

Transient security assessment methods

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Abstract

The paper provides a comparative study of three methods for transient security assessment of power systems: step-by-step simulation, Lyapunov's direct method and an empirical method based on the kinetic energy of the system. Different ways of formulating the Lyapunov algorithm are shown, and the basic concepts are summarised in a simple form. Particular emphasis is given to the application of the methods to relatively large practical problems with case studies and comparative results provided.

List of symbols

- D_i = damping of the i th machine
- $E_i = E_i \angle \delta_i$ = complex value of internal machine voltage behind the transient reactance
- I_i = complex value of injected current
- M_i = inertia, $M_0 = \sum_{i=1}^n M_i$
- P_{mi} = mechanical power of the i th machine
- P_{ei} = electrical power of the i th machine
- $[V], [I]$ = vectors of nodal voltages and currents
- $[Y]$ = admittance matrix
- $Y_{ik} = Y_{ik} \angle \theta_{ik} = G_{ik} + jB_{ik}$ = admittance value (reduced matrix)
- n = number of machines in multimachine systems
- δ_i = internal machine angle, $\delta_{ik} = \delta_i - \delta_k$
- ω_i = absolute machine speed, $\omega_{ik} = \omega_i - \omega_k$
- ω_0 = synchronous speed
- $p = d/dt$ = derivative
- $\sum_{i,k} = \sum_{i=1}^{n-1} \sum_{k=i+1}^n$
- S superscript implies value at the post-fault stable singular point.
- Re = real part
- $i \in j$ = subset of nodes directly connected by branches to node i in the reduced network

1 Introduction

The fundamental problem of power-system operation is to ensure that user demands are met at the lowest cost compatible with adequate continuity in supply and sufficiently small frequency and voltage deviation. The system operator is responsible for the minute-by-minute control of the system, and there are several problems which arise from carrying out such operation. With modern advances in electronics and telecommunication, a significant and relevant support for online operating decisions can be obtained from digital computers using advanced mathematical methods.

The basic problem for the power-system operator is to meet power-system commitments in the most economical way, making allowance for probable component outages. Fig. 1 illustrates the basic structure of a supervisory control scheme in which security assessment plays a vital role. The first function of security analysis is to determine whether the normal system is secure or not. The second function is to determine what corrective action should be taken when the system is insecure.

Two types of security assessment can be identified, the steady state and the transient. The steady-state security assessment examines the steady-state response of the system under credible outage conditions.⁸ Each contingency in the steady-state security analysis causes transients which can result in very undesirable electro-mechanical oscillation, loss of synchronism of generators or areas, trips of protection relays etc., which in turn can lead to dangerous conditions; to prevent such difficulties, fast corrective action must be taken.

With modern advancement in sparsity programming, diakoptical concepts and physically-oriented simplifications like the decoupling power effect, the steady-state security assessment can be carried out in a reasonable time on modern computers for on-line applications.

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However, the same does not apply to transient security assessment because of its dynamic nature.

A complete analysis of a single transient using step-by-step simulation makes demands on computer storage and time for a large system. Alternative analysis methods are available, but very few have the potential for use in a security-assessment mode. Fig. 2 illustrates the basic alternatives available and the paper discusses in particular two alternative energy direct methods and compares them against a step-by-step implicit simulation method using practical examples.

2 Model

The transient security assessment is evaluated for the system, operating at a particular loading condition, after it has been determined secure in the steady-state condition. Since the transient security analysis is carried out for different fault conditions and fault clearance times, and must be carried out in a limited calculation time, it is necessary to compromise on the accuracy of the model and to use simple machine representation with a constant voltage behind a transient reactance interconnected through a passive linear network. The

