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Improved efficient static weighted routing strategy on two-layer complex networks

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Global static routing is one kind of important routing algorithms for complex networks, especially in large communication networks. In this paper, we propose a heuristic global static routing algorithm to mitigate traffic congestion on two-layer complex networks. The proposed routing algorithm extends the relevant static weighted routing algorithm in the literature [Y. Zhou, Y. F. Peng, X. L. Yang and K. P. Long, *Phys. Sci.* **84**, 055802 (2011)]. Our routing path is constructed from a proper assignment of edge weights by considering the static information of both layers and an adjustable parameter α. When this routing algorithm is adopted on BA–BA two-layer networks with an appropriate parameter α, it can achieve the maximum network traffic capacity compared with the shortest path (SP) routing algorithm and the static weighted routing algorithm.

Keywords: Layered complex network; routing strategy; scale-free network; network capacity; average path length.

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1. Introduction

Since the discovery of small-world networks by Watts and Storogatz[1] and scale-free networks by Barabási and Albert,[2,3] many large scale communication networks such as the Internet and World Wide Web have been found that their degree distributions follow the power-law distribution.[4] These networks can be properly described as complex networks, and their transmission capacity is of great importance to people's works and lives. Therefore, the study of dynamic process taking place in large complex networks has attracted a great deal of attention from statistical physics and engineering researchers, such as a relief of traffic congestion on complex networks and so on.[5]

Typically, there are three ways to enhance traffic capacity and relieve network congestion: modifying underlying network topology, reallocating network traffic resources and developing new routing strategies. The former two methods are considered as the "hard" strategies, because they require changing of the topological structure and the network resources. In the past few years, a lot of influential works have been presented.[6–27] However, changing network topology or reallocating network resources usually costs remarkable expense, manpower or even energy. On the contrary, improving routing strategy has been proved to be an effective method which can be easily implemented by software methods. The traditional shortest path (SP) routing is widely used in various types of communication networks.[28,29] However, this routing strategy often easily leads to congestions on hub nodes. To solve this problem, many heuristic routing strategies have been proposed in Refs. 30–47. Yan *et al.* proposed a routing strategy named the efficient routing in which the path between nodes i and j is denoted as $P_{ij} = \min \Sigma_{x=0}^{l} k_x^{\beta}$, where k_x is the degree of node x, l is the path length and β is a tunable parameter.[30] Naganuma and Igarashi proposed a routing strategy using neural networks and adjustive rules of the connection weights.[39] Kawamoto and Igarashi investigated a new efficient packet routing strategy which mitigates traffic congestion on complex networks by minimizing the maximum betweenness.[42] In this routing strategy, the iteration steps for convergence is much less than the algorithm proposed by Danila *et al.*[35,36]

Most previous studies are applied only to one-layer networks. However, a lot of communication networks actually revealed to have a two-layer or multilayered structure.[48–50] Several networks mutually interact and depend on each other, and the structure of each layer may be different. For example, the peer-to-peer network is mapped on the underlying IP network.

In previous researches, Zhuo *et al.* proposed a novel efficient static weighted routing (SWR) strategy on two-layer complex networks. The weights are defined as follows: (1) the weight of the logical edges mapped on the hub nodes at the physical layer is assigned a higher weight according to the degree of nodes; (2) packets choose the routing path with the minimum sum of logical edges' weights.[49] In SWR strategy, there is an adjustable optimization parameter β, and the optimum value in BA on BA networks is $\beta = 0.8$. In this paper, we propose an improved static weighted routing (ISWR) strategy based on the static information of both layers. Under our routing strategy, the packets are able to bypass the large degree nodes in logical-layer and physical-layer. The distribution of packets becomes evener. It can effectively relieve the congestion and improve the network capacity of the two-layer complex network.

The paper is organized as follows. In the Sec. 2, the models are introduced. In Sec. 3, the ISWR strategy is presented in detail. In Sec. 4, simulation results of traffic dynamics are provided. Finally, we conclude the paper in Sec. 5.

