Robust Direct Adaptive Fuzzy Control of Flexible Joints Robots with Time-Varying Stiffness/Damping Parameters

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Summary/Abstract

In this article, we address the problem of controlling unknown flexible-joint robots with unknown time-varying stiffness and damping parameters. We propose a Robust Direct Adaptive Fuzzy Control (RDAFC) strategy that accommodates the dynamics anonymity and joints stiffness/damping variations. The RDAFC strategy relies on the synergy of the concepts of fuzzy logic approximation and the Sliding Mode Control (SMC). The fuzzy logic approximation relaxes the need for knowing the robot dynamics and the SMC accommodates the parameters variations. We also modify the RDAFC strategy to be suited to the KUKA Lightweight Robot (LWR) and propose a control strategy that can accommodate dynamics anonymity, uncertainty and joints elasticity variations. Experimental results are performed on a KUKA LWR moving in free space with its joints stiffness and damping vary with time in sine and cosine waveforms respectively. From the experiments, we can see that excellent tracking performance is obtained when using the RDAFC strategy despite the joints elasticity parameters time-variance and the robot dynamics unavailability.

1 Introduction

Flexible Joints Robots (FJR) are applied nowadays in many vital applications like industry, medicine, space, etc. Joints flexibility adds safer operation of robots for different applications. However, such joints flexibility results in more complex control situations. Despite their complexity, the derivation of the control strategies for the FJR attracted interests of many researchers from all over the world.

In the framework of singular perturbation, Spong et al. proposed a control strategy that relies on the concept of integral manifold in realizing the strategy and promising results were obtained [1]. In the FJR, the number of variables is more than the control actions and this urged researchers of using model reduction along with the singular perturbation in enhancing the control performance of the FJR [2]. Simplified PD controller was efficiently used in controlling the FJR with compensating the joints friction torques [3]. In [4, 5, 6], adaptive control was employed in accommodating possible parameters uncertainty in the control problem of the FJR. $H_{\infty}$ control design was suggested for the FJR and the effect of the external disturbance was attenuated below a certain level [7]. In [8, 9, 10], universal approximators like fuzzy and neural systems were used in building control strategies for such robot systems and good performance was obtained despite the dynamics uncertainties. Online gravity compensation is proposed along with a PD controller and only the motor side position and velocity are needed [11]. Passivity-based control strategy is suggested that can guarantee stable performance for the FJR using the motor side position and stiffness torque feedback [12].

One of the most important aim of almost all researchers is to enable the robot mimicking the human in its behavior since such resemblance would enable the robot of doing more complex tasks. If we consider the assembly tasks as an application example, we can see that human operator can perform such tasks simply because of the human high capability of the environment recognition and excellent arm muscles control. If we focus on the arm muscles control, we can notice that a human operator changes his arm joints elasticity with time so that smoother and safer tasks are performed and possible environment elasticity are accommodated [13, 14]. Of course such a change in joints elasticity are done unintentionally and acquired with human experience. Furthermore, if the human performs a certain task and shifts to another one, then his arm joints elasticity may also be varied (unintentionally) to accommodate such a change (if required). Stemming from this motivation one may need the variation of the joints elasticity parameters in a robot according to the task requirements. A FJR is a nonlinear system and when we have the elasticity terms are time-varying, then we would have a time-varying nonlinear system.

In this article we suggest a Robust Direct Adaptive Fuzzy Control (RDAFC) strategy for unknown FJR with time-varying stiffness/damping parameters. The fuzzy logic approximation and the Sliding Mode Control (SMC) are used in accommodating the dynamics anonymity and the time-variance of the joints elasticity. Then we modify the RDAFC strategy to be suited to the KUKA Lightweight Robot (LWR) with unknown time varying joints elasticity parameters. We will consider the current joint specific controller as a diffeomorphism that relates the the joints state variables (joints position and velocity) to the input added torques which results in an unknown mapping with time-varying parameters and the RDAFC strategy would be used in controlling such a robot system. The rest of the paper is organized as follows. In section 2, we describe the control problem in hand and section 3 will lodge preliminary concepts and definitions. In sec-
tion 4, the RDAFC strategy will be presented and section 5 will contain the RDAFC strategy for the KUKA LWR. Experimental validation will be shown in section 6 and section 7 will conclude the article with pinpointing the recommendations for future works.