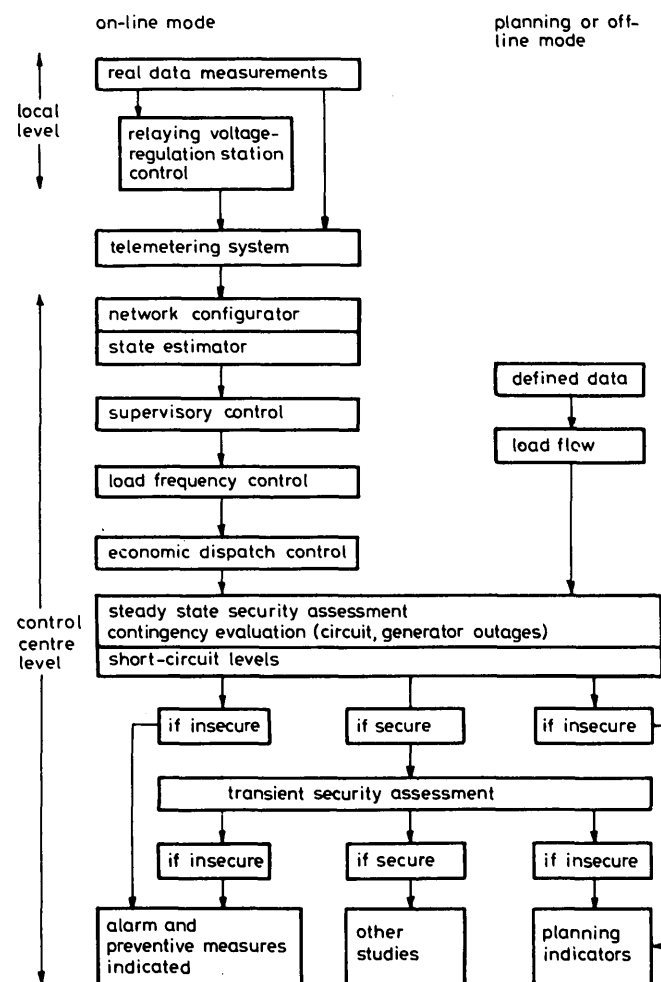


Fig. 1 Overall supervisory control scheme

appertaining equations are:

(a) Swing dynamic, differential equations:

$$M_i p^2 \delta_i + D_i p \delta_i = P_{mi} - P_{ei} \quad i = 1, 2, \dots, n \quad (1a)$$

(b) Electrical power output, nonlinear algebraic equations:

$$P_{ei} = \text{Re}(V_i I_i^*) \quad i = 1, 2, \dots, n \quad (1b)$$

(c) Kirchoff's nodal law, linear algebraic equations:

$$[Y][V] = [I] \quad (1c)$$

The set of eqns. 1 may be simplified, by network reduction, to the n internal machine nodes, and by substitution it reduces to $2n$ equations:

$$M_i p \omega_i + D_i \omega_i = P_{mi} - P_{ei} \quad (2a)$$

$$p \delta_i = \omega_i \quad i = 1, 2, \dots, n \quad (2b)$$

where

$$P_{ei} = E_i^2 Y_{ii} \cos \theta_{ii} + \sum_{j \neq i}^n E_i E_j Y_{ij} \cos(\delta_{ij} - \theta_{ij}) \quad (2c)$$

When uniform damping is considered, only $2n - 2$ state equations are necessary to represent the system adequately for control and stability purposes.⁵ If one machine, generally the one with the largest inertia, is specified arbitrarily as a reference, there are left $n - 1$ relative angles and $n - 1$ relative speeds as state variables. Taking the n th machine as reference, the new system of equations may be expressed as

$$p \delta_{in} = \omega_{in}$$

$$p \omega_{in} = M_i^{-1} (P_{mi} - E_i^2 G_{ii}) - M_n^{-1} (P_{mn} - E_n^2 G_{nn})$$

$$- E_i E_n Y_{in} \{ M_i^{-1} \cos(\delta_{in} - \theta_{in}) - M_n^{-1} \cos(\delta_{in} + \theta_{in}) \}$$

$$+ \sum_{j=1}^{n-1} \{ M_n^{-1} E_n E_j Y_{nj} \cos(\delta_{nj} - \theta_{nj})$$

$$- M_i^{-1} E_i E_j Y_{ij} \cos(\delta_{ij} - \theta_{ij}) \} - \frac{D_i}{M_i} \omega_{in}$$

$$i = 1, 2, \dots, n - 1 \quad (3)$$

$$\frac{D_i}{M_i} = \text{constant for all } i$$

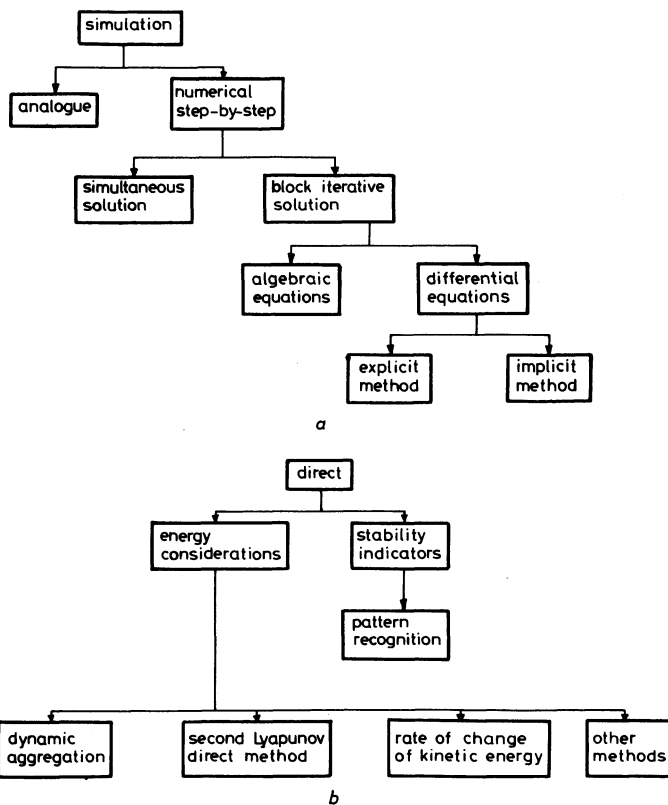


Fig. 2 Basic alternative methods for transient security assessment

a Simulation
b Direct

Eqn. 3 is a set of $2n - 2$ first-order nonlinear differential equations which may be stated in concise matrix form as

$$p[x] = [f(x)] \quad (4)$$

where x is the state vector of variables x_i and $[f(x)]$ is a vector of nonlinear functions $f_i(x)$.