2. The Models

2.1. *Two-layer network model*

In two-layer complex network model, the underlying layer is called physical-layer and the upper layer is called logical-layer. The logical-layer builds on the physical-layer. The nodes of logical-layer send packets to the destination nodes according to the given logical-layer routing table. Each logical link between two nodes on the logical-layer is mapped to the physical-layer routing path between the two nodes on the physical-layer. It seems that the logical-layer treats the physical-layer as a "black box" that provides a stable network connection. Meanwhile, the physical-layer does not care about the logical-layer routing table, that is, it is only in charge of sending packets from one end of the logical link to the other end according to the given physical-layer routing table. A simple example illustrates the above descriptions in Fig. 1. For simplicity, here we assume the routing strategy is the SP for both two layers. The routing path between node 1 and node 4 is $RP_{1,4} = \{1, 2, 3, 4\}$ on the logical-layer, the logical links $l_{1,2}$, $l_{2,3}$ and $l_{3,4}$ are mapped to the physical-layer SP $\{1, 6, 2\}$, $\{2, 5, 3\}$ and $\{3, 4\}$, respectively. The physical-layer packet flow reaches its destination along $RP'_{1,4} = \{1, 6, 2, 5, 3, 4\}$.

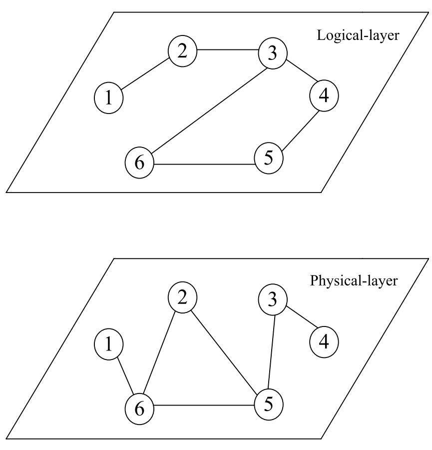

Fig. 1. Illustration of a two-layer network model, the upper layer is logical-layer and the lower layer is physical-layer.

2.2. *Network topology*

Recent researches on complex networks indicate that in many complex networks including the Internet, WWW and metabolic networks, degree distributions obey the power-law distribution $p(k) \sim k^{-r}$. The Barabsi–Albert (BA) scale-free network model is a well-known model which can construct networks with a power-law degree distribution. In our study, we generate the underlying traffic networks by using the BA network model. This model can be defined as follows: starting from m_0 fully connected nodes as seeds in an initial network, at each time-step, a new node with m edges ($m \leq m_0$) is added into the existing network according to the preferential attachment, i.e. the probability that a new node connects with an existing node x is proportional to the degree k_x of the node, $P(k_x) = k_x/\Sigma_y k_y$.

2.3. *Traffic model*

In this paper, there are R packets with randomly chosen sources and destinations generated in the logical-layer at each time-step. The logical-layer calculates the logical paths for each packet according to the logical-layer routing strategy. We assume that each node on the physical-layer can both receive and send packets at the same time. The queue length of each node in the physical-layer is infinite. The first-in-first-out (FIFO) packet forwarding discipline is applied to each node's queue. Once a packet is generated, it is placed at the end of the queue. The physical-layer is in charge of sending packets from one end of the logical link to the other end according to the physical-layer routing strategy. When the packet arrives at the last node of a logical link, the next logical link is selected by the logical-layer until the packet arrives its destination. At each time-step, the nodes of physical-layer choose C packets from the front of their queue and send them to the next nodes according to the given physical paths. Here, C is called the nodes delivering capacity (we set $C = 2$ for all physical nodes). Once one packet arrives at its destination, it will be removed from the system.

