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On U-BG-Filter of a U-BG-BH-Algebra

Husein Hadi Abbass and Alaa Abdulla Hamza

Department of Mathematics
Faculty of Education for Girls
University of Kufa, Najaf, Iraq

Abstract

In this paper, we introduce the notion of U-BG-filter in U-BG-BH-algebra and observed that every filter of a U-BG-BH-algebra is a U-BG-filter. A necessary and sufficient condition is derived for every $U-BG-filter$ of $U-BG-BH-algebra$ to become a filter. Some properties of $U-BG-filter$ are studied with respect to homomorphism, Cartesian products and quotient $U-BG-BH-algebra$.

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Keywords: BH-algebra, BG-algebra, U-BG-BH-algebra, U-BG-filter, filter, Cartesian products and quotient $U-BG-BH$-algebra

1 Introduction

The notion of U-BG-BH-algebra was introduced and extensivelys studied by H.H.Abass and L.S.Mahdi ([3]), in 2014. This class of U-BG-BH-algebra was introducsd as a combination of the classes of BH-algebra and BG-algebra.In 1980, E.Y.Deeba ([6]) introduced the notion of filters and in the setting of bounded implicative BCK-algebra constructed quotient algebra via a filter.Also E.Y.Deeba and A.B.Thaheem ([7]) studied afilters in BCK-algebra in 1990. In

1991 C.S.Hoo ([8]) was presented the filters in BCI-algebra . In 1996, J.Meng ([10]) introduced the notion of BCK-filter in BCK-algebra. In 2012 H.H.Abass and H.A.Dahham ([1]) discussed the concept of completely closed filter of a BH-algebra, and completely closed filter with respect to an element of BH-algebra. In this paper, the notion of U-BG-filter of U-BG-BH-algebra is introdused.

2 Preliminary Notes

In this section, some basic concepts about a BG-algeba, BH-algebra, U-BG-BH-algrbra, filter, U-BG-filter, subalgebra, normal subset and quotient U-BG-BH-algebra are given.

Definition 2.1. *([9]) A BG-algebra is a non-empty set X with a constant 0 and a binary operation * satisfying the following axioms:for all $x, y, z \in X$:*
(I) $x * x = 0$,
(II) $x * 0 = x$,
(III) $(x * y) * (0 * y) = 0$,

Lemma 2.2. *([9]) Let $(X, *, 0)$ be a BG-algebra. Then*

(i) *The right cancellation law holds in X, i.e. $x * y = z * y$ implies $x = z$,*

(ii) $0 * (0 * x) = x$, $\forall x \in X$.

(iii) *If $x * y = 0$, then $x = y, \forall x, y \in X$.*

(iv) *If $0 * x = 0 * y$, then $x = y$, $\forall x, y \in X$.*

(v) $(x * (0 * x)) * x = x$, $\forall x \in X$.

Definition 2.3. *([11]) A BH-algebra is a nonempty set X with a constant 0 and a binary operation "$*$"satisfying the following conditions:*

(I) $x * x = 0, \forall\, x \in X$.

(II) $x * y = 0$ *and* $y * x = 0$ *imply* $x = y$, $\forall\, x, y \in X$.

(III) $x * 0 = x, \forall\, x \in X$.

Proposition 2.4. *([9]) Every BG-algebra is a BH-algebra.*

Definition 2.5. *([3]) A U-BG-BH-algebra is defined to be a BH-algebra X in which there exists a proper subset U of X, such that:*

(U_1) $0 \in U$, $|U| \geq 2$.

(U_2) U *is a BG-algebra.*

Definition 2.6. *([5]) A nonempty subset* S *of a BH-algebra* X *is called a BH-subalgebra or subalgrbra if* $x * y \in S$, $\forall\ x,\ y \in S$.

Definition 2.7. *([1]) Let* X *be a BH-algebra, a nonempty subset* N *of* X *is said to be normal of* X *if* $(x * a) * (y * b) \in N$ *for any* $x * y$ *and* $a * b \in N$, $\forall\ x,\ y,\ a,\ b \in X$.

Definition 2.8. *([2]) A BH-algebra* X *is called medial if* $x * (x * y) = y$, $\forall\ x,\ y\ \in X$.

Definition 2.9. *A filter of a BH-algebra* X *is a non-empty subset* F *of* X *such that:*

(F_1) *If* $x \in F$, *and* $y \in F$, *then* $y * (y * x) \in F$ *and* $x * (x * y) \in F$.