The transient-stability concepts used by power-system engineers have the character of stability defined in the Lyapunov sense which refers to equilibrium or singular points over which perturbations are applied. A CIGRE definition states:

'A power system is said to be in a condition of transient stability with respect to a given disturbance if following this disturbance it returns to a condition of steady-state synchronous operation. The disturbance and both the initial and the final conditions of the power system must be completely defined'.

This postfault steady-state synchronous condition corresponds to an equilibrium or a singular point of the system, and the system condition at clearance time corresponds to the perturbed Lyapunov condition. The equilibrium points are the stationary points defined by $p[x] = 0$, that is, $[f(x^s)] = 0$ for all t values.

A nonlinear system has several such points, and for any one of them a change of the variables can be made such that $[f(o)] = 0$, and the point is then called the origin.

3 Step-by-step simulation

For comparison purposes, the simplified model expressed by eqn. 1 and an efficient numerical implicit trapezoidal integration procedure are used, giving numerical stable characteristics for large constant integration steps. The step length is halved after the switching events to improve accuracy, and is adjusted to cater for different switching times. The nonintegrable variable P_e is estimated at the beginning of every step, making use of linear extrapolation formulas to improve the rate of convergence of the iterative solution process. Efficient sparsity techniques are used in the network solution.

4 Lyapunov method

Lyapunov stability theorems are formulated as a comprehensive control and mathematical theory.⁹ For power-system application, stability is analysed around the postfault condition. If it is stable for small perturbations (steady-state stable), it is important to evaluate how stable it is for larger perturbations and how large the perturbations may be.

Lyapunov defines a real scalar function $V(x)$ of the system variables to establish stability theorems, which can be summarised as follows: The origin of the system $p[x] = [f(x)]$ is asymptotically stable if in its neighbourhood there is a function $V(x)$ such that

- (a) $V(x)$ has continuous partial derivatives
- (b) $V(o) = 0$
- (c) $V(x) > 0$ if $[x] \neq 0$
- (d) $pV(x) < 0$ if $[x] \neq 0$

The asymptotic stability character extends to all the bounded region where the above conditions apply, these being only sufficient conditions of stability. The concepts may be visualised by an approximate graphical interpretation as shown in Fig. 3.

Assuming that the graphs in Fig. 3 represent the equilibrium points in a system where point S is the origin or the postfault steady-state point of interest, then it can be easily seen that for small perturbations

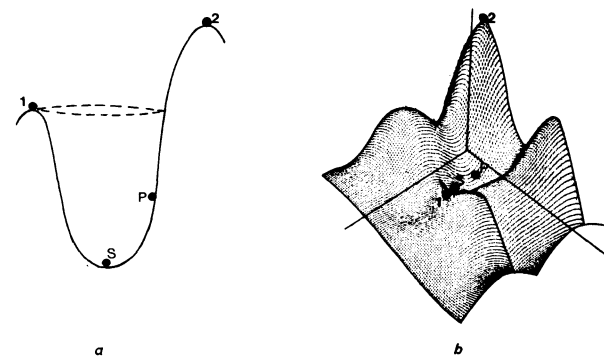


Fig. 3 Approximate graphical interpretation of stability criteria

a Planar representation
b Three-dimensional representation

point S is asymptotically stable. If, after fault clearance, the system is left at point P, it will eventually return to the equilibrium point. If the system is allowed to go further than the unstable equilibrium point 1, it may not necessarily return to S, and if it goes beyond the unstable equilibrium point 2, then it will not return to S. Point 1 is associated with the boundary of V_{min} which encloses the region where the Lyapunov conditions apply.

The difficulty is that the region of asymptotic stability obtained from the stated conditions clearly depends on the particular Lyapunov function chosen. Generally, it is only a subdomain of the total domain of interest. The larger the domain represented by the function, the more accurately can the stability margin be evaluated. If the function selected does not represent a sufficiently large domain, the results obtained are pessimistic, i.e. the system may be stable beyond the calculated V_{min} , but the stability criteria thus predicted are on the safe side for security-assessment purposes.

To obtain reasonably accurate and safe results, it is important to select a suitable Lyapunov function. From many functions mentioned in the literature, three most suitable energy-like functions are considered.^{3,6}

$$V_1 = \sum_{i,k} (\frac{1}{2} M_i M_k \omega_{ik}^2 - M_0 E_i E_k Y_{ik} \cos \delta_{ik} - \delta_{ik} F_{ik}) + V_k \quad (5)$$

$$V_2 = \sum_{i,k} (\frac{1}{2} M_i M_k \omega_{ik}^2 - M_0 E_i E_k B_{ik} \cos \delta_{ik} - \delta_{ik} S_{ik}) + V_k \quad (6)$$

where

$$F_{ik} = P_i M_k - P_k M_i$$

$$S_{ik} = M_0 E_i E_k B_{ik} \sin \delta_{ik}^S$$

$$P_i = P_{mi} - E_i^2 G_{ii}$$

and V_k makes the function null at the origin.