In communication networks, the maximum capacity for normal data transmission and processing is called the network capacity. This network capacity can be evaluated by the critical packet generation rate R_c. At $R = R_c$, the network undergoes a phase transition from a free-flow state to a congestion state. When $R < R_c$, i.e. packet generation rate is less than the critical value R_c, the number of generated packets roughly equals to the number of removed packets, and therefore the network is in a free-flow state. When $R > R_c$, the network is congested, and the generated packets number exceeds the network processing limitation. In each time-step, only a certain number of packets can reach their destination nodes. Meanwhile, other packets will continue accumulating in the network, causing a congestion in the whole network system. To accurately measure the network capacity of the two-layer complex network, we use the order parameter introduced in Ref. 51:

$$\eta(R) = \lim_{t\to\infty} \frac{C}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}, \tag{1}$$

where $\Delta N_p = N_p(t+\Delta t) - N_p(t)$, $\langle \ldots \rangle$ indicates the average over time windows of width Δt, and $N_p(t)$ denotes the number of packets in the network at time t. When $R < R_c$, $\eta = 0$, it means that the network system is in the free-flow state. While $R > R_c$, $\eta > 0$, it means that the packets are accumulating in the network system and the network is congested. Therefore, R_c is the maximal packet generating rate under which the system can maintain a free-flow state.

3. Routing Strategy

We have known that each layer has its own schemes. The physical-layer provides services to its logical-layer. In this paper, the numbers of nodes in the logical-layer and the physical-layer are assumed to be same. While node i^l and node j^l are connected in the logical-layer, in the physical-layer there may exists a different path between node i^p and node j^p. In physical-layer, the path of node i^p to node j^p can be notated as $P(i^p \to j^p) := i^p \equiv x_0^p, x_1^p, \ldots, x_{n-1}^p, x_n^p \equiv j^p$, and the edge between any two connected node x^p and y^p is given a weight $\omega_{x^p y^p} = (k_{x^p})^\alpha + (k_{y^p})^\alpha$ (k is the degree of node of physical-layer and α is a tunable parameter). Then for any path from node i^p to node j^p in the physical-layer, the weight can be denoted

$$\Omega_{i^p j^p} = \sum_{m=0}^{n} \omega_{x_m^p x_{m+1}^p} = \omega_{x_0^p x_1^p} + \cdots + \omega_{x_{n-1}^p x_n^p}, \tag{2}$$

where n is the path length. The optimal path we choose for delivering data packets is the one which minimizes the weight Ω. If there are more than one optimal path, one will be chosen randomly. The process of physical-layer routing table is described as follows:

- Calculate the degree of each node of the original physical-layer networks, $[k_{1^p}, k_{2^p}, \ldots, k_{n-1^p}, k_{n^p}]$.
- Set the weight of each edge $\omega_{i^p j^p}$ as $((k_{i^p})^\alpha + (k_{j^p})^\alpha)$.
- Calculate the physical-layer routing table of the weighted network through the use of the SP routing strategy with the use of $\Omega_{i^p j^p}$ in Eq. (2).

The data packets are generated at logical-layer. The physical-layer is responsible for sending and receiving data packets. In both logical-layer and physical-layer, the packets tend to move to the hub nodes and accumulate on those nodes. Therefore, the routing paths of logical-layer are necessary to bypass these hub nodes. In the logical-layer, the edge between any two connected nodes x^l and y^l is given a weight $\omega_{x^l y^l} = (k_{x^l})^\alpha + (k_{y^l})^\alpha + \Omega_{x^p y^p}$ (k is the degree of node of the logical-layer, α is a tunable parameter and $\Omega_{x^p y^p}$ is the physical path weight between node x^p and y^p). And the path weight from node i^l to node j^l is defined as

$$\Omega_{i^l j^l} = \sum_{m=0}^{n} \omega_{x_m^l x_{m+1}^l} = \omega_{x_0^l x_1^l} + \cdots + \omega_{x_{n-1}^l x_n^l}. \tag{3}$$

The optimal path we choose the one which makes the above weight a minimum. Similarly to the process of the physical-layer routing table, the process of the logical-layer routing table can be described as follows:

- Calculate the degree of each node of the original logical-layer networks, $[k_{1^l}, k_{2^l}, \ldots, k_{n-1^l}, k_{n^l}]$.
- Calculate the any path weight of physical-layer, notated as $\Omega_{i^p j^p}$.
- Set the weight of each edge of logical-layer $\omega_{i^l j^l}$ as $((k_{i^l})^\alpha + (k_{j^l})^\alpha + \Omega_{i^p j^p})$.
- Calculate the logical-layer routing table of the weighted network through the use of the SP routing strategy with the use of $\Omega_{i^l j^l}$ in Eq. (3).

