Gait Generation for Inchworm-Like Robot Locomotion Using Finite State Model

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ABSTRACT
The gait of a multi-segment inchworm robot is a series of actuator actions that will change the shape of the robot to generate net motion. In this article, we model the multi-segment inchworm robot as a finite state automaton. Gait generation is posed as a search problem on the graph described by the automaton with prescribed state transitions. The state transitions are defined based on the kinematics of robot locomotion. The auxiliary actuator concept is introduced. Single-stride and multi-stride gait generations are discussed. Single-stride gaits exhibit fault-tolerant and real-time computation features that are necessary in actual applications. Both computer simulation and experimental hardware platform are developed for various aspects of the gait generation and planning.

1. INTRODUCTION
An inchworm-like robot is a mobile robot that imitates the locomotion pattern of a natural inchworm. This type of robots usually consist of interconnected actuating modules that can either deform in the direction of travel (extensors) or produce friction against the environment (grippers). The locomotion of the robot is through a series of cyclic actuator actions to change the shape of the robot termed gaits. The gaits exploit the constrained nature of a robot's interaction with its environment to generate net body motion. This kind of robots are especially useful in travelling and conducting tasks in narrow and highly constrained environments, such as pipes and conduits in industrial plants, or slender vessels and intestines in human body.

In this article, we study the gait generation problem for a class of multi-segment inchworm robots. Various types of gaits exhibit different kinematic and dynamic behaviors, which are crucial to the application environment. The gait generation algorithms can be applied to the control of inchworm-type robotic endoscopes for medical inspection [1,10] and pipe inspection systems [4]. Hyper-redundant robot (snake-like or inchworm-like robot) locomotion has been studied in [2,8]. It is formulated based on the continuous "backbone curve" of the robot. Locomotion gaits are generated based on the wave theory. The works of [7] and [9] further describe robot locomotion in terms of the geometric phase associated with a connection on a principal bundle and develop local trajectory generation strategy. Basically, the mechanical systems studied in above works have continuous states. For inchworm robots used as inspection tools or material delivery systems, precise control of the robot's position is usually not necessary. Therefore, simple actuators with only binary actions, i.e., on and off, are frequently used in the design of the grippers and the extensors. The condition between the on and off states is considered as transist behavior, which will settle within a very short time. Based on this fact, we can use the finite automaton model [8] to study the inchworm robot with only binary actuation actions. The action of the inchworm actuators can be described as states. A gait becomes a sequence of state transitions that follow the kinematics of locomotion. Since a finite automaton can be expressed as a directed graph with the states as the nodes and the transitions as the arcs, the gait generation problem becomes a graph search problem. The advantage of using this approach is that it reduces the effort in gait generation from a continuous state space to a binary state space. Also, online gait generation and planning can be achieved directly with additional structures imposed on the finite automaton model.

Single-stride gait and multi-stride gait generations are both investigated. The concept of auxiliary grippers and extensors are introduced and used in both cases. The strategy for single-stride gait allows both forward and backward motions, and has fault-tolerant feature. On-line gait generation and gait change can be achieved through single-stride gait. Multi-stride gait generation leads to the standing wave gait, which is the fastest gait in terms of the number of state transitions in a complete gait cycle. Computer simulation of various gait generation algorithms is implemented. A simple experimental inchworm robot platform based on the solenoid actuators is built to verify the result of simulation and to study the dynamic behavior of the gaits.

2. SIMPLE INCHWORM ROBOT MODEL
As actual robots may adopt different mechanical design, here we assume a simplified model of the inchworm robot actuators as depicted in Figure 1. A robot with n segments has n extensors and (n+1) grippers. Each gripper or extensor can be actuated independently. All grippers employ identical...
modular mechanical design. The main function of the gripper is to provide friction for robot locomotion. Without loss of generality, the width of the gripper is assumed to be a constant. Similar modular design concept is applied to the extensor as well. The stroke lengths of all extensors are the same. The fully retracted and fully stretched lengths are denoted as $l_{\text{min}}$ and $l_{\text{max}}$ respectively. The intermediate length of the extensor, $l_i$, is within the range of $l_{\text{min}} \leq l_i \leq l_{\text{max}}$. We assume that the robot moves in an environment with one-dimensional topology, for example, a straight pipe with rigid wall, or curved human intestine with elastic wall. The actual robot design can make the gripper and extensor modules negotiate the bend in the constrained environment and compliant to the shape of the surroundings.