The two functions differ only in the term F_{ik} or S_{ik} and both neglect the transfer conductances of the post-fault reduced admittance matrix. V_2 neglects them by considering $Y_{ik} = jB_{ik}$ and V_1 by considering $Y_{ik} = jY_{ik}$. A third function, similar to V_2 in character but equal to V_1 in structure, is

$$V_3 = \sum_{i,k} (\frac{1}{2} M_i M_k \omega_{ik}^2 - M_0 E_i E_k Y_{ik} \cos \delta_{ik} - \delta_{ik} F_{ik}) + V_k \quad (7)$$

where

$$F'_{ik} = P'_i M_k - P'_k M_i$$

$$P'_i = P'_{ei} - E_i^2 G_{ii}$$

In programming the Lyapunov method on a digital computer, three separate stages were defined as shown in Fig. 4.

To improve computational efficiency and reduce storage the algorithm always uses reduced admittance matrices, although the Lyapunov function only requires it in the postfault stage. The actual reduction is carried out once only in the pre-fault stage using very efficient sparsity routines.¹⁰ The reduced but full matrix is then modified, with very little extra computation time, for different faults using simple diakoptical procedure.² The greatest area of interest is in the postfault stage where a number of simplifications must be made.

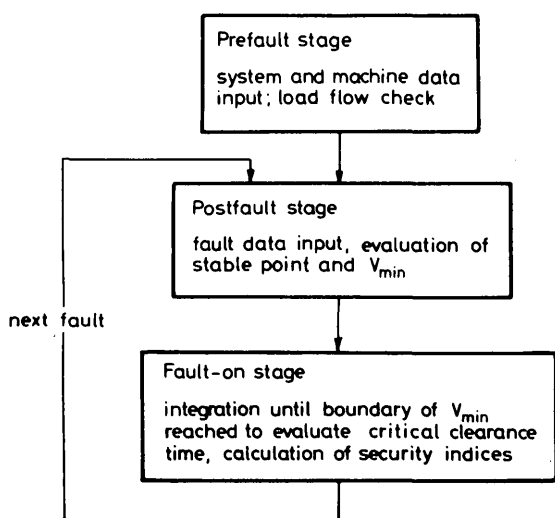


Fig. 4
Three stages in security analysis using Lyapunov approach

In addition to neglecting transfer conductance, other assumptions must be made in the search for the boundary of V_{min} , the unstable equilibrium point which gives a minimum V value. There may be $2(2^{n-1} - 1)$ of such points, and searching for them with an iterative technique is cumbersome and very time-consuming for large systems. Traditionally, points suggested by the one-machine-infinite-busbar system, $(\pi - \delta^S)$ and $(-\pi - \delta^S)$, have been used as starting points in the search. If these starting points are assumed to be relatively close to the real equilibrium points, then an approximate boundary V_{min} may be evaluated directly from them and the search restricted to the specific point which provides the minimum with sufficient accuracy.⁴ Further assumptions are made to determine which of the starting points are more likely to define V_{min} . The algorithm developed for the comparison test included one-machine-going-unstable cases only, since tests with two machines proved always to give larger V values.

Two alternative approaches, based on the Newton-Raphson method, were used to find the singular points; one neglected the transfer conductances (RNR) but was consistent with the Lyapunov function, and the other included conductances (FNR). The postfault stage of the security-analysis program may be summarised as shown in Fig. 5.