(F_2) *If* $x \in F$ *and* $x * y = 0$ *then* $y \in F$.

Remark 2.10. *Let* $(X, *_X, 0_X)$ *and* $(Y, *_Y, 0_Y)$ *be BH-algebra. A mapping* $f : X \longrightarrow Y$ *is called a Homomorphism if* $f(x *_X y) = f(x) *_Y f(y)$ *for any* $x, y \in X$*.A homomorphism* f *is called a monomorphism (resp., epimorphism) if it injective (resp., surjective). A bijective homomorphism is called an isomorphism. Two BH-algebra X and Y are said to be isomorphic, written* $X \cong Y$, *if there exists an isomorphism* $f : X \longrightarrow Y$. *For any homomorphism* $f : X \longrightarrow Y$, *the set* $\{x \in X : f(x) = 0_Y\}$ *is called the kernel of* f, *denoted by Ker(f), the set* $\{f(x) : x \in X\}$ *is called image of* f, *denoted by* $Im(f)$. *Notice that* $f(0_X) = 0_Y$*.([11]), and the set* $\{x \in X : f(x) = y,\ for\ some\ y \in Y\}$*,is preimage of* f, *denoted by* $f^{-1}(Y)$ *([4]).*

Remark 2.11. *([9]) Let (X,*,0) be a BG-algebra and let N be a normal subalgebra of X. Define a relation* $\sim_N$ *on X by* $x \sim_N y$ *if and only if* $x * y \in N$, *where* $x, y \in X$. *Then it is easy to show* $\sim_N$ *is an equivealence relation on X. Denote the equivealence class containing x by* $[x]_N$, *i.e.* $[x]_N = \{y \in X : x \sim_N y\}$ *and let* $X/N = \{[x]_N : x \in X\}$. *If* $*'$ *denoted on* X/N *by* $[x]_N *' [y]_N = [x * y]_N$. *Then* $(X/N, *', [0]_N)$ *is a BG-algebra and it is called qutient bg-algebra of X by N. The authers in ([1]) generalized this concept to BH-algebra to obtain* $(X/N, *', [0]_N)$ *qutient BH-algebra of X by N .*

Remark 2.12. *Let* $\{(X_i, *_{X_i}, 0_{X_i}) : i \in \lambda\}$ *be a family of* $U_i - BG - BH - algebra$. *Define the cartesian product of all* X_i, $i \in \lambda$ *to be the structure* $\prod_{i\in\lambda} X_i = (\prod_{i\in\lambda} X_i, \circledast, (0_{X_i}))$, *where* $\prod_{i\in\lambda} X_i$ *is the set of tuples* $\{(x_i) : \forall i \in \lambda$ *and* $x_i \in X\}$, *and whose binary operation* $\circledast$ *is give by* $(x_i) \circledast (y_i) = (x_i *_{X_i} y_i), \forall i \in \lambda$ *and* $x_i,\ y_i \in X_i$. *Note that the binary operation* $\circledast$ *is componentwise.*

3 Main Results

In this section, we introduce the concepts of a $U-BG-filter$ of a $U-BG-BH-algebra$. Also, we study some properties of it with examples.

Definition 3.1. *A non-empty subset F of a $U-BG-BH-algebra$ X is called a $U-BG-filter$ of X, if it satisfies (F_1) and*
(F_3) If $x \in F$ and $x * y = 0$ then $y \in F$, $\forall\, y \in U$.

Example 3.2. *Consider the $U-BG-BH-algebra(X; *, 0)$, where $X = \{0, 1, 2, 3\}$ and * is the binary operation define by the following table:*

*	0	1	2	3
0	0	2	1	0
1	1	0	2	0
2	2	1	0	0
3	3	3	3	0

and $U = \{0, 1, 2\}$. The subset $F = \{1, 2\}$ is U-BG-filter, but the subset $F = \{1, 3\}$ is not a U-BG-filter of X, since $3 * (3 * 1) = 0 \notin F$.