3. KINEMATICS OF INCHWORM LOCOMOTION

We first use an inchworm robot with continuous states to illustrate the principle of locomotion. For an $n$-segment robot shown in Figure 1, the kinematic constraint to allow the robot move in a 1-D space is given by

$$\hat{s}_1 + \sum_{i=1}^{n} l_i - s_2 = 0 \quad (1)$$

where $s_1$ and $s_2$ are the positions of the rear gripper and the front gripper. According to [7], $(s_1, s_2) \in G = (R_+ \times R_+)$ are the group variable that reflects the position of the robot as a subgroup of the group of rigid motion and $(l_1, l_2, \ldots, l_n) \in M = R^n$ are the base space variables describe the body motion. The expression of (1) is a connection on the trivial principal bundle $Z = M \times G$. It describes how trajectories in the base space $M$ are related to the motion in the group $G$. The gait is thus a closed path in $M$ that will generate net motion in $G$ [9]. Figure 2 and 3 demonstrate the gaits of a two-segment and a three-segment inchworm robots and their corresponding closed paths in $M$. Both gaits will generate a net motion with one stride length ($l_{\text{max}} - l_{\text{min}}$) along the x-direction. Other types of gaits with different stride lengths can be achieved with the choice of initial conditions of the base space variables and the closed path to be taken in $M$. One will notice that as the number of the segment increase, the dimension of the base space increase as well. To generate suitable gaits (or the closed paths) in high dimensional spaces become difficult as the number of possible paths to be taken increases.

4. MODELING INCHWORM AS AN AUTOMATON

Based on the simple mechanical design and actuation principle, we can model a multi-segment inchworm robot whose grippers and extensors have only simple binary actuation actions as a finite automaton. The extensor and gripper each has only binary value states “0” and “1”.

**DEFINITION 1:** The extensor state of an $n$-segment inchworm robot is an $n$-tuple vector with binary numbers, $q = (x_1, x_2, \ldots, x_n)$, where $x_i \in \{0, 1\}$. When $x_i = 0$, Extensor $i$ is retracted; $x_i = 1$, Extensor $i$ is fully stretched.

**DEFINITION 2:** The gripper state of an $n$-segment inchworm robot is an $(n+1)$-tuple vector with binary numbers, $p = (y_1, y_2, \ldots, y_n, y_{n+1})$, where $y_i \in \{0, 1\}$. When $y_i = 0$, Gripper $i$ is released; $y_i = 1$, Gripper $i$ is fully activated.

Because the locomotion of the inchworm robot is achieved by the deformation of the extensors, the inchworm gait can be defined solely on the extensor states $q$. The gripper state $p$ can be determined by the transition function between the consecutive extensor states [5].

**DEFINITION 3:** The gait of an $n$-segment inchworm robot is a sequence of extensor states $q_0, q_1, \ldots, q_f$ such that $q_0 = q_f$. Definition 3 implies that the gait is a closed trajectory in the state space of the extensor $\{0, 1\}^n$ (or a “digitized” base space $M$). For instance, the gait of the two-segment robot shown in Figure 2 is $(1,1) \rightarrow (0,1) \rightarrow (1,0) \rightarrow (1,1)$. The gait in Figure 3 is $(0,0,0) \rightarrow (0,0,1) \rightarrow (0,1,0) \rightarrow (1,0,0) \rightarrow (0,0,0)$. Note that the transition between any state of the robot’s extensors must satisfy the kinematic constraints imposed by the environment.

**DEFINITION 4:** The gait generator of an $n$-segment inchworm robot is a 5-tuple $A = (Q, \Sigma, \delta, q_0, F)$, where $Q$ is a finite set of inchworm states, $\Sigma$ is a non-empty set of event labels called an input alphabet, $q_0 \in Q$ is the initial state, $F \subseteq Q$ is the set of final states, and $\delta$ is the transition function mapping $\delta : Q \times \Sigma \rightarrow Q$. That is, $\delta(q, a)$ is a state for each state $q$ and input symbol $a \in \Sigma$. 

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Alternatively, $A$ can be represented by a directed graph in which the nodes are the states in $Q$, the arcs are the transition functions defined by the function $\delta$, and the set of labels for the arcs are the alphabets in $\Sigma$. The gait generation problem becomes finding a path (a sequence of arcs) from the root node (denoted the initial state $q_0$) to the desired node (denoted the final state $q_f$) as illustrated in Figure 4.

