Permeability of microscale fibrous porous media using the lattice Boltzmann method

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The permeabilities of microscale fibrous porous media were calculated using the multiple-relaxation-time (MRT) lattice Boltzmann method (LBM). Two models of the microscale fibrous porous media were constructed based on overlapping fibers (simple cubic, body-centered cubic). Arranging the fibers in skew positions yielded two additional models comprising non-overlapping fibers (skewed simple cubic, skewed body-centered cubic). As the fiber diameter increased, the fibers acted as granular inclusions. The effects of the overlapping fibers on the media permeability were investigated. The overlapping fibers yielded permeability values that were a factor of 2.5 larger than those obtained from non-overlapping fibers, but the effects of the fiber arrangement were negligible. Two correlations were obtained for the overlapping and non-overlapping fiber models, respectively. The effects of the rarefaction and slip flow are also discussed. As the Knudsen number increased, the dimensionless permeability increased; however, the increase differed depending on the fiber arrangement. In the slip flow regime, the fiber arrangement inside the porous media became an important factor.

1. Introduction

Synthetic porous media are widely used in modern industry and engineering applications, such as heat exchangers, filters, catalysts, and fuel cell electrodes, and natural porous media are relevant to the function of aquifers and the liver. Transport in porous media has been intensively investigated in a variety of fields, including environmental, chemical, and biological engineering. There are three types of porous media: granular, fibrous, and network. Among those, the fibrous porous media composed of woven or non-woven fibers are useful in microstructured reactors and biofiltration systems. The design of engineering devices that include porous media components must consider the permeability of the porous component, since the permeability directly affects the flow in porous media, as described by Darcy’s law.

Numerous experiments (Fowler and Robertson, 1991; Kumar et al., 2010; Lu et al., 1994; Masalmeh, 2003; Poulsen and Kueper, 1992) have been conducted to measure the permeability properties of such porous media, as well as numerical simulations solving the Navier–Stokes equation (Delisée et al., 2010; Ghaddar, 1995; Higdon and Ford, 1996; Lee and Yang, 1997; Martin et al., 1998; Wang et al., 2007). However, the simulation of flow through microscale pores, based on the Stokes equation, is computationally expensive because the flow passages can be very narrow. Poor convergence and numerical instabilities can occur upon application of ordinary Navier–Stokes codes in simulations of the flow inside porous media. The lattice Boltzmann method (LBM) provides an alternative technique and is a very efficient and effective way to simulate flow inside porous media (Sukop and Thorne, 2006; Maier and Bernard, 2010). It is free to distribute the inclusions (solid nodes) in any manner without requiring the modification of the grid geometry or the solver part of the program code. The LBM is also well-known for its capability of efficient parallelization. Clague et al. (2000) and Jeong et al. (2006) utilized 2D and 3D LBM to obtain the permeability of porous media with randomly distributed fibrous inclusions, and both studies yielded good agreement with the N–S solution of Higdon and Ford (1996). Rong et al. (2007) conducted 3D LBM simulations of randomly distributed fibrous porous media and compared their results with experimental data. Hao and Cheng (2009) and Gao et al. (2012) used the multiple-relaxation-time (MRT) LBM models to estimate the permeability of 3D carbon paper models. It is known that the MRT LBM enhances the stability of the LBM code, and its details are shown in Section 2.

LBM studies of the permeability of the fibrous porous media have been applied mainly to randomly distributed fibers, approximating the non-woven fibrous porous media; however, models of randomly distributed fibers are limited in their applicability. Since the fiber overlapping volume is not controlled, it is not possible to
compensate the effect of the non-dimensionalized permeability \( K/\eta D^2 \), where \( D \) is the fiber diameter. Therefore, the overlapping volume must be controlled in order to achieve clear relationship between the fiber diameter \( D \) and the porosity \( \varepsilon \). One way to control the overlap volume is to use ordered fiber models, such as the simple cubic model employed by Higdon and Ford (1996). In such cases, the ordered overlapping fibers can be regarded as a network porous structure (Wang and Pan, 2008; Wang, 2012).