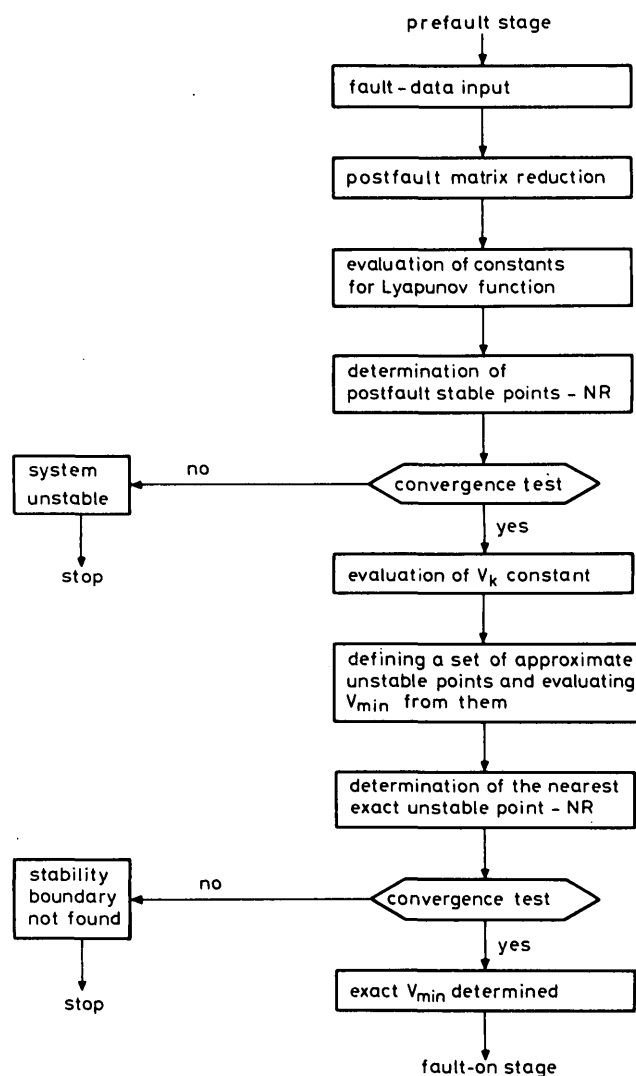


Fig. 5
Post-fault stages in security analysis using Lyapunov approach

5 Rate of change of kinetic energy method

Over the last 20 years the transient-stability behaviour of a power system has been analysed in some detail both theoretically and numerically. Although considerable progress has been made in understanding it, there is still a great deal of effort put in to finding a new approach for direct stability indices.

By empirically studying the different power-system energy forms and their interaction when a disturbance is present, several conclusions may be obtained, some of them clearly pointing in favour of the established Lyapunov method. Another important observation can be made between the system behaviour as a whole and the rate of change of its kinetic energy RKE .¹ When a fault occurs, the gener-

ators close to the fault speed up, and thus increase their kinetic energy, while the rest decelerate, until fault clearance when a new interaction begins. The rate of absorption or giving up of kinetic energy of each machine plays an important role in defining the final system behaviour. Empirically it can be shown that *RKE* at fault clearance time has a maximum negative value if it corresponds to the critical clearance time.

Practical results in some cases support the argument but the accuracy depends on machine loading, power factor and values of line reactance. Results giving underestimated and overestimated stability margins were obtained. For example, in a two-machine system the best results were obtained for a full load and a leading power factor. The kinetic energy of a rotating body is expressed by

$$KE = \frac{1}{2} I \omega^2 \quad (8)$$

The kinetic energy of a synchronous machine due to a disturbance is given by

$$KE_D = \frac{1}{2} M(\omega - \omega_0)^2 \quad (9)$$

and its rate of change is

$$RKE = pKE_D = M(\omega - \omega_0)p\omega \quad (10)$$

Assuming that synchronous speed does not change substantially, and neglecting damping, the rate of change for the machine can be expressed as

$$RKE = (\omega - \omega_0)(P_m - P_e) \quad (11)$$

and for a multimachine system, the total *RKE* is

$$RKE = \sum_{n=1}^n RKE_i$$

The *RKE* must be evaluated at fault clearance time with the system representation already in the postfault conditions. Fig. 6 shows the relation between *RKE* and clearance time for two different faults in a 12-machine system.

The program used for testing *RKE* is based on the program used for the Lyapunov test, and by integrating continuously the faulted system the postfault *RKE* equivalent to a clearance time can be obtained without actually clearing the fault. Integration is stopped when *RKE* becomes positive, and the time when it has a maximum negative value is taken as the critical clearance time.

Table 1
CHARACTERISTICS OF TEST SYSTEMS

System	Number of machines	Number of nodes	Ratio of largest and next largest inertia	Reference
A	4	6	33.34	3
B	5	12	3.46	Fig. 7
C	7	13	∞	Fig. 8
D	9	34	1.10	6
E	12	49	4.17*	7
F	33	195	11.6*	7

*Ratio between the average of the two largest and the next largest inertia

Table 2
CRITICAL CLEARANCE TIMES (s), FOR SMALL SYSTEMS

System	Faulted Busbar	Simulation*	Lyapunov method					<i>RKE</i> method
			RNR			FNR		
			V_1	V_2	V_3	V_1	V_2	
A	3	0.505	0.496	0.496	0.495	0.497	0.507	0.50
B	6	0.22	0.199	0.211	0.190	0.207	0.153	0.275
C	5	0.15	0.140	0.139	0.118	0.144	0.146	0.25
D	11	0.60	0.551	0.545	0.566	0.554	0.502	0.66

*Step-by-step trapezoidal integration method
For system B see Fig. 7, for system C see Fig. 8

Table 3
MACHINE GOING OUT OF STEP, FOR SMALL SYSTEMS

System	Faulted busbar	Simulation	Lyapunov method				
			RNR			FNR	
			V_1	V_2	V_3	V_1	V_2
A	3	3	3	3	3	3	3
B	6	generator 3	generator 3	generator 3	generator 3	generator 3	generator 3
C	5	busbar 15 Δ	busbar 15 Δ	busbar 15 Δ	busbar 15 Δ	busbar 15 Δ	busbar 15 Δ
D	11	1	9 Δ	9 Δ	1	9 Δ	1

Δ indicates the machine decelerates in relation to the rest.