According to the above descriptions, the edge connects to high-degree nodes has higher weight, and the traffic load of central edges will be mitigated significantly under our strategy. If there are more than one paths, one of them will be chosen randomly.

In Ref. 49, under the SWR strategy, for a physical path $P(i^p \to j^p) := i^p \equiv x_0^p, \ldots, x_n^p \equiv j^p$, the weight of the physical path associated with the logical link is defined as $\Omega_{i^p j^p} = L(P(i^p \to j^p) : \alpha) = \min \Sigma_{m=0}^{n} (k(x_m^p))^\alpha$, where $k(x_m^p)$ is the degree of node x_m^p at the physical-layer and α is an adjustable parameter. The weight definition of logical link is similar to the efficient path of efficient routing strategy. For every packet from source node i to destination node j, the routing path of the logical-layer is defined as $P_{i^l j^l} = \min \Sigma_{m=0}^{n} \Omega_{i^p j^p}$, where $\Omega_{i^p j^p}$ is the weight of the logical link between node i and node j and n is the path length of the logical-layer.

Since the topologies of physical-layer and logical-layer are different, they have different center nodes. Under the SWR strategy, physical-layer routing can redistribute heavy traffic loads from center nodes to other noncenter nodes to improve traffic capacity of the whole network. For the logical-layer, the routing strategy can bypass the center nodes of the physical-layer. However, it cannot bypass the center nodes of the logical-layer. Since the SWR strategy do not consider center nodes of logical-layer, the weight of logical link cannot accurately reflect the hub nodes on the logical-layer network. In our proposed strategy, the weight of logical link is $\omega_{x^l y^l} = (k_{x^l})^\alpha + (k_{y^l})^\alpha + \Omega_{x^p y^p}$. The routing paths of logical-layer can bypass hub nodes of both layers, so it is more suitable for the two-layer networks than SWR. In the next section, the simulation results will be illustrated.

4. Simulation Results

In this section, we first investigate the variations of order parameter η versus packet generating rate R with various α under ISWR routing in Fig. 2(a). We use the BA on BA two-layer network with network size $N = 400$, and $m = m_0 = 4$ for simulation. One can see that for relatively small R and various α, η is approximately equal to zero, which implies that the networks are in the free-flow state; while R is larger than the critical packet generating rate R_c, η suddenly increases, which means that the

