rohrleitungen. Mitteilungen über Forschungsarbeiten, Heft 89.

REIHER, H.
1925. Wärmeübergang von strömender Luft on Rohre und Röhrenbündel im Kreuzstrom. Forschungsarbeiten, Heft 269.

SCHULZE, E.
1928. Versuche zur Bestimmung der Wärmeübergangszahl von Luft und Rauchgas in Technischen Rohren. Mitteilung Nr. 117 der Wärmestelle des Vereins deutscher Eisenhüttenleute.

THOMAS, H. K., and WADLOW, E. C.
1929. Some Investigations into the Performance of Tubular Radiators for Motor Vehicles. The Automobile Engineer, 19 : 147.

This paper originally appeared in the University of California Publications in Engineering, Vol. 2, No. 13, pp. 443-461, October 17, 1930.