Another factor must be carefully considered: the presence of narrow pores results in a mean free path that is comparable to the fiber diameter (the characteristic length). The Knudsen number (\( Kn \)) is defined as the ratio of the mean free path to the characteristic length. The Knudsen number for the lattice Boltzmann simulation purposes. The lattice Boltzmann scheme is governed by (d’Humières et al., 2002)

\[
\begin{align*}
\tilde{f}_{s}(\mathbf{x} + \mathbf{e}_{s} \delta t, t + \delta t) - \tilde{f}_{s}(\mathbf{x}, t) &= -\frac{1}{\tau + 0.5} (f_{s} - f_{s}^{eq})|_{(x,t)},
\end{align*}
\]

where \( f_{s} \) the particle distribution in the direction \( x \), \( x \) the position, \( \mathbf{e}_{s} \) the discrete velocity in the direction \( \alpha \), \( \delta t \) the time step, and \( \tau \) the non-dimensional relaxation time.

In this study, the three-dimensional 15-velocity (D3Q15) lattice Boltzmann model is used (Wang et al., 2010), following the convention of Qian et al. (1992). A schematic diagram of the D3Q15 model is shown in Fig. 1. In this model, the discrete velocities \( \mathbf{e}_{s} \) are given by

\[
\begin{align*}
\mathbf{e}_{s} &= \begin{cases} 
0 & \alpha = 0 \\
(\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1) & \alpha = 1, 2, \ldots, 6; \text{ group I} \\
(\pm 1, \pm 1, \pm 1) & \alpha = 7, 8, \ldots, 14; \text{ group II}
\end{cases}
\end{align*}
\]

indicating only three discrete velocity values: 0 for \( \alpha = 0, 1 \) for group I, and \( \sqrt{3} \) for group II. The speed of sound, \( c_{s} \), is given as \( 1/\sqrt{3} \) (Qian et al., 1992). The equilibrium distribution function \( f^{eq} \) may be expressed as (Succi, 2001),

\[
f^{eq}_{s} = \omega_{s} \rho \left[ 1 + \frac{\epsilon_{s} u_{i} u_{j}}{c_{s}^{2}} + \frac{(\epsilon_{s} \epsilon_{s} - c_{s}^{2} \delta_{ij}) u_{i} u_{j}}{2 c_{s}^{4}} \right],
\]

where \( \omega_{s} \) denotes the weighting factor of the system, \( \rho \) the density, \( u_{i} \) the macroscopic velocity, and \( c_{s} \) the speed of sound. The values of \( \omega_{s} \) are given as (Qian et al., 1992)

\[
\begin{align*}
\omega_{s} &= \begin{cases} 
2/9 & \alpha = 0 \\
1/9 & \alpha = 1, 2, \ldots, 6; \text{ group I} \\
1/72 & \alpha = 7, 8, \ldots, 14; \text{ group II}
\end{cases}
\end{align*}
\]

In the actual simulations, Eq. (1) was applied in two steps: collision and streaming. The equation was separated out into the corresponding forms as

\[
\begin{align*}
\text{Collision step} : \tilde{f}_{s}(\mathbf{x}, t) - f_{s}(\mathbf{x}, t) &= -\frac{1}{\tau + 0.5} (f_{s} - f_{s}^{eq})|_{(x,t)},
\end{align*}
\]

and

\[
\begin{align*}
\text{Streaming step} : f_{s}(\mathbf{x} + \mathbf{e}_{s} \delta t, t + \delta t) &= \tilde{f}_{s}(\mathbf{x}, t),
\end{align*}
\]

where \( \tilde{f} \) denotes the post-collision state of the distribution function \( f \). The simulation on the single variable \( f \) permits the extraction of the macroscopic flow properties, such as the density, pressure, and momentum, by

\[
\rho = \sum_{s} f_{s}, \quad p = \sum_{s} \epsilon_{s} f_{s}, \quad \rho u = \sum_{s} \mathbf{e}_{s} f_{s}.
\]