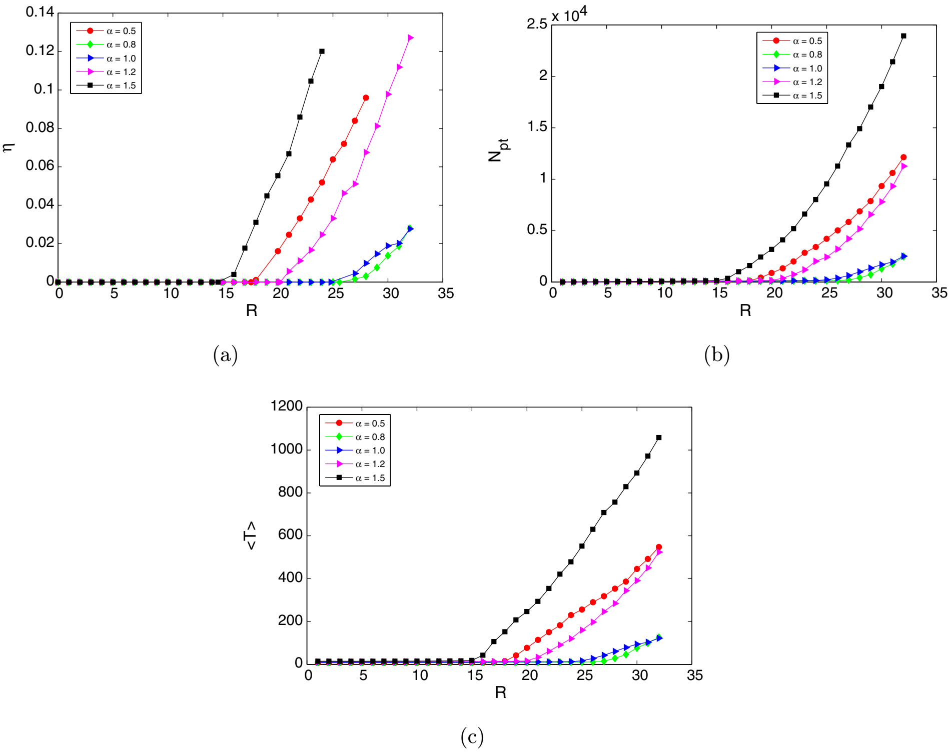

Fig. 2. (Color online) (a) η versus packet generating rate R with different α under ISWR routing; (b) packet number N_{pt} in the network versus packet generating rate R with different α under ISWR routing and (c) average packet travel time $\langle T \rangle$ versus packet generating rate R with different α under ISWR routing. The BA on BA networks' parameters are $N = 400$ and $m = m_0 = 4$.

packets accumulate in the networks. Obviously, for the different α the network capacity is not the same. Figures 2(b) and 2(c) show the results of the packet number $N_{\rm pt}$ in the network and average packet travel time $\langle T \rangle$ with changing R. When R is larger than the critical packet generating rate R_c, traffic congestion occurs in the network, and therefore $N_{\rm pt}$ and $\langle T \rangle$ increases with R. We can see that the critical values of packet generating rate R_c are the same as in Fig. 2(a). As shown in Fig. 2, when $\alpha = 0.5$, $R_c \approx 17$; when $\alpha = 0.8$, $R_c \approx 26$; when $\alpha = 1.0$, $R_c \approx 25$; when $\alpha = 1.2$, $R_c \approx 20$ and when $\alpha = 1.5$, $R_c \approx 15$. We can find R_c is not a monotonic function of α, and there is an optimal value of parameter α corresponding to the largest R_c.

Figure 3 shows the results of the R_c as a function of parameter α at BA on BA two-layer networks under the ISWR routing (the size of network is $N = 400$, and $m = m_0 = 4$). One can see that R_c increases with the parameter α at first and then decreases. The maximum value of R_c can be obtained for $\alpha \approx 0.8$. From the simulation results of Fig. 3, we can conclude that the optimal value of parameter α is

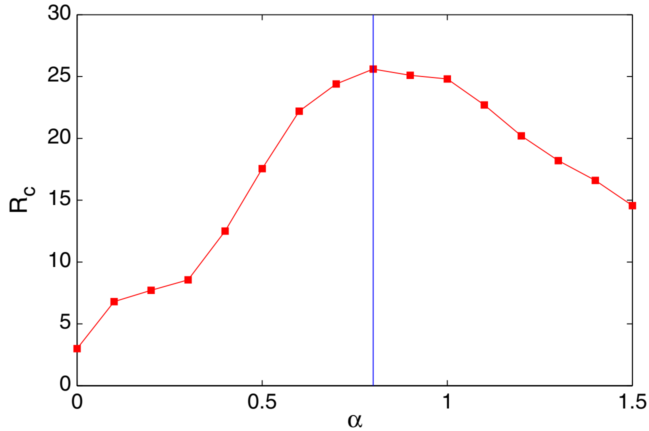

Fig. 3. (Color online) R_c versus α at BA on BA two-layer networks under the ISWR routing. These values are the averages of more than 20 realizations of BA on BA scale-free networks. The BA on BA networks' parameters are $N = 400$ and $m = m_0 = 4$.

approximately equal to 0.8, and we will use $\alpha = 0.8$ for our routing strategy in the following simulations.