6 Results

Several power systems, representing different system sizes and degrees of interconnection, were analysed and are shown in Table 1.

Current literature reports investigation on the Lyapunov method based on relatively small system sizes of up to nine machines, giving accurate results for critical clearance time (Table 2) and for the identity of the machine going out of step (Table 3). The three Lyapunov functions given by eqns. 6, 7 and 8 using the Newton-Raphson formulation, as well as a number of alternative functions not discussed in the paper, provide the same range of accuracy. The *RKE* method gives results with a wider range of error as shown in Table 2.

Systems E and F are more representative of actual practical problems, but unfortunately there is very little available information on the application of the Lyapunov method to such systems.

Tests carried out on the larger practical problems using the RNR formulation, combined with any of the three Lyapunov functions, show that it is not sufficiently accurate or reliable for security-assessment purposes. For example, the 12-machine system functions V_1 and V_3 are found to be negative in the neighbourhood of most of the post-fault stable points, while V_2 either gives overconfident, unsafe results or is found to be negative. Although for the 33-machine system V_3 gave somewhat better results than when V_1 and V_2 were used, it suggested starting values for the search of V_{min} points which gave no convergence.

The FNR formulation, on the other hand, produces in general more satisfactory results, although there is a considerable increase in the pessimism or underestimation of the stability margin in relation to the results obtained for the smaller systems. The underestimation may be acceptable since such an assessment is on the safe side as shown in Table 4. Function V_3 is equivalent to V_1 if a FNR formulation is used.

The *RKE* method for some networks gives much more accurate results, but for others it can under- or overestimate and for this reason, in its present form, cannot be used for security assessment purposes.

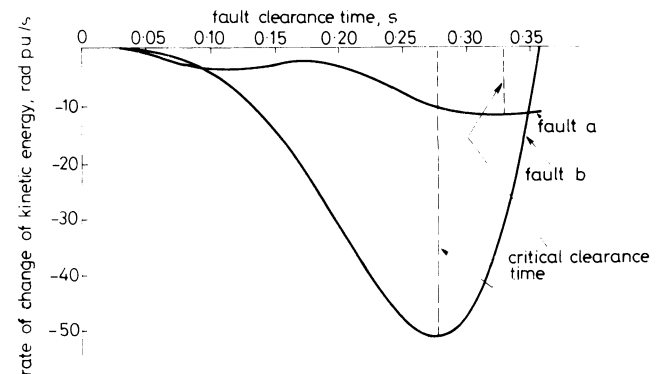


Fig. 6
Relation between *RKE* and clearance time for a 12-machine system

It can be seen from the results that in two cases function V_1 gives optimistic or overconfident results which makes it unsuitable for security assessment. The overestimation is clearly the result of the empirical simplifications in the search for V_{min} . This leaves the function V_2 with the FNR formulation as the only viable alternative for security assessment.

Results of machine-going-unstable for system E as evaluated by the Lyapunov method are shown in Table 5.

It is very difficult to assess which machine goes unstable first with a step-by-step simulation; the only means is to simulate cases and to study their behaviour. The difficulty arises when two unstable cases give different results. Fig. 9 represents the 12-machine-system behaviour for different fault durations at the same busbar resulting in a different machine leading the rest in each case.

7 Program characteristics

The three programs were written with the specific aim of minimising storage and computation time. The actual storage required for a 50-machine system with up to 250 busbars and 350 branches without overlay using 60-bit words on the CDC7600 computer is shown in Table 6.

Table 4
CRITICAL CLEARANCE TIME (s), FOR LARGER SYSTEMS

System	Faulted busbar	Simulation	Lyapunov method		RKE method
			V_1	V_2	
E	30	0.51	0.325	0.135	0.38
	167	0.30	0.171	0.070	0.32
	101	0.28	0.335	0.219	0.32
	169	0.25	0.149	0.071	0.30
	162	0.24	0.154	0.069	0.30
	163	0.37	0.218	0.083	0.34
	166	0.30	0.200	0.102	0.32
	151	0.15	0.047	0.036	0.26
	46	0.32	0.240	0.124	0.33
F	173	0.35	0.204	0.153	0.35
	6	0.28	0.130	0.137	0.30
	28	0.37	0.488	0.123	0.35
	188	0.38	unstable	0.104	0.35

Table 5
MACHINE GOING OUT OF STEP, FOR LARGE SYSTEMS

System	Faulted busbar	Simulation	Lyapunov FNR method	
			V_1	V_2
E	30	29	103	101
	167	103	103	101
	101	101	101	101
	169	101	101	101
	162	103	101	101
	163	163	101	101
	166	166	101	101
	151	101	101	101
	46	46	103	101