Remark 3.3. *If X is a U-BG-BH-algebra. Then $\{0\}$ and X are a U-BG-filter of X, called trivial U-B-filters of X. A U-BG-filter F of X is called a proper U-BG-filter of X if $F \neq X$.*

Theorem 3.4. *Let X be a U-BG-BH-algebra and S is a subalgebra of X, satisfies the right cancellation low in X. Then S is a U-BG-filter of X.*

Proof. **(i)** Let $x, y \in S$, then $x * y \in N$ and $y * x \in N$, using Definition(2.6). So $y * (y * x) \in S$ and $x * (x * y) \in S$.
(ii) Let $x \in S, x * y = 0, y \in U$, then $x * y = y * y$, [by Definition(2.1)(I)]. We obtian $x = y$, [by using the right cancellation low], so $y \in S$. Therefore S is a U-BG-filter of X. □

Proposition 3.5. *Let X be a U-BG-BH-algebra. Then every filter of X is a U-BG-filter of X.*

Proof. *Is obvious. [Since $U \subseteq X$ and F is a filter of X].* □

Remark 3.6. *The convers of proposition (3.5) is not correct in general as in the following example. Consider the U-BG-BH-algebra X in example(3.2). The subset $F = \{1, 2\}$ is a $U-BG-filter$ of X, but it is not a filter since $1 \in F$ and $1 * 3 = 0$ but $3 \notin F$.*

Theorem 3.7. *Let X be a medail U-BG-BH-algebra. Then every a non-empty subset A of X is a U-BG-filter of X.*

Proof. Let A be a non-empty subset of X.
(i) Let $x,\ y\ \in A$. Then $x = y*(y*x)$[By Definition(2.8)]. Thus $y*(y*x) \in A$. Similarly, $x*(x*y) \in A$.
(ii) Let $x \in A, x*y = 0,\ y \in U$. Then $y = x*(x*y)$ [By Definition(2.8)], imply that $y = x*0$,then $y = x$ [By definition(2.3)(III)], so $y \in A$. Therefore, A is a U-BG-filter of X. □

Theorem 3.8. *Let X be a U-BG-BH-algebra, and F be a U-BG-filter of X such that $x*y \neq 0$, $\forall y \notin F$ and $x \in F$. Then F is a filter of X.*

Proof. Let F be a U-BG-filter of X such that $y \in X$ and $x \in F$,
(i) Let $x,\ y\ \in F$,then $y*(y*x)$, $x*(x*y) \in F$ [By definition(3.1)(F_1)],
(ii) Let $x \in F, x*y = 0$,. Then we have two cases. **Cases(I):** If $y \in U$, then $y \in F$ [By definition(3.1)(F_3)]. **Cases(II):** If $y \notin U$ then either $y \notin F$ or $y \in F$. Suppose $y \notin F$, then $x*y \neq 0$, this a contradiction. Thus $y \in F$ Therefore , F is a filter of X. □

Proposition 3.9. *Let X be a U-BG-BH-algebra and let $\{F_i,\ i \in \lambda\}$ be a family of U-BG-filters of X.Then $\bigcap\limits_{i\in\lambda} F_i$ is a U-BG-filter of X.*

Proof. Let $\{F_i,\ i \in \lambda\}$ be a family of U-BG-filters of X. To prove $\bigcap\limits_{i\in\lambda} F_i$ is a U-BG-filter of X.
(i) If $x,\ y \in \bigcap\limits_{i\in\lambda} F_i$, then $x,\ y \in F_i$, $\forall i \in \lambda$. Hence $y*(y*x)$, $x*(x*y) \in F_i$ [since F_i is a U-BG-filter of X, $\forall\, i \in \lambda$, by definition(3.1)(F_1)]. Then $y*(y*x)$, $x*(x*y) \in \bigcap\limits_{i\in\lambda} F_i$.
(ii) Let $x \in \bigcap\limits_{i\in\lambda} F_i$ such that $x*y = 0$, $y \in U$. Then $x \in F_i\ \forall\, i \in \lambda$. Thus $y \in F_i$,[Since F_i is a U-BG-filter of X,$\forall\, i \in \lambda$, by definition(3.1)(F_3)]. Therefore, $\bigcap\limits_{i\in\lambda} F_i$ is a $U-BG-filter$ of X. □

Remark 3.10. *The union of $U-BG-filters$ of U-BG-BH-algebra may be not a $U-BG-filter$ as in the following example.*

Example 3.11. *Consider the U-BG-BH-algebra $X = \{0,1,2,3,4\}$ with binary operation $''*''$ defined by the following table:*

*	0	1	2	3	4
0	0	1	2	0	0
1	1	0	1	4	3
2	2	2	0	1	1
3	3	1	2	0	2
4	4	3	1	2	0

where U={0,1,2}. $F_1 = \{0,4\}$ *and* $F_2 = \{0,3\}$ *are two* $U - BG - filters$ *of* X*, The union of the* $U - BG - filters$ *is not a U-BG-filter of* X*. Since* $3, 4 \in F_1 \bigcup F_2$*, but* $3 * (3 * 4) = 2 \notin F_1 \bigcup F_2$.