2 Problem Statement

A Flexible Joint Robot (FJR) can be described by the following dynamics [15]:

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = K(q_m - q) + D(\dot{q}_m - \dot{q}) \]  
\[ B\ddot{q}_m + K(q_m - q) + D(\dot{q}_m - \dot{q}) = \tau_m \]  

(1)  

(2)

Where \( q \in \mathbb{R}^n \) is the links position vector, \( M(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^n \) is the centripetal and Coriolis vector, \( G(q) \in \mathbb{R}^{n \times n} \) is the gravity vector, \( \tau_m \in \mathbb{R}^n \) is the torque vector produced by the actuator, \( K \in \mathbb{R}^{n \times n} \) is a diagonal matrix whose main diagonal elements \( k_i \) are the joints stiffness, \( q_m \in \mathbb{R}^n \) is the actuator side position, \( D \in \mathbb{R}^{n \times n} \) is a diagonal matrix whose main diagonal elements \( d_i \) are the joints damping. Suppose that \( x_1 = q, x_2 = \dot{q}, x_3 = \theta, \) and \( x_4 = \dot{\theta} \). One can describe (1) as:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(X) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(X) + g\tau_m + K(x_1 - x_3) + D(\dot{x}_1 - \dot{x}_3)
\end{align*}
\]  

(3)

where \( X \in \mathbb{R}^{4 \times n} : X = (x_1, x_2, x_3, x_4)^T, f_1(X) = M^{-1}(x_1)(K(x_3 - x_1) + D(x_4 - x_2) - C(x_1, x_2)x_2 - G(x_1)), f_2(X) = B^{-1}(K(x_1 - x_3) + D(x_2 - x_4)), \) and \( g = B^{-1} \). (3) can be written in the following compact form:

\[ \dot{X} = F(X) + G(\tau_m + K(x_1 - x_3) + D(x_2 - x_4)) \]  

(4)

with \( F(X) \in \mathbb{R}^{4 \times n} : F(X) = (x_2, f_1(X), x_4, f_2(X)) \) and \( G = (0, 0, 0, g)^T \). For time-varying damping/stiffness parameters, (4) would be:

\[ \dot{X} = F(X) + G(\tau_m + K(t)(x_1 - x_3) + D(t)(x_2 - x_4)) \]  

(5)

For unknown robot dynamics, (5) results in unknown time-varying nonlinear system and the objective of this article is to propose a robust adaptive fuzzy controller for such kind of robot systems. We will consider the case of a KUKA Lightweight Robot (LWR) as a case study and modify the suggested strategy to be applicable to such a robot system with time-varying joints stiffness and damping parameters.

3 Preliminaries

Before exhibiting the main control strategy suggested in this paper, we will explain the concept of fuzzy logic approximators along with other preliminary concepts, properties and assumptions.

3.1 Fuzzy Logic Approximators

One of the vital applications of the fuzzy set theory is the functions approximation. It gives a feasible way of approximating unknown smooth functions through the use of T-S fuzzy models. Suppose that we desire to approximate the control action of (5), and consider that \( (q_1, \dot{q}_1, ..., q_n, \dot{q}_n) = (u_1, u_2, ..., u_{2n}) \). Let’s assume that the output of each mapping, that will be approximated, is \( y_f \). Such approximation would be feasible in the context of fuzzy If-Then rules as:

Controller Rule i:

\[ y_f = y_{f_i} (u) \]  

with:

\[ y_{f_i} = \theta^T h(u) \]  

(7)

with:

\[ \mu_i(u) = \prod_{j=1}^{2n} A_{ij}(u_j) \]

\[ h(u) = \left( \frac{\mu_1(u)}{\sum_{i=1}^{L} \mu_i(u)}, \frac{\mu_2(u)}{\sum_{i=1}^{L} \mu_i(u)}, ..., \frac{\mu_L(u)}{\sum_{i=1}^{L} \mu_i(u)} \right) \]

\[ \mu_i(u) \geq 0 \]

\[ \sum_{i=1}^{L} \mu_i(u) > 0 \]

and:

\[ \theta = (y_{f_1}^T, y_{f_2}^T, ..., y_{f_L}^T) \]

Hence, the control action \( \tau_m \) of (5) can be approximated through a fuzzy logic controller \( \tau_f = (y_{f_1}, ..., y_{f_n}) \) and this is called direct fuzzy control [17]. That is:

\[ \tau_f(q, \dot{q} | \theta) = \theta^T h(q, \dot{q}) \]  

(8)
3.2 Properties and Assumptions

Below properties are common between robot manipulators [15]:

**P1.** For all robot manipulators, $M(q)$ is a positive definite and symmetric matrix.

**P2.** For all robot manipulators, the matrix $\dot{M}(q) - 2C(q, q)$ is a skew symmetric matrix, that is for all $\dot{x} : \dot{x} \in \mathbb{R}^n$ and $\dot{x} \neq 0$, we have $\dot{x}^T(\dot{M}(q) - 2C(q, q)) \dot{x} = 0$.