![Figure 4: Search on the graph formed by $A$](image)

One can draw the analogy between the continuous inchworm robot model stated in Section 2 and the finite-state inchworm model. The base space $M$ corresponds to the binary state space $\{0,1\}^n$; the kinematic constraints are equivalent to the state transition functions that follow kinematic rules that we will study in the following section. Trajectory generation (gait generation) in the base space then becomes a search problem on the directed graph $A$. Through this transformation, standard graph search algorithms can be utilized in the gait generation.

5. SINGLE-STRIDE GAIT GENERATION

5.1 Auxiliary grippers and extensors

For an $n$-segment inchworm robot, one auxiliary extensor and one auxiliary gripper are added to the front and rear of the robot respectively as shown in Figure 5 for gait generation. Physically the auxiliary actuators do not exist. Initially, the two auxiliary extensors are set to the retracted position (0) and the stretched position (1) respectively depending on the direction of travel. For example, if the robot shown in Figure 5 moves to the right, the initial setting of the right and the left auxiliary extensors should be 1 and 0 respectively. The right one is then called the front auxiliary extensor and the left one the rear auxiliary extensor. As one gait cycle is completed, the physical extensors return to the initial states but the front auxiliary extensor will be retracted and the rear one will be extended (Figure 5). The front and rear auxiliary grippers remain stationary through out the entire gait. When a new gait cycle starts the front and rear auxiliary extensors are reset to 1 and 0 respectively. If the robot moves to the left, the initial setting of the right and the left auxiliary extensors should be 0 and 1. The roles of the front and rear extensors are swapped. Obviously, the auxiliary grippers remain stationary and serve as the reference frame for the locomotion. The inchworm robot moves one-stride length with respect to the auxiliary front and rear grippers.

**Definition 5:** The augmented extensor state of an $n$-segment inchworm robot is an $(n+2)$-tuple binary vector: $q' = (x_0, x_1, \ldots, x_n, x_{n+1})$. The states of the right and the left auxiliary extensors are $x_0$ and $x_{n+1}$ respectively.

Based on this definition, an inchworm gait becomes a sequence of augmented extensor states $(q_0, q_1, \ldots, q_f)$ such that $q_0 = (x_0, x_0, x_{n+1})$, and $q_f = (x_f, q_0, x_{n+1})$, where $q_0$ is the given extensor initial state. If $(x_0, x_{n+1}) = (0,1)$, then $(x_f, x_{n+1}) = (1,0)$. If $(x_0, x_{n+1}) = (1,0)$, then $(x_f, x_{n+1}) = (0,1)$. The gait generator becomes $A = (Q', \Sigma, \delta, q_0', F')$, where $Q'$ is the set of the augmented extensor states; $F' \subseteq Q'$ is the final state set.

![Figure 5: Auxiliary grippers and extensors](image)

5.2 Simple one-stride gaits

A one-stride gait will make the inchworm robot move one-stride length when one gait cycle is completed. Based on the simplified inchworm robot model, the positions of every gripper will move one-stride length as well. To make a gripper move, it is necessary to coordinate the motion of the two neighboring extensors with simultaneous stretching and retraction actions. In State 1 of Figure 2, Gripper 2 cannot move because both Extensors 1 and 2 are stretched to the limit. Gripper 2 can only move one-stride length to the right from State 2 to State 3. This action is coordinated by the stretching of Extensor 1 and the contraction of Extensor 2 while the grippers at both ends remain stationary to provide friction for locomotion. Re-looking at state 1, Gripper 1 can be similarly coordinated by the extension of the rear auxiliary extensor, although it does not exist physically, and the contraction of Extensor 1 from state 1 to state 2. In this case, the rear auxiliary gripper and Gripper 2 remain stationary.

In terms of the binary actuator states, the coordinated stretching and contraction of neighboring extensors makes the state of the neighboring extensors change their value from $(0,1)$ to $(1,0)$ if the robot moves to the right, and vice versa, if the robot moves to the left. Based on this observation, we can use the following strategy to generate inchworm gaits for the forward motion (right):

![Diagram showing the gait generation process](image)
Possible gaits with the same initial condition. Figure 6 label sequence represents the sequence of the extensor states tree with the root node as nodes as Figure 6. Exhaustive search on the graph, we are able to find all node to a pendant node is a valid gait (Figure 4). Using the motion.