Proposition 3.12. *Let* X *be a U-BG-BH-filter and let* $\{F_i,\ i \in \lambda\}$ *be a chain of U-BG-filters of* X*.Then* $\bigcup_{i\in\lambda} F_i$ *is a U-BG-filter of X.*

Proof. Let $\{F_i,\ i \in \lambda\}$ be a chain of U-BG-filters of X. To prove $\bigcup_{i\in\lambda} F_i$ is a U-BG-filter of X.
(i) If $x, y \in \bigcup_{i\in\lambda} F_i$, $\forall i \in \lambda$, then there exist $F_j, F_k \in \{F_i\}_{i\in\lambda}$ such that $x \in F_j$ and $y \in F_k$. So, either $F_j \subseteq F_k$ or $F_k \subseteq F_j$. If $F_j \subseteq F_k$, then $x \in F_k$ and $y \in F_k$, we have $y * (y * x) \in F_k$ and $x * (x * y) \in F_k$ [since F_k is a U-BG-filter of X, $\forall\, i \in \lambda$, by definition(3.1)(F_1)]. Similarly, if $F_k \subseteq F_j$. Then $y * (y * x)$, $x * (x * y) \in \bigcup_{i\in\lambda} F_i$.
(ii) Let $x \in \bigcup_{i\in\lambda} F_i$ such that $x * y = 0$, $y \in U$.There exists $j \in \lambda$ such that $x \in F_j$. Hence $y \in F_j$,[Since F_i is a U-BG-filter of X,$\forall\, i \in \lambda$, by definition(3.1)(F_3)]. Thus $y \in \bigcup_{i\in\lambda} F_i$. Therefore, $\bigcup_{i\in\lambda} F_i$ is a U-BG-filter of X. □

Proposition 3.13. *Let* X *and* Y *be U-BG-BH-algebras and* $f : (X, *_X, 0) \longrightarrow (Y, *_Y, 0_Y)$ *be a BH-homomorphism. Then* $ker(f)$ *is a U-BG-filter of X.*

Proof. **(i)** Let $x, y \in ker(f)$. Then $f(x) = 0_Y, f(y) = 0_Y$, so $f(y *_X (y *_X x)) = f(y) *_Y (f(y) *_Y f(x) = 0_Y$. Thus $y *_X (y *_X x) \in ker(f)$ Similarly, $x *_X (x *_X y) \in ker(f)$
(ii) Let $x \in ker(f)$ and $y \in U$. such that $x *_X y = 0_X$. Then $f(x) = 0_Y$. Now, $f(x *_X y) = f(x) *_Y f(y) = f(0_X) = 0_Y$. [By Proposition(2.10)]. So, $0_Y *_Y f(y) = f(y) *_Y f(y)$, [by Definition(2.1)(I)], we obtain $f(y) = 0_Y$,[by Lemma(2.2)(i)]. Therefore, $y \in ker(f)$, [By Remark(2.10)]. Then $ker(f)$ is a U-BG-filter of X. □

Theorem 3.14. *Let* $f : (X, *_X, 0_X) \longrightarrow (Y, *_Y, 0_Y)$ *be a U-BG-BH-monom orphism, and let* F *be a* $U - BG - filter$ *of* X*,such that* $f(U)$ *is a BG-algebra of X. Then* $f(F)$ *is a* $f(U) - BG - filter$ *of* Y*.*

Proof. Let F be a U-BG-filter of X.
(i) *Let* $x,\ y \in f(F)$*.Then there exist* $a,\ b \in F$ *such that* $x = f(a),\ y = f(b)$*. Then* $y *_Y (y *_Y x) = f(b) *_Y (f(b) *_Y f(a) = f(b) *_Y (f(b *_X a)) = f(b *_X (b *_X a)) \in f(F)$*.[Since* $b *_X (b *_X a) \in F$*, by Definition(3.1)(*F_1*)]. Hence* $y *_Y (y *_Y x) \in f(F)$*. Similarly,* $x *_Y (x *_Y y) \in f(F)$*.*
(ii) *Let* $x \in f(F)$ *such that* $x *_Y y = 0_Y, y \in f(U)$*.Then there exist* $a \in F$ *and* $b \in U$ *such that* $x = f(a)$ *and* $y = f(b)$*. Now,* $x *_Y y = f(a) *_Y f(b) = f(a *_X b) = 0_Y = f(0_X)$*. Then* $a *_X b = 0_X$*, [since f is an injective]. Thus,* $b \in F$*, [by definition(3.1)(*F_3*)]. So,* $y = f(b) \in f(F)$*. Therefore, f(F) is a U-BG-filter of X.* □