Define the error vector to be:

$$\ddot{x}_{1,3} = x_{1,3} - x_{1,3}$$

and consider the filtered error vector to be described as:

$$s = s_{1,3} + \gamma \ddot{x}_{1,3}$$

with $\gamma > 0$. (10) can be rewritten as:

$$s = \ddot{x}_{1,3} - \dot{s}_{r,1,3}$$

where:

$$\dot{s}_{r,1,3} = \ddot{x}_{1,3} - \ddot{x}_{r,1,3}$$

**Note 1.** It has been shown that the filtered error described by (10) has the following properties: (i) the equation $s(t) = 0$ defines the time-varying hyperplane in $\mathbb{R}^n$, on which the tracking error vector $\ddot{x}_{1,3}$ decays exponentially to zero. (ii) if $\ddot{x}_{1,3}(0) = 0$ and $|s(t)| \leq \varepsilon$ with constant $\varepsilon$, then $\ddot{x}_{1,3}(t) \in \Omega_{\varepsilon} = \{ \ddot{x}_{1,3}(0) \leq 2^{i-1} - 2^{i-2}, i = 1, 2 \}$ for all $t \geq 0$ and (iii) if $\ddot{x}_{1,3}(0) \neq 0$ and $|s(t)| \leq \varepsilon$ then $\ddot{x}_{1,3}(t)$ will converge to $\Omega_{\varepsilon}$ within a time constant of $\frac{(n-1)}{\gamma}$ [18].

Taking the time derivative of (11), we obtain:

$$\dot{s} = \ddot{x}_{1,3} - \dot{s}_{r,1,3}$$

Despite the robustness of the SMC, a possible chattering may deteriorate the control performance and may even drive the system to be unstable. Therefore, a modified filtered error [19] is introduced that can be expressed as:

$$s_{\varepsilon} = s - \varepsilon \tanh(\frac{s}{\varepsilon})$$

Let’s define $k(t) = (k_1(t), ..., k_n(t))^T$ and $d(t) = (d_1(t), ..., d_n(t))^T$. All joints stiffness and damping are assumed to be bounded. That is:

$$|k_i(t)| \leq k_{ui}$$

and

$$|d_i(t)| \leq d_{ui}$$

Suppose that $KG$ and $DG$ are the upper bounds of $KG$ and $DG$ respectively. We will design the control strategy relying on the modified filtered error (14). However, before we proceed in explaining the suggested control strategy, below assumptions are needed to be satisfied:

**A1.** The signals $x_{1,3}, \dot{x}_{1,3}$, and $\ddot{x}_{1,3}$ are available for measurement.

**A2.** The signals $x_{d1,d3}, \dot{x}_{d1,d3}$, and $\ddot{x}_{d1,d3}$ are bounded and piecewise continuous.

**A3.** All joints stiffness and damping parameters are bounded.

4 Robust Direct Adaptive Fuzzy Control (RDAFC) Design

Since the dynamics of the robot is assumed to be unknown then $F(X)$, $G$, $K(t)$, and $D(t)$ would be unknown. We use the fuzzy logic in approximating $F(X)$ and we will denote such an approximation as $\hat{F}(X|\theta)$. Suppose that the approximation error is $w$, that is:

$$w = \hat{F}(X|\theta) - F(X)$$

The minimum approximation error $w^*$ is defined to be:

$$w^* = \hat{F}(X|\theta^*) - F(X)$$

Where $\theta^*$ is the optimal parameter vector of $\theta$ that is defined as:

$$\theta^* = \arg \min_{\theta \in \Omega_{\theta}} \left[ \sup_{X \in \Omega_X} \hat{F}(X|\theta) - F(X) \right]$$

and

$$\hat{F}(X|\theta) = \theta^T h(q, \dot{q})$$

Let’s introduce the following control action:

$$\tau_m = \hat{G}^{-1}[-(\dot{\bar{w}} + \hat{KG}\bar{x}_1 - x_3) + \hat{DG}\ddot{x}_1$$