Table 6
COMPUTER STORAGE REQUIREMENT, 50 MACHINES, 250 BUSBARS, 350 BRANCHES

Program	Step-by-step trapezoidal integration	Lyapunov method	RKE method
	Program	8.1 k	7.5 k
Data common block	10.9 k	14.5 k	15.7 k
Total	19.0 k	22.0 k	22.2 k

Table 7
COMPARATIVE EXECUTION TIME

System	Number of machines	Number of faults	Integration time step (s)	Execution time (s)		
				Simulation method*	Lyapunov method	RKE method
E	12	9	0.01	47.6	1.9	3.2
F	33	14	0.05	40.8	18.0	24.5

*Step-by-step trapezoidal integration method using three trials for every fault to determine the critical clearance time.

The execution time required to determine the critical clearance time for a given number of faults is shown in Table 7.

The absolute figures for the evaluation of the critical clearance time are relatively small but the same computer requires only 0.68 s to execute a fast-decoupled load flow for the 33-machine system. It is estimated that on an IBM 360/370 computer the execution time will be 20 to 25 times higher.

Although the Lyapunov and the RKE method are faster than the simulation, it is important not to overlook the power of the implicit trapezoidal method when large integration steps are used. However, an important limitation of the simulation method for security-assessment analysis is the time required to examine the unprocessed information, which may be considerable.

8 Stability index

The results of a security-assessment study must be concisely presented to the power-system operator so that he may be able to take any necessary corrective action effectively. Such information may be presented in terms of simple indices.

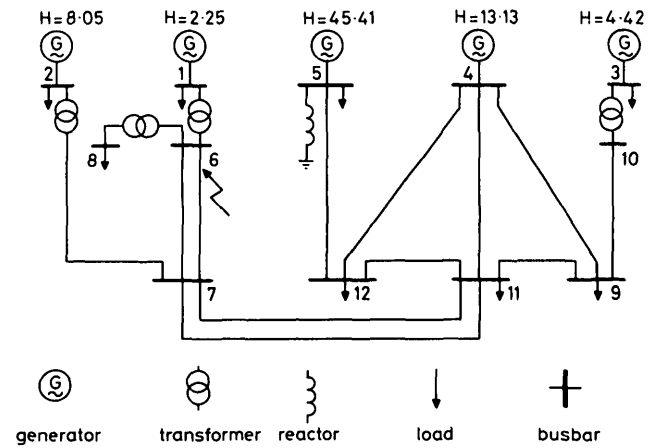


Fig. 7
5-machine test system B

H = inertia constant

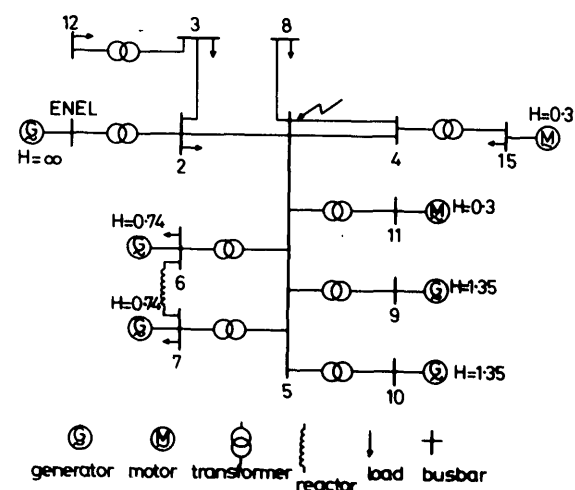


Fig. 8
7-machine test system C

H = inertia constant

The planning engineer traditionally uses a step-by-step simulation and observes the relative-angle values to ascertain the stability of the system. Direct methods may provide more sophisticated information; for example, they can evaluate the critical time clearance and provide simple indices which directly indicate the degree of stability of the system. Both of the two described methods may be used to evaluate such indices.

A simple index can be obtained by comparing the pattern at a specific fault clearance (s.f.c.) with that of the critical time clearance, time value expressed as a percentage. For example, using the Lyapunov V function

$$\eta = \frac{V_{s.f.c.}}{V_{min}} \times 100 \text{ per cent}$$

where values close to 100% represent critical conditions.