Theorem 3.15. *Let* $f : (X, *_X, 0_X) \longrightarrow (Y, *_Y, 0_Y)$ *be a U-BG-BH-epimor phism, such that* $f^{-1}(U)$ *is a BG-algebra of X. If* F *be a U-BG-filter of* Y*. Then* $f^{-1}(F)$ *is* $f^{-1}(U) - BG$*-filter of* X*.*

Proof. Let F be a U-BG-filter of Y.
(i) Let $x,\ y \in f^{-1}(F)$. Then $f(x),\ f(y) \in F$.
So $f(y) *_Y (f(y) *_Y f(x)) \in F$,[since F is a U-BG-filter of Y]. Thus, $f(y) *_Y (f(y) *_Y f(x)) = f(y *_X (y *_X x)) \in F$,[since F is a U-BG-filter of Y]. Therefore, $y *_X (y *_X x) \in f^{-1}(F)$. Similarly, $x *_X (x *_X y) \in f^{-1}(F)$.
(ii) Let $x \in f^{-1}(F)$ such that $x *_X y = 0_X, y \in f^{-1}(U)$. Then $f(x) \in F$ and $f(x *_X y) = f(x) *_Y f(y) = f(0_X) = 0_Y, f(y) \in U$, Hence $f(y) \in F$. Thus $y \in f^{-1}(F)$. Therefore, $f^{-1}(F)$ is a U-BG-filter of X. □

Theorem 3.16. . *Let* X *be a* $U - BG - BH-$ *algebra,* N *be a normal subalgebra of* X *and* U/N *is a BG-algebra, such that* $(X/N, *', [0]_N)$ *is a* $U/N - BG - BH-$ *algebra. If* F *is a U-BG-filter of* X*, then* F/N *is a* $U/N - BG-$*filter of* X/N*.*

Proof. Let X be a U-BH-BH-algebra, and let F be a U-BG-filter of X. To prove F/N is a $U/N - BG - filter$ of X/N.
(i) Let $[x]_N, [y]_N \in F/N$, then $[y]_N *' ([y]_N *' [x]_N) = [y]_N *' [y * x]_N, = [y * (y * x)]_N$, Hence $[y]_N *' ([y]_N *' [x]_N) \in F/N$[Since $y * (y * x) \in F$, F is a U-BG-filter of X]. Similarly, $[x]_N *' ([x]_N *' [y]_N) \in F/N$.
(ii) Let $[x]_N \in F/N$ and $[y] \in U$, $[x]_N *' [y]_N = [0]_N$.
Since $[x]_N *' [y]_N = [0]_N$, then $[x * y]_N = [0]_N$, Hence $(x * y) * 0 \in N$. [By Definition(2.11)], So $x * y \in N$, then $y \in [x]_N$. We obtain $[y]_N = [x]_N$, then $[y]_N \in F/N$. Therefore, F/N is a $U/N - BG - filter$ of X/N. □

Theorem 3.17. *Let* $\{(X_i, *, 0_i) : i \in \lambda\}$ *be a family of* $U_i-BG-BH-$*algebras. Then* $(\prod_{i\in\lambda} X_i, \circledast, (0_i))$ *is a* $\prod_{i\in\lambda} U_i - BG - BH-$*algebra.*

Proof. 1. To prove $(\prod_{i\in\lambda} X_i, \circledast, 0_{X_i})$ is a BH-algebra.