To move the robot backward, one can simply reset the front and rear auxiliary extensors with 1 and 0 respectively. The other extensors can also show that it will take n+1 state transitions to complete the single-stride gait for an n-extensor inchworm robot. (Auxiliary actuators are not shown.) Five different to remain stationary. The unaffected grippers still hold on to the environment with friction contact. With a given q0 = (0, q0, 1), a sequence of labels in Σ can be generated from δf to satisfy the final state qf = (1, q0, 0) ∈ F'. This label sequence represents the sequence of the extensor states of a gait. Because there are a number of possible state transition functions allowed for state qf at any step, the forward gait generator A can be represented by a directed tree with the root node as qf = (0, q0, 1) and all the pendant nodes as qf = (1, q0, 0). Every directed path from the root node to a pendant node is a valid gait (Figure 4). Using the exhaustive search on the graph, we are able to find all possible gaits with the same initial condition. Figure 6 illustrates the one-stride gait generation of a tree-extensor robot. (Auxiliary actuators are not shown.) Five different gaits can be produced with identical initial conditions. Note that the number of steps in each of the gait is the same. We will use fewer steps to complete a gait when it has faulty transitions to complete the gait cycle. In principle, the robot has no faulty extensor [Condition (d)], it will take 4 transitions to complete the gait cycle. In principle, the robot will use fewer steps to complete a gait when it has faulty extensors. Since the faulty extensor is being dragged along during the movement, actual locomotion speed will be affected.

### <Forward motion gaits>

- **Initialization**
  Given an inchworm robot with extensor state q0, set the initial augmented state q0 = (0, q0, 1).

- **State transition function**
  Let q' = (x0, x1, \ldots, x_n) and q' = (x0, x1, \ldots, x_n). The state transition q' = δf(q', i) is defined as:

  \[
  (x_i, x_{i+1}) = (1, 0) \quad \text{and} \quad x^k = x^k \quad \text{for} \quad k \neq i, i+1 \quad \text{else} \quad q'_i = q'_i
  \]

- **Final state**
  One complete gait cycle will be terminated at qf = (1, q0, 0).

Note that each one-stride state transition only allows two extensors to move simultaneously. The other extensors remain stationary. The unaffected grippers still hold on to the environment with friction contact. With a given q0 = (0, q0, 1), a sequence of labels in Σ can be generated from δf to satisfy the final state qf = (1, q0, 0) ∈ F'. This label sequence represents the sequence of the extensor states of a gait. Because there are a number of possible state transition functions allowed for state qf at any step, the forward gait generator A can be represented by a directed tree with the root node as qf = (0, q0, 1) and all the pendant nodes as qf = (1, q0, 0). Every directed path from the root node to a pendant node is a valid gait (Figure 4). Using the exhaustive search on the graph, we are able to find all possible gaits with the same initial condition. Figure 6 illustrates the one-stride gait generation of a tree-extensor robot. (Auxiliary actuators are not shown.) Five different gaits can be produced with identical initial conditions. Note that the number of steps in each of the gait is the same. We will use fewer steps to complete a gait when it has faulty transitions to complete the gait cycle. In principle, the robot has no faulty extensor [Condition (d)], it will take 4 transitions to complete the gait cycle. In principle, the robot will use fewer steps to complete a gait when it has faulty extensors. Since the faulty extensor is being dragged along during the movement, actual locomotion speed will be affected.

### <Backward motion gaits>

- **Initialization**
  Given an inchworm robot with extensor state q0, set the initial augmented state q0 = (1, q0, 0).

- **State transition function**
  Let q' = (x0, x1, \ldots, x_n) and q' = (x0, x1, \ldots, x_n). The state transition q' = δf(q', i) is defined as:

  \[
  (x_i, x_{i+1}) = (0, 1) \quad \text{and} \quad x^k = x^k \quad \text{for} \quad k \neq i, i+1 \quad \text{else} \quad q'_i = q'_i
  \]

- **Final state**
  One complete gait cycle will be terminated at qf = (1, q0, 0).

This one-stride gait generation strategy also allows the gaits to be changed during any phase of the movement. One can interrupt the robot at any state and make the current state as the new initial state and reset the front and rear auxiliary extensors with 1 and 0 respectively. A new forward gait can be generated based on this new initial state using the identical transition function. Likewise, the robot can move backward immediately by resetting the front and rear auxiliary extensors to 0 and 1 respectively and using the backward transition function to generate the backward gait. Since the interrupt can be issued at any instance of the motion, real-time on-line gait generation can be achieved.