2.2. The D3Q15 MRT lattice Boltzmann scheme

The D3Q15 MRT LBM is governed by (d’Humières et al., 2002)

\[
f(r_{i} + \mathbf{e}_{s} \delta t, t + \delta t) - f(r_{i}, t) = -M^{-1} \mathbf{S}(\mathbf{m}(r_{i}, t) - \mathbf{m}^{eq}(r_{i}, t)),
\]

where \( f \) and \( \mathbf{m} \) are the vectors of the particle distribution functions and the kinetic parameters respectively. The kinetic parameters for D3Q15 model are

\[
\mathbf{m} = [\rho, \mathbf{e}, \mathbf{c}, \mathbf{c} \cdot \mathbf{c}, \mathbf{c} \cdot \mathbf{E}, \mathbf{c} \cdot \mathbf{B}, \mathbf{c} \cdot \mathbf{c} \cdot \mathbf{B}, \mathbf{c} \cdot \mathbf{E}, \mathbf{E} \cdot \mathbf{B}, \mathbf{c} \cdot \mathbf{E} \cdot \mathbf{B}, \mathbf{E}^{2}, \mathbf{B}^{2}, \mathbf{E} \cdot \mathbf{B}]^{T}.
\]

The collision matrix is in lieu of the relaxation time \( \tau \) of the BGK model, and it is given by

\[
\mathbf{S} = \text{diag}(0, s_{1}, s_{1}, 0, s_{4}, 0, s_{4}, 0, s_{4}, s_{4}, s_{5}, s_{5}, s_{5}, s_{5}, s_{5}, s_{14}),
\]

where the indices are the relaxation frequencies for the macroscopic quantities in the vector \( \mathbf{m} \), with \( s_{0} = s_{14} \). The viscosity of the fluid is determined by

\[
\nu = \frac{1}{3} \left( \frac{1}{s_{9}} \frac{1}{2} \right) = \frac{1}{3} \left( \frac{1}{s_{9} s_{11}} \frac{1}{2} \right).
\]

2.3. Implementation of the slip effect

The slip flow effects were implemented by defining the Knudsen number for the lattice Boltzmann simulation purposes. The
Knudsen number represents the ratio of the mean free path \( l \) to the characteristic length \( H \) according to

\[
\text{Kn} = \frac{l}{H}.
\]

(12)

where the mean free path \( l \) approximates a particle’s distance traveled during the relaxation time \( \tau \) with a lattice speed \( c = \delta x/\delta t \) according to (Lim et al., 2002),

\[
l = c \tau = \frac{\delta x}{\delta t} \tau.
\]

(13)

where \( \tau (= \delta t/\delta t) \) is the non-dimensional relaxation time. Substituting Eq. (13) into Eq. (12) yields the expression of Kn,

\[
\text{Kn} = \frac{l}{H} = \frac{\delta x}{H}.
\]

(14)

The use of Eq. (14) with the implicit LBM yields a solution that is accurate to second order (Lee and Lin, 2005). Furthermore, because the mean free path is inversely dependent on the pressure, Kn is not a constant. The local Kn can be represented as

\[
\text{Kn}(x, t) = \text{Kn}_0 \frac{P_o}{P(x, t)}.
\]

(15)

where Kn0 and P0 are the Knudsen number and the pressure at the outlet, respectively.

2.4. Calculating the permeability

The flow in porous media is governed by

\[
\frac{dP}{dx} = \frac{\mu U}{K} + C \rho U |U|,
\]

(16)

where \( \mu \) denotes the dynamic viscosity of the fluid, \( K \) the permeability of the porous medium, \( C \) the Forchheimer constant, \( \rho \) the density of the fluid, and \( U \) the mean flow velocity. The Forchheimer constant is given by \( C = C_E/K \), where \( C_E \) is the Ergun constant. Eq. (16) can thus be rewritten as

\[
\frac{dP}{dx} = \left( \frac{D^2}{K} + F \text{Re}_D \right) \frac{\mu U}{D^2},
\]

(17)

where \( \frac{D^2}{K} \) represents the Darcy drag and \( F \text{Re}_D \) the Forchheimer drag. The two dimensionless drag terms together form the Darcy–Forchheimer (DF) drag. In evaluating the drag, the calculation of \( \text{Re}_D \) uses \( \rho_n \) and \( U_m \) the arithmetic means of the inlet and the outlet. Note that the inverse of the Darcy drag is equal to the non-dimensionalized permeability.