(i) Let $(x_i) \in \prod_{i\in\lambda} X_i$, $\forall i \in \lambda$, and $x_i \in X_i$. Then $(x_i)\circledast(x_i) = (x_i \circledast_{X_i} x_i) = (0_{X_i})$, [Since $x_i *_{X_i} x_i = 0_{X_i}, \forall\, i \in \lambda$ and $x_i \in X_i$],

(ii) Let $(x_i), (y_i) \in \prod_{i\in\lambda} X_i, \forall\, i \in \lambda$ and $x_i,\, y_i \in X_i$

such that $(x_i) \circledast (y_i) = (0_{X_i})$, and $(y_i) \circledast (x_i) = (0_{X_i})$, then $(x_i *_{X_i} y_i) = (0_{X_i})$, and $(y_i *_{X_i} x_i) = (0_{X_i})$. Then $x_i *_{X_i} y_i = 0_{X_i}$ and $y_i *_{X_i} x_i = 0_{X_i}$. So, $x_i = y_i$, $\forall\, i \in \lambda$, $x_i \in X_i$. Therefore, $(x_i) = (y_i)$.

(iii) Let $(x_i) \in \prod_{i\in\lambda} X_i$, $\forall i \in \lambda$, and $x_i \in X_i$. So, $(x_i)\circledast(0_i) = (x_i *_{X_i} 0_i) = (x_i)$, [Since $x_i *_{X_i} 0_i = x_i$, $\forall\, i \in \lambda$ and $x_i \in X_i$, by definition(2.3)(III)].

Therefore, $(\prod_{i\in\lambda} X_i, \circledast, (0_i))$ is a BH-algebra.

2. $|\prod_{i\in\lambda} U_i| \geq 2,$ [Since $|U_i| \geq 2$].

3. To prove $\prod_{i\in\lambda} U_i$ is a BG-algebra. Let $(x_i) \in \prod_{i\in\lambda} U_i, \forall\, i \in \lambda$ and $x_i \in U_i$. It is clear that **(i)** $(x_i) \circledast (x_i) = (0_i)$ and **(ii)** $(x_i) \circledast (0_i) = (x_i)$, $\forall\, i \in \lambda$, $x_i \in X_i$. Now, **(iii)** Let $(x_i), (y_i) \in \prod_{i\in\lambda} X_i$, $\forall i \in \lambda$, x_i, $y_i \in X_i$, So $((x_i) \circledast (y_i)) \circledast ((0_i) \circledast (y_i)) = (x_i *_{X_i} y_i) \circledast (0_i *_{X_i} y_i) = ((x_i *_{X_i} y_i) *_{X_i} (0_i *_{X_i} y_i)) = (x_i)$, [since U_i is a BG-algebra]. So $\prod_{i\in\lambda} U_i$ is a BG-algebra. Therefore, $(\prod_{i\in\lambda} X_i, \circledast, (0_i))$ is a $\prod_{i\in\lambda} U_i - BG - BH-$algebra.

□

Theorem 3.18. *Let* $(\prod_{i\in\lambda} X_i, \circledast, (0_{X_i}))$ *is a* $\prod_{i\in\lambda} U_i - BG - BH-$*algebra. If* $\{F_i : i \in \lambda\}$ *be a family of* $U_i - BG-$*filters of* X_i*. Then* $\prod_{i\in\lambda} F_i$ *is a* $\prod_{i\in\lambda} U_i - BG - filter$ *of the product algebra* $\prod_{i\in\lambda} X_i$*.*

Proof. **(i)** Let $x = (x_i),\ y = (y_i) \in \prod_{i\in\lambda} F_i$, $\forall\, x_i,\ y_i \in F_i$, and $i \in \lambda$,

$y \circledast (y \circledast x) = (y_i) \circledast ((y_i) \circledast (x_i)) = (y_i *_{X_i} (y_i *_{X_i} x_i)) \in \prod_{i\in\lambda} F_i$, [Since $y_i *_{X_i} (y_i *_{X_i} x_i) \in F_i$,by Defintion(3.1)($F_1$)],

(ii) Let $(x_i) \in \prod_{i\in\lambda} F_i$, and $(y_i) \in \prod_{i\in\lambda} U_i$ such that $(x_i) \circledast (y_i) = (0_{X_i})$, $\forall i \in \lambda$, x_i, $y_i \in X_i$,

Then $(x_i *_{X_i} y_i) = (0_{X_i})$, $y_i \in U_i, \forall\, i \in \lambda$.

So $x_i \in F_i,\ x_i * y_i = 0_i,\ y_i \in U_i, \forall\, i \in \lambda$, Hence $y_i \in F_i$, [Since F_i is a $U_i - BG - filter$ of X_i], then $(y_i) \in \prod_{i\in\lambda} F_i$. Therefore, $\prod_{i\in\lambda} F_i$ is a $\prod_{i\in\lambda} U_i - BG - filter$ of $\prod_{i\in\lambda} X_i$. □

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