In the above series of simulations, we compare different parameters η versus R, $N_{\rm pt}$ versus R, $\langle T \rangle$ versus R and R_c versus α under the same ISWR routing strategy. In Fig. 4, the order parameter η versus the packet generating rate R is shown for three different routing strategies. For the sake of comparison, the two other routing strategies are SP and SWR. In our extensive simulations at BA on BA two-layer networks with average degree $\langle k \rangle = 8$ and network size $N = 400$, the critical packet generating rate R_cs of SP, SWR, ISWR are 2.5, 15 and 25.6, respectively. The R_c of

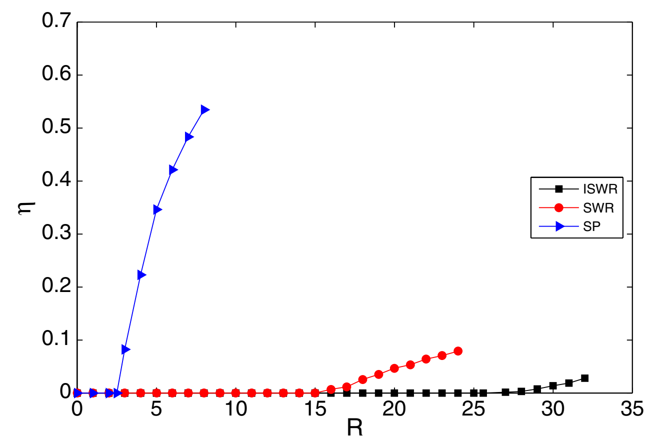

Fig. 4. (Color online) The order parameter η as a function of packet generating rate R for the two-layer networks under three routing strategies. The BA on BA networks' parameters are $N = 400$ and $m = m_0 = 4$. These values are the averages of more than 20 realizations.

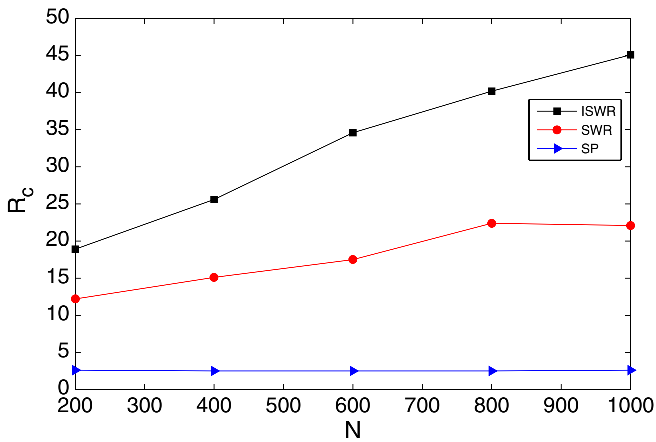

Fig. 5. (Color online) R_c versus network size N with average degree $\langle k \rangle = 8$ under three different routing strategies. These values are the averages of more than 20 realizations.