3. Geometry and boundary conditions

Because the objective of the current study was to investigate the flow characteristics and permeability of microscale fibrous porous media, a three-dimensional cubic domain with the mesh size of \( 100 \times 100 \times 100 \) lattice was selected with periodic boundary conditions in all three spatial directions. The periodic boundary conditions permitted simulation of the fluid flow inside a complex fibrous structure using a simple unit fiber geometry in the cubic domain. To induce the flow, pressure difference is applied between the inlet and the outlet boundaries, as in the work of Jeong et al. (2006). On the solid surface, the halfway bounce-back boundary condition (Sukop and Thorne, 2006) is applied for no-slip treatment. The porosity of the fibrous structure is then calculated regarding the solid–fluid interface located on the halfway between the solid nodes and the fluid nodes as in Fig. 2. Following the work of Higdon and Ford (1996), two distinct fiber arrangement models, a simple cubic and a body-centered cubic model, were constructed as shown in Fig. 3. In these arrangements, the fibers overlapped at the center of the cubic domain; thus, the models are referred to as overlapping fiber models.

For engineering purpose, the permeability \( K \) is generally non-dimensionalized by dividing it by the square of the characteristic length scale. For the fibrous porous media, it is customary to use the fiber diameter \( D \) as the characteristic length. For the granular porous media, however, it is reasonable to use \( D^2 \), where \( D_0 \) is the effective inclusion diameter calculated by (Jeong et al., 2006; Jeong, 2010)

\[
D_i = \left( \frac{6V_i}{\pi} \right)^{1/3},
\]

(18)

and where \( V_i \) is the volume of an inclusion in the model. \( D_i \) is essentially the diameter of a sphere with a volume equal to that of an inclusion in the model. Because all models included an ordered fiber arrangement, the volume of each fiber could be calculated. Thus, \( D_i \) could be defined in the same way, even though the inclusions were fibers. This important fact enabled a direct comparison of the dimensionless permeabilities of the granular and fibrous porous media; however, in this study, the use of \( D_i \) was limited only by comparing the permeability results of the fibrous porous media and those of the granular porous media. The fiber diameter \( D \) was regarded as more pertinent to the fibrous porous media models of interest, and was used subsequently.

The overlapping fiber models used here were identical to those examined by Higdon and Ford (1996), thereby providing a means for directly validating the simulation results; however, distinct models with non-overlapping fibers required construction. These non-overlapping models are illustrated in Fig. 4. The three fibers in the simple cubic model became skewed so that overlap did not occur until the dimensionless fiber diameter \( D' = D/L \) reached 0.5, where \( L \) is the length of a side of the cubic domain. Rearranging the four fibers to remove any overlap became much more complicated in the body-centered cubic model. As shown in Fig. 4, the skewed body-centered cubic model appeared to be intricate, but this model comprised nothing more than the composite of four distinct fibers, as depicted in Fig. 5. The fiber (a) in the original body-centered cubic model shown in Fig. 3 remained in position, whereas the other fibers were translated so that the fibers (b), (c), and (d) exited the cubic domain through the center of the domain boundary normal to the \( x, y \), and \( z \) axes, respectively. The six tiny regions in the domain of (a) and the two small fiber regions in the domains of (b), (c), and (d) facilitated smooth fiber connections under the periodic boundary conditions in all directions of the cu-
bic domain. Note that each fiber was connected without resulting in a change of slope or volume loss. Overlap in the skewed body-centered cubic model did not occur until $D^* \approx 0.35$. A treatment of the overlap regime, $D^*/C_3 > 0.35$, is beyond the scope of this paper.