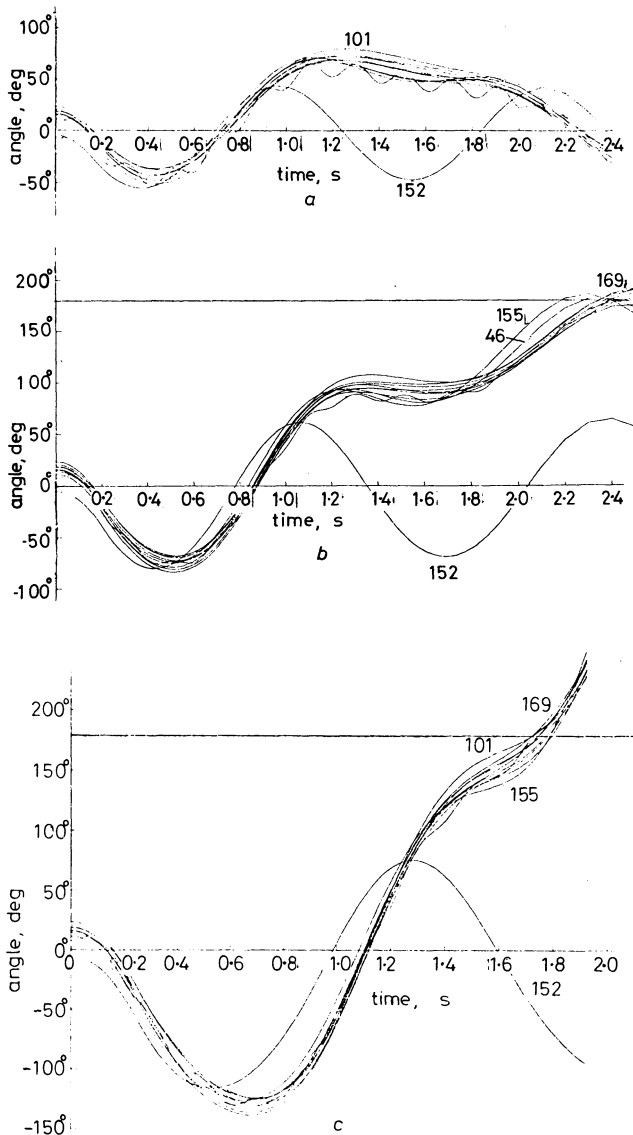


Fig. 9
12-machine system behaviour for a fault at the same busbar 151

a 0.15 s clearance time
b 0.20 s clearance time
c 0.25 s clearance time

Although this index does not represent the system dynamic behaviour, it may provide alternative information for the operator or the planning engineer for a quick assessment of the degree of stability.

9 Conclusions

Three methods for transient security assessment have been tested and compared. The various levels of assessment in the order of increasing complexity may be summarised as follows:

- (a) stability analysis for a given clearance time
- (b) providing information on machines going unstable
- (c) calculation of stability index
- (d) calculation of critical clearance time
- (e) defining preventive measures to avoid emergency state.

As complexity increases a direct approach like the Lyapunov or the *RKE* method became more important and the simulation less useful, the actual choice depending on the specific requirement.

The *RKE* method, although giving promising results, is not sufficiently reliable and requires further research before it can be applied for general security assessment.

The Lyapunov methods using energy-like functions are more reliable, comparing very favourably in execution time with alternative methods. The necessary empirical simplifications introduced in order to make the difficult task of searching for V_{min} easier can produce optimistic, overconfident and unsafe assessment. Out of the alternative functions reported in the paper and many others tried, only one is sufficiently useful for the practical test systems examined.

In general, it is expected when using the V_2 function with the FNR formulation that about 90% of the critical assessment times may be accepted with confidence. The remaining 10% marginal but underestimated cases need to be examined using a more accurate step-by-step simulation method. The Lyapunov method with this particular function always gave a fast, pessimistic, underestimated but safe assessment for the particular test systems.

10 Acknowledgments

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11 References

- 1 AL-AZZAWI, F.J., 'Methods for assessing power-system transient stability'. Ph.D. thesis, University of Manchester Institute of Science and Technology, 1976
- 2 BRAMPELLER, A., JOHN, M.N., and SCOTT, M.R., 'Practical diakoptics for electrical networks' (Chapman and Hall, 1969)
- 3 GUPTA, C.L., and EL-ABIAD, A.H., 'Determination of the closest unstable equilibrium state for Lyapunov methods in transient stability studies', *IEEE Trans.*, 1976, PAS-95, pp. 1699-1712
- 4 PRABHAKARA, F.S., and EL-ABIAD, A.H., 'A simplified determination of transient stability regions for Lyapunov methods', *ibid.* 1975, PAS-94, pp. 672-689
- 5 RIBBENS-PAVELLA, M., 'Critical survey of transient stability studies of multimachine power systems by Lyapunov's direct method'. Ninth annual Allerton Conference on circuits and system theory, Monticello, University of Illinois, 1971, pp. 751-767
- 6 RIBBENS-PAVELLA, M., and LEMAL, B., 'Fast determination of stability regions for on-line transient power-system studies', *Proc. IEE*, 1976, 123, (7), pp. 689-696
- 7 RUDNICK, H., 'A.C. transient security assessment'. M.Sc. dissertation, University of Manchester Institute of Science and Technology, 1977
- 8 VERVLOET, F.G., and BRAMPELLER, A., 'A.C. security assessment', *Proc. IEE*, 1975, 122, (9), pp. 897-902
- 9 WILLEMS, J.L., 'Stability theory of dynamic systems' (Nelson, 1970)
- 10 BRAMPELLER, A., ALLAN, R.N., and HAMAM, Y.M., 'Sparsity' (Pitman, 1976)