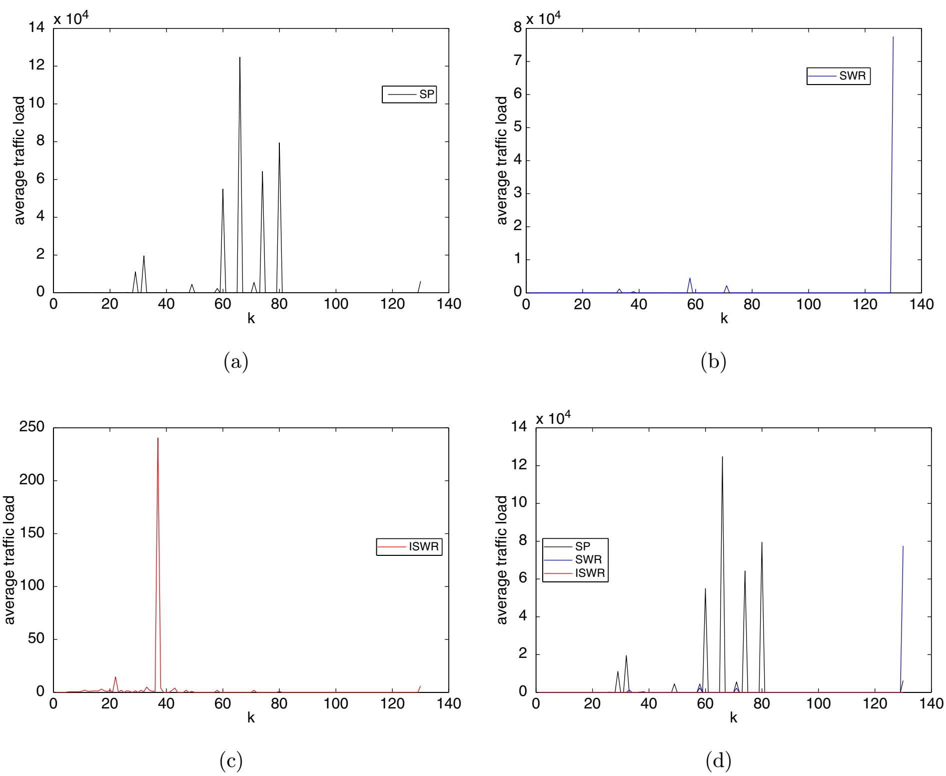

Fig. 6. (Color online) Distribution of average traffic load versus node degree k in logical-layer, (a) under SP routing; (b) under SWR routing; (c) under ISWR routing and (d) under three routing.

our ISWR routing strategy is 10.24 times more than that of the SP, while the R_c of the SWR is 6 times more than that of the SP. Obviously, one can see that our routing strategy has better performance.

In Fig. 5, we investigate the scalability of our routing strategy by the growth trends of R_c when network size N evolves. Conceptually, the scalability can be viewed as an important designing objective of routing strategy. Figure 5 presents the R_c values for BA on BA two-layer networks (the network size N ranges from 200 to 1000 and average degree $\langle k \rangle = 8$) under three different routing strategies. One can see that the R_c value of SP remains quite stable; while the R_cs of SWR and ISWR increase with the network size N for a fixed network average degree. For the R_c of ISWR, the larger network size is, the larger R_c is. So, our strategy is more suitable for large scale complex systems. From the comparisons, it is shown that our ISWR routing strategy can improve the network capacity better than other two routing strategies. This can be explained easily because the ISWR strategy can redistribute the heavy traffic loads on several different paths to avoid packets pass through the central nodes for both logical-layer and physical-layer. Figures 6 and 7 show the