### Example

For any \( i \in \Sigma = \{0, 1, \ldots, n\} \),

\[
\begin{align*}
\text{If} \quad & (x_i^a, x_{i+1}^a) = (1, 0) \\
\text{Then} \quad & (x_i^b, x_{i+1}^b) = (0, 1) \quad \text{and} \quad x_k^b = x_k^b \quad \text{for} \quad k \neq i, i+1 \\
\text{else} \quad & q'_i = q'_i
\end{align*}
\]

5.3 Fault-tolerant gaits

The single-stride gait has a robust fault-tolerant feature. If any extensor in the robot failed, the robot can still move around by using the rest of the extensors as long as the grippers are functioning. In this situation, we can treat the robot as an under-actuated system. When using the state transition δf or δb to determine the next valid move, the state of the failed extensor will be omitted. The fault-tolerant gait generation is shown in Figure 7 (auxiliary actuators not shown). Conditions (a), (b) and (c) have the same initial condition but with different faulty segments. In Condition (a), Extensor 1 is failed, so it is treated as a two-extensor robot. It takes only 3 transitions to complete this gait. If the robot has no faulty extensor [Condition (d)], it will take 4 transitions to complete the gait cycle. In principle, the robot will use fewer steps to complete a gait when it has faulty extensors. Since the faulty extensor is being dragged along during the movement, actual locomotion speed will be affected.
6. MULTI-STRIDE GAITS

6.1 Double-stride gaits

A double-stride gait makes the inchworm robot move two-stride length when one gait cycle is completed. Based on the analogy of single-stride gait generation strategy, the double-stride gait requires 4 extensors to be actuated simultaneously (Figure 8). Three intermediate grippers will be released and moved by the stretching of extensors 1, 2 and the contraction of extensors 3, 4. To generate such type of gaits two auxiliary extensors and grippers have to be added to the each end of the robot. With the double-stride gaits, the robot can complete one gait cycle with fewer numbers of state transitions and larger stride motion but at the expense of larger power consumption as compared to the single-stride gaits. Triple-stride gaits and even n-stride gaits can be defined similarly. Nevertheless, to use these types of gaits, the initial conditions of the extensors must be paired up to enable the subsequent motion that may not be available all the time.

![Figure 8: Double-stride gait](image)

6.2 Standing wave gaits

The other type of multi-stride gaits is the standing wave gait, which requires all the extensors to be actuated at the same time and the neighboring extensors to be activated in opposite way (Figure 9). Therefore, it requires only two state transitions to complete the cycle. Similar to the double-stride gaits, the initial condition usually cannot start the standing wave pattern. Nevertheless, based on the augmented gripper and extensor concept, we propose the following strategy to generate standing wave gaits.

![Figure 9: Standing wave gait generation](image)
standing wave gait loop. The kinematic constraints of locomotion are maintained throughout the gait.

7. IMPLEMENTATION

To validate the gait generation algorithms, a computer simulated inchworm locomotion program based on the finite automata is developed. The gaits shown in Figures 7 and 8 are generated by the program. It is coded in Mathematica running on a PC with Pentium II 233 MHz processor. In addition to computer simulation, we also design and build an experimental platform for the gait generation and planning of the inchworm robot. This is to investigate the dynamic characteristics of various gaits and their interaction with the environment that cannot be observed in the computer simulation. The robot is designed based on modular concept as shown in Figure 10. Each module has a cart-like geometry moving along a horizontal track. There are two solenoid actuators on each of the cart: one for gripping action (emulating the gripper) and the other for extension action (emulating the extensor). The natural state of the gripper is retracted. It can be opened to hold on the wall. Viscous layers are adhered to the walls of the track to provide enough friction. The extensors are naturally retracted as well. Wheels are built under all carts to support the weight of the robot. The overall system is shown in Figure 11. The control of the robot is through a data acquisition interface connected to a PC. The gait generation algorithm is then downloaded from the result of Mathematica simulation. The experiments on various types of gait generation are demonstrated in the video [11]. Because the single-stride gait generation strategy exhibits online gait change and fault-tolerant features, further investigation on sensor-based gait generation and planning is possible.

Figure 10: Modular design

8. SUMMARY

This article is our modest attempt to systematically investigate the gait generation problem for an n-segment inchworm-like robot. We model the inchworm robot as a finite automaton. The gait is a sequence of state transitions that follows the mechanics of inchworm locomotion. The gait generation problem becomes an exhaustive search on the directed graph defined by the automaton. The generation algorithms for various gaits are investigated, namely, single-stride gaits with backward motion and fault-tolerant features, and multi-stride gaits with standing wave feature. These algorithms can be applied to the control of various inchworm robot systems for inspection and material delivery purpose. Computer simulation of the gait generation is developed. An experimental inchworm robot platform is designed and constructed for further investigation of the dynamic characteristics of various gaits and their interaction with the environment. Further study on real-time on-line gait generation and planning will be carried out in near future.

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