$$\bar{x}_3] \tanh(\frac{s_{\varepsilon}}{\varepsilon}) - K_d s - F(\hat{X}|\theta) + \hat{X}_d]$$

with $K_d = \text{diag}(k_{d1}, ..., k_{d2n})$; $k_{d1}, ..., k_{d2n}$ are positive constants. The control action (21), with parameters update laws described by (22)-(26) below, can be shown to provide globally stable performance for the FJR described by (5). The parameters update laws are:

$$\dot{\theta} = \eta_1 s_{\varepsilon} h(q, \dot{q})$$

$$\dot{\hat{G}} = \eta_2 s_{\varepsilon} \tau_m$$

Suppose that $\hat{KG}$ and $\hat{DG}$ are the upper bounds of $KG$ and $DG$ respectively. We will design the control strategy relying on the modified filtered error (14). However, before we proceed in explaining the suggested control strategy, below assumptions are needed to be satisfied:

**A1.** The signals $x_{1,3}, \dot{x}_{1,3}$, and $\ddot{x}_{1,3}$ are available for measurement.

**A2.** The signals $x_{d1,d3}, \dot{x}_{d1,d3}$, and $\ddot{x}_{d1,d3}$ are bounded and piecewise continuous.

**A3.** All joints stiffness and damping parameters are bounded.
The minimum approximation error $w^*$ is defined to be:

$$w^* = \tau_f(q, \dot{q}, \theta^*) - \tau_m^*$$  

(30)

Where $\theta^*$ is the optimal parameter vector of $\theta$ that is defined as:

$$\theta^* = \arg \min_{\theta \in M_\theta} \left\{ \sup_{q \in M_q, \dot{q} \in M_\dot{q}} \left( \tau_f(q, \dot{q}, \theta^*) - \tau_m^* \right) \right\}$$  

(31)

$$\dot{w} = \tau_f(q, \dot{q}, \theta) - \tau_m^*$$  

(29)

The minimum approximation error $w^*$ is defined to be:

$$w^* = \tau_f(q, \dot{q}, \theta^*) - \tau_m^*$$  

(30)

Where $\theta^*$ is the optimal parameter vector of $\theta$ that is defined as:

$$\theta^* = \arg \min_{\theta \in M_\theta} \left\{ \sup_{q \in M_q, \dot{q} \in M_\dot{q}} \left( \tau_f(q, \dot{q}, \theta^*) - \tau_m^* \right) \right\}$$  

(31)

$$\dot{\hat{\theta}} = -\eta_\hat{\theta} \hat{\theta} (q, \dot{q})$$  

(37)

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(37)

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(37)

$$\dot{\hat{\theta}} = -\eta_\hat{\theta} \hat{\theta} (q, \dot{q})$$  

(37)
Grammed the joints stiffness and damping to vary in sine searchers to apply their own control schemes in control-ware that makes its joints control strategy customizable. For research purposes, a Fast Re- strategy, we used it in controlling a KUKA Lightweight In order to see the performance of the suggested RDAFC 6 Experimental Results

In order to see the performance of the suggested RDAFC strategy, we used it in controlling a KUKA Lightweight Robot (LWR) with time-varying joints stiffness and damping parameters. The KUKA LWR is a 7-DOF industrial robot and the key features of the KUKA LWR is detailed in [22]. For research purposes, a Fast Research Interface (FRI) is available in the robot hardware that makes its joints control strategy customizable by the user through a C++ platform hence allowing researchers to apply their own control schemes in controlling the robot joints [20]. Through its platform, we programmed the joints stiffness and damping to vary in sine and cosine waveforms respectively, that is for the \(i\)th joint \(k_i(t) = 20 + 10\sin(t)\) and \(d_i(t) = 20 + 10\cos(t)\). Figure 1.a shows the KUKA LWR that we used in our experiments and Figure 1.b shows the waveforms of the variations of the joints stiffness and damping parameters. Figure 2.a-f show the desired pose signals of the KUKA LWR end effector and the corresponding joint position signals are shown in Figure 3.a-g. The control actions when using the RDAFC strategy in commanding the joints of the given robot are shown in Figure 3.h-n. The RDAFC strategy was used with the following details: 

\[
K_d = \text{diag}(15, 12, 15, 10, 5, 4)
\]

\[
\varepsilon^T = [0.005, 0.07, 0.06, 0.05, 0.04, 0.09, 0.04]^T
\]

\[
\gamma^T = [80, 80, 80, 50, 12, 12, 5]^T
\]

\[
M_k^T = [2, 3, 3, 2, 0, 3, 4, 2, 2, 2, 2, 2]^T
\]

\[
M_d^T = [2, 1, 3, 0, 3, 0, 2, 0, 2, 0, 2, 0, 1, 7]^T
\]

\[
M_w^T = [0.002, 0.017, 0.007, 0.0005, 0.0007, 0.00012, 0.00007]^T
\]

\[
\eta_1 = 0.001, \eta_2 = 0.01, \eta_3 = 0.0001, \text{ and } \eta_4 = 0.0001.
\]