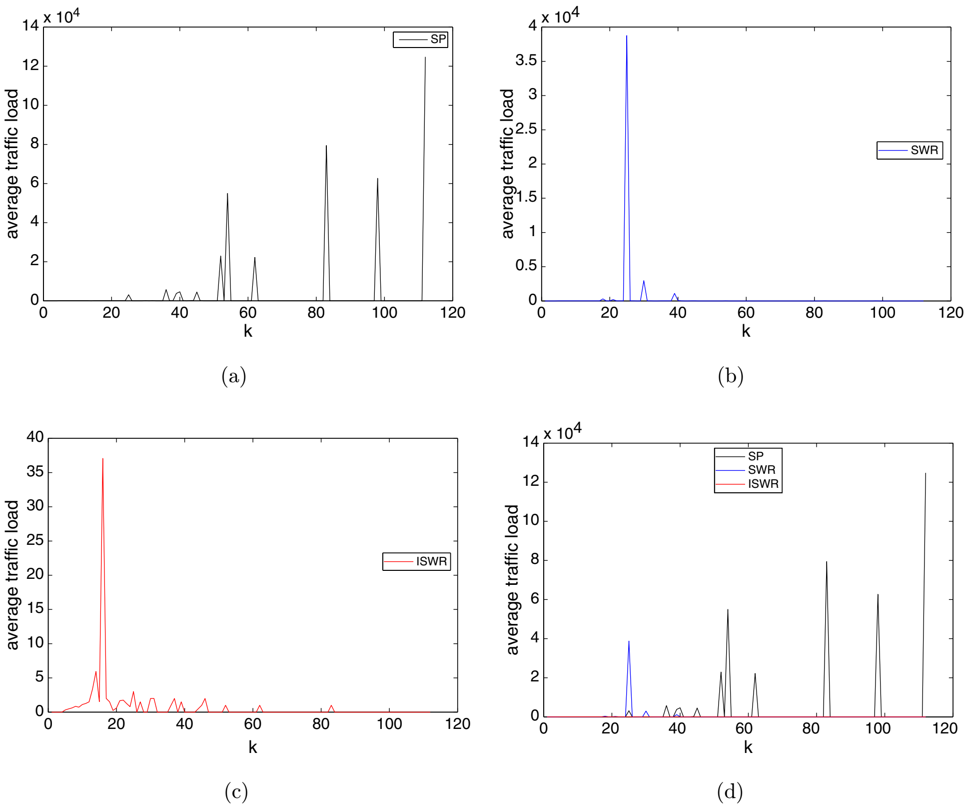

Fig. 7. (Color online) Distribution of average traffic load versus node degree k in physical-layer, (a) under SP routing; (b) under SWR routing; (c) under ISWR routing and (d) under three routing.

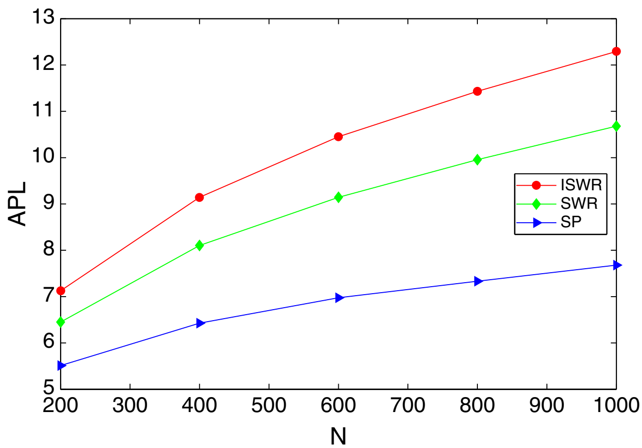

Fig. 8. (Color online) The average SP length APL versus network size N for different routing strategies. The BA networks' parameters are $m_0 = m = 4$. The simulation results are the averages by running over 20 independent networks.

distribution of average traffic load versus node degree k in logical-layer and physical-layer under congested state ($R = 50$, BA on BA network size $N = 1000$ and average degree $\langle k \rangle = 8$). One can see that the average traffic load of our strategy is significantly lower than that of other two routing strategies.

Finally, we compare the average path length (APL) versus N under the three different routing strategies at BA on BA two-layer networks with $m = m_0 = 4$ in Fig. 8. It can be seen that ISWR routing strategy increases APL a little in comparison with other strategies. While a system requires a large network capacity, such a sacrifice of APL is worthwhile.

5. Conclusions

In summary, we proposed a heuristic routing strategy called the ISWR strategy to enhance the network capacity of the two-layer complex networks. Knowing the fact that the traffic congestion tends to appear on the high-degree nodes in both the logical-layer and the physical-layer, for the sake of performance, a more suitable assignment of the weight of the edges and a tunable parameter α were thus introduced in our proposed routing path algorithm. By adjusting the value of α, two-layer complex networks have optimal network capacities. Moreover, we found that there exists an optimal value of α leading to the maximal network capacity. In the simulations, we analyzed the critical values of R_c, the number of packets in the networks, the average packet travel time, average path length and so on. The extensive simulation results confirmed the effectiveness of our ISWR strategy when compared with the SP routing strategy and the static weighted routing strategy. The network capacity was improved several times at the expense of increasing a bit average length. We hope that our work may be useful for designing and optimizing of static routing strategies on some real two-layer networks, for example, peer-to-peer network on IP network. In future work, we will take the dynamic information of network into account to improve the two-layer complex network performance further.

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