4. Results and discussion

The flow characteristics in four models of fibrous porous media were simulated using the lattice Boltzmann scheme. This section describes the model validation analysis, the no-slip flow characteristics, and the permeability results in the slip-flow regime. Before presenting the results, we shall revisit the permeability calculation discussed in Sections 2 and 3. The dimensionless permeability could be obtained by taking the inverse of the Darcy drag in Eq. (17). The accuracy of this calculation relies on a negligible Forchheimer drag, which is achieved by reducing the Reynolds number without modifying the fiber geometries or fluid properties. The dimensionless permeability is expected to converge to its original value as the Reynolds number decreases to zero. Convergence is indeed observed, as depicted in Fig. 6. Also, invoking the MRT scheme yields constant permeability for various values of the fluid viscosity. The variable $\phi(=1 - \varepsilon)$ in the abscissa represents the solid volume fraction. Note that the dimensionless permeability was very sensitive to changes in the solid volume fraction, and the graph was plotted with the ordinate in log scale. Two models for different fiber arrangements yielded the same result, but the results began to deviate at $D^*/C_2 \approx 0.6$. As the solid volume fraction increased further, the two models yielded totally different permeability values because the fibers overwhelmed the flow and the permeable area became extremely small. Thus, the narrow flow passages became dependent on the fiber arrangements, resulting in a huge difference in the dimensionless permeability.

4.1. Validation

The permeability results of the simple cubic and body-centered cubic models were compared with the results reported by Higdon and Ford (1996) using the same models. Fig. 8 clearly shows that the results of this study agreed well with those of Higdon and Ford (1996). Here, the permeability $K$ was non-dimensionalized by the square of the fiber radius $R$. The variable $\phi(=1 - \varepsilon)$ in the abscissa represents the solid volume fraction. Note that the dimensionless permeability was very sensitive to changes in the solid volume fraction, and the graph was plotted with the ordinate in log scale. Two models for different fiber arrangements yielded the same result, but the results began to deviate at $\phi \approx 0.6$. As the solid volume fraction increased further, the two models yielded totally different permeability values because the fibers overwhelmed the flow and the permeable area became extremely small. Thus, the narrow flow passages became dependent on the fiber arrangements, resulting in a huge difference in the dimensionless permeability.

4.2. No-slip flow

4.2.1. Flow characteristics

The flow variables were reconstructed using the simulated particle distribution functions for the simple cubic model, as described in Eq. (7). The contours of four major flow variables, $u$, $v$, $w$, and $p$, are shown in Fig. 9. The bold arrows indicate the major flow direc-
tion, which was normal to the x-axis. Because a no-slip condition on the fiber surfaces was adopted, the contours were similar to those of the flow-past cylinders. The periodic boundary conditions were applied to all surfaces of the cubic domain. The symmetric fiber model was combined with the periodic boundary conditions to demonstrate the absence of fluid flow through the boundaries normal to the y or z axes of the domain; however, because all fibers were considered to be smoothly connected with the virtual fibers of the neighboring domains, nonzero \( v \) and \( w \) occurred on the boundaries normal to the \( z- \) and \( y- \) axes, respectively.

4.2.2. Flow regimes

Because the fiber shapes were similar to the granular inclusions for a large fiber radius, the flow behavior and permeability were predicted to converge to the corresponding values for the granular porous media. The permeability studies of the granular porous media by Jeong et al. (2006) and Jeong (2010) support this conjecture. In those studies, spheres and cubes were disposed in staggered and in-line arrangements to form granular porous media.

Fig. 5. Four distinct fibers in the skewed body-centered cubic model.

Fig. 6. Convergence of the dimensionless permeability at low Reynolds number.