Gauss membership functions of the form:

\[
A_{ij}(u_j) = \exp\left(-\frac{(u_j - c)^2}{2\sigma^2}\right)
\]  (39)

are used in the premise of the \(i\)th If-Then rule of the RDAFC. \(c\) and \(\sigma\) are the center and width of the gauss membership function. The joints position and velocity, say \(q\) and \(\dot{q}\) respectively, are considered the input variables for the fuzzy logic controller, and each one of those state variables is assigned with two membership functions for the premise part of the if-then rules. For simplicity, we will describe each Gauss membership function described by (39), with an ordered pair \((c, \sigma)\). Below are the parameters of the fuzzy sets of the variables considered in the RDAFC, say \(q\) and \(\dot{q}\):

\[
q_1 : (-0.7, 0.0849) \text{ and } (-0.9, 0.0849)
\]

\[
q_2 : (-0.4, 0.0849) \text{ and } (-1, 0.0849)
\]

\[
q_3 : (0.3, 0.0849) \text{ and } (0.2, 0.0849)
\]

\[
q_4 : (1.6, 0.0849) \text{ and } (1.2, 0.0849)
\]
Figure 2: The manipulated object signals: (a) x (in mm); (b) y (in mm); (c) z (in mm); (d) Θ (in degree); (e) Ψ (in degree); (f) Φ (in degree); (g) e_x (in mm); (h) e_y (in mm); (i) e_z (in mm); (j) e_Θ (in degree); (k) e_Ψ (in degree); (l) e_Φ (in degree).

Figure 3: Joints position and velocity signals: (a) q_{d1} (in mrad); (b) q_{d2} (in mrad); (c) q_{d3} (in mrad); (d) q_{d4} (in mrad); (e) q_{d5} (in mrad); (f) q_{d6} (in mrad); (g) q_{d7} (in mrad); (h) \tau_1 (in N.m); (i) \tau_2 (in N.m); (j) \tau_3 (in N.m); (k) \tau_4 (in N.m); (l) \tau_5 (in N.m); (m) \tau_6 (in N.m); (n) \tau_7 (in N.m); (o) \dot{q}_1 (in mrad); (p) \dot{q}_2 (in mrad); (q) \ddot{q}_1 (in mrad); (r) \ddot{q}_4 (in mrad); (s) \dot{q}_5 (in mrad); (t) \dot{q}_6 (in mrad); (u) \dot{q}_7 (in mrad).

As per using the suggested RDAFC control strategy in section 5, the joints control actions were graphed in Figure 3. h-n that resulted in the joints position error signals shown in Figure 3. o-u. We can see that the RDAFC strategy is of excellent joint space performance. Figure 2. g-l show the pose error signals and we can notice that the excellent joint space tracking performance had a direct reflection on that of the task space. Furthermore, we graphed the filtered error and the modified filtered error and as shown in Figure 4. Figure 5 shows the estimate of the i^{th} joint approximation error \hat{w}_i, stiffness bound k_{ui}, and damping bound d_{ui}. We can see that all signals involved in the design are bounded and excellent tracking performance is achieved despite the time variance of the
7 Conclusion

The control problem of the Flexible Joint Robots (FJR), with unknown dynamics and unknown time-varying joints stiffness and damping, was addressed and a Robust Direct Adaptive Fuzzy Control (RDAFC) strategy is proposed for such robot systems. RDAFC strategy is a synergy of the concepts of fuzzy logic approximation and the Sliding Mode Control (SMC). The fuzzy approximator relaxes the need for knowing the robot dynamics and the SMC accommodates the variations of the joints elasticity parameters. Then we modify the RDAFC strategy to be suited for the KUKA Lightweight Robot (LWR) with the joints stiffness and damping parameters to be time-variant. The stable performance of the suggested RDAFC strategy is shown and experiment is performed on the KUKA LWR with varying the joints stiffness and damping in sine and cosine waveforms and the efficiency of the suggested approach is shown.

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References