Fig. 7. Comparison between the permeability–viscosity relations of the BGK model and the MRT model.
One such arrangement, the staggered cube model, was introduced in Fig. 10. The 3D cubic domain contained one whole cube at its center and eight small cubes at each of the domain corners. Under the periodic or symmetric boundary conditions, the small cubes formed whole cubes through connections with the corresponding small cubes of the neighboring domains. Thus, a porous medium composed of a staggered arrangement of identical cubic inclusions was modeled. This model was used in the current study to validate the simulation results, including simulations of the slip flow, which will be discussed later.

The research of Jeong et al. indicated that the permeability of an isotropic granular porous media with the spherical and cubic inclusions positioned in staggered or in-line arrangement can be represented by (Jeong et al., 2006):

\[
\frac{K}{D_s^2} = \exp \left[ 0.709 \ln \left( \frac{e^{11/3}}{(1 - e)} \right) - 5.09 \right],
\]  

(19)

where \( e \) is the porosity. Note that the permeability \( K \) was non-dimensionalized using \( D_s \), the effective diameter calculated as described in Eq. (18). The permeability of the skewed simple cubic model was compared with the values predicted according to Eq. (19), as shown in Fig. 11. The abscissa was chosen as

\[
\beta = \frac{e^{11/3}}{(1 - e)^2},
\]  

(20)

so that the correlation reported by Jeong et al. (2006) was linear on the log–log scale whereas the skewed simple cubic model yielded a curve. The skewed simple cubic model behavior resembled that of the granular porous media for low values of \( \beta \), but the permeability deviated from the linear correlation of Jeong et al. (2006) as \( \beta \) increased. Thus, the fibrous porous media could be categorized into three distinct regions, as depicted in Fig. 11, and the criteria are listed in Table 1.

4.2.3. Permeability

The permeabilities of the four fiber models were calculated over a range of porosity values. The permeabilities were non-dimensionalized using the fiber diameter \( D \), and the results are shown in Fig. 12. Fiber overlap generally increased the permeability by up to a factor of 2.5 relative to the permeability provided by non-overlapping fibers. Thus, two distinct correlations were observed for the models with overlapping fibers and the models with non-overlapping fibers. The correlation for the model with overlapping fibers could be described according to

\[
\frac{K}{D^2} = \exp \left[ -0.007 (\ln \beta)^2 + 0.77 \ln \beta - 3.73 \right],
\]  

(21)

and the corresponding relation for the model with non-overlapping fibers was found to be
\[ K_0 = \exp\left(-0.009 (\ln b)^2 + 0.81 \ln b - 4.19\right), \]  
\[ (22) \]

where \( D \) represents the fiber diameter.

Eqs. (21) and (22) accurately described the trends over a wide range of porosity, 0.37 \( \leq \varepsilon \leq 0.99\). The two overlapping fiber models did not agree, however, for porosity values below 0.37 (that is, \( b \leq 0.0658\)). As discussed in the context of the validation studies, these deviations from the trend corresponded to flow through very narrow passages. Note that in this region, the non-overlapping fibers did not yield measurable flow because the low porosity (as a result of the large fiber diameter) resulted in fiber overlap, even though the fibers were skewed. Eqs. (21) and (22) are useful in that they permit calculation of the dimensionless permeability of the fibrous porous media using only the porosity. If the diameter of the fibers in the porous media is known, the dimensional \( K \) may also be calculated.

### Table 1

<table>
<thead>
<tr>
<th>Variables</th>
<th>Granular region</th>
<th>Intermediate region</th>
<th>Fibrous region</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>( \leq 31.8 )</td>
<td>31.8–394.3</td>
<td>( \geq 394.3 )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>( &lt; 0.864 )</td>
<td>0.864–0.954</td>
<td>( \geq 0.954 )</td>
</tr>
<tr>
<td>( D' (=D/L) )</td>
<td>( &gt; 0.24 )</td>
<td>0.14–0.24</td>
<td>( \leq 0.14 )</td>
</tr>
</tbody>
</table>

The effects of slip flow were investigated using the modified lattice Boltzmann method described in Section 2, and the non-dimensional relaxation time \( \tau \) was varied along the spatial position. As expected based on Arkilic et al. (1997) and Jeong et al. (2006), the permeability of the slip flow increased as \( Kn_o \) increased. This result was compared with the slip flow of Jeong et al. (2006) in Fig. 13. The flow results agreed well, and the permeability appeared to grow linearly as \( Kn_o \) increased, for \( Kn_o \leq 0.08\). Note that the permeability \( K \) was non-dimensionalized using the effective diameter \( D_e \), since Fig. 13 shows the staggered cube model, which assumed a granular shape.

The non-overlapping fibers represented actual fibrous porous media more accurately than the overlapping fibers; therefore, the slip flow studies were conducted using the non-overlapping fiber models. Fig. 14 shows the permeability–\( Kn_o \) relations for various porosities and for the two models with non-overlapping fiber arrangements. The permeability greatly decreased as the porosity decreased; moreover, the graph clearly shows that the permeability increased to a greater extent for the skewed body-centered cube model than for the other models. This indicated that the fiber arrangement affected \( K \), especially for large values of \( Kn_o \). The skewed body-centered cubic model permeability was higher than...
that of the skewed simple cubic model by as much as 30% at lower values of $\beta$.

The permeability–porosity relationship in the slip flow regime was examined using the non-overlapping fiber models for $0.5 < \varepsilon < 0.9$. In this region, the dimensionless permeability as a function of $\beta$ was linear on a log–log scale,

$$ K/D^2 = \exp[C_1 \ln \beta - C_0] $$

(23)

Variations in the coefficients $C_1$ and $C_0$, as well as $Kn_0$, are depicted in Figs. 15 and 16 with their corresponding least-squares correlation fits. Note that the coefficients for the skewed simple cubic model and the skewed body-centered cubic model differed. The correlations for the coefficients were:

for skewed simple cubic:

$$ C_0 = 47Kn_0^2 - 14.7Kn_0 + 4.22 $$

$$ C_1 = 10Kn_0^2 - 2.4Kn_0 + 0.79 $$(24)

for skewed body-centered cubic:

$$ C_0 = 63Kn_0^2 - 19Kn_0 + 4.24 $$

$$ C_1 = 12Kn_0^2 - 2.6Kn_0 + 0.78 $$

(25)

as shown in Figs. 15 and 16. Eq. (23), along with the coefficients in Eqs. (24) and (25), provide a global correlation for the permeability relationships in most fibrous porous media over a range of porosities, $0.5 < \varepsilon < 0.9$.

5. Conclusions

The flow behavior and permeability in microscale fibrous porous media were investigated using the D3Q15 lattice Boltzmann method. Four distinct fibrous porous models comprising different fiber arrangements were examined. The fibrous porous media tended to behave as the granular porous media at low porosities, apparently because the fiber shapes resembled the shapes of granular inclusions for large fiber diameters, for a fixed intra-fiber distance; however, as the fiber diameter decreased, the permeability behavior differed from that of the granular porous media, as expected. The permeability depended strongly on the porosity of the porous media, with a lower dependence on the fiber arrangement geometry. The models of porous media formed from overlapping fiber inclusions yielded permeabilities up to 2.5 times larger than those predicted by the models with non-overlapping fibers. Thus, two distinct permeability correlations were observed for the overlapping and non-overlapping fiber models. The correla-
tions are applicable to a range of porosities, \(0.37 \leq \varepsilon \leq 0.99\), and they do not require any information except for the porosity. The slip flow effects, which are applicable to microscale gas flows, were also studied. The permeability in a non-overlapping fiber model was calculated under slip flow conditions. The porosity range was limited to \(0.5 \leq \varepsilon \leq 0.9\), yielding a linear relationship between the dimensionless permeability \((K/D^2)\) and \(\beta\). The correlation was recast in a simpler form, and the coefficients were presented as functions of \(K_n\). The skewed body-centered cubic model was more sensitive to the slip flow effects, and the results differed substantially from those of the skewed simple cubic model at high \(K_n\). This indicated that the fiber arrangement is important for rarefied gas flow in a porous medium.

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