ADAPTIVE SINGLE-TONE WAVEFORM DESIGN FOR TARGET RECOGNITION IN COGNITIVE RADAR

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Keywords: Cognitive radar, Waveform design, Single-tone, Sequential detection

Abstract

Cognitive Radar is a recently proposed system concept, one of whose most important characteristics is the closed-loop operation. The feedback structure from the receiver to the transmitter enables the optimization of transmission waveforms based on the latest knowledge about targets and environments. In this paper, we propose an improved waveform design method for target recognition in cognitive radar by restricting the waveforms to be single-tone with constant envelope which are commonly used in practical transmitters. Comparing with a previously proposed method using arbitrary waveforms, our method provides almost the same performance with highly reduced computational cost in sequential transmissions.

1 Introduction

Cognitive Radar is a newly proposed system of active sensors, in which waveform-agile sensing can be realized with the featured feedback structure [1]. Based on the prior knowledge about the targets and the environments, waveforms can be adaptively optimized to improve system performance and efficiency.

Recently, many attempts have been focusing on targets recognition using waveform-adaptation.

In Ref. [2], Goodman proposes the integration of waveform design with a sequential-hypothesis testing framework [3] that controls when hard decisions may be made with adequate confidence [4]. He also compared two different waveform design techniques for use with active sensors operating in a target recognition application. One is considered by Bell in [5] based on maximizes the mutual information between a random target ensemble and the echo signal, and the other is based on eigenvector of weighted autocorrelation matrix proposed in [6], [7]. The target hypotheses are further extended to statistical characterization by power spectral densities in [8]. Thus, the waveforms are matched to the target class rather than to individual target realizations.

Since most existing radar systems can only emit constant envelope waveforms, the methods in [2], [8] using arbitrary waveforms are hard to be implemented. To exploit the waveform diversity in practical systems, we propose an improved waveform design method for target recognition by restricting the waveforms to be single-tone with constant envelope.

In the next section, we define the problem statement and system model. The structure of Sequential Hypothesis Test is described in Section 3. In Section 4, the Eigen-solution method from [2] is reviewed. Our proposed method is detailed in Section 5. Simulation results are shown in Section 6. Finally, Section 7 concludes the paper.

2 Problem statement and system model

We consider the target recognition problem in which one of $M$ possible targets is known to be present [2]. The objective is to identify the target as soon as possible (with fewer transmissions). Since most of the radar systems are digitized, and for the convenient of computer simulation, the system is modeled in discrete-time. Each target hypothesis has a fixed impulse response $h_i(n), n=1,2,\ldots,L_i, i=1,2,\ldots,M \in \{1,2,\ldots,M\}$ which is exactly known. $x(n), n=1,2,\ldots,L_0$ is the transmission waveform with constant energy $E = x^H x$. The echo signal is

$$y = x \ast h_i + g, \quad i \in \{1,2,\ldots,M\}$$

where $g$ is AWGN with average power $\sigma^2_n$; $\ast$ denotes the convolution operator. The convolution operator can be replaced by matrix multiplication if define

$$Q_i = \begin{bmatrix}
    h_i(1) & 0 & \cdots & \cdots & 0 \\
    h_i(2) & h_i(1) & \ddots & \cdots & 0 \\
    \vdots & \vdots & \ddots & \ddots & \vdots \\
    h_i(L_0) & h_i(L_0-1) & \cdots & h_i(1) & 0 \\
    0 & h_i(L_0) & h_i(L_0-1) & \cdots & h_i(1) \\
    \vdots & \vdots & 0 & h_i(L_0) & \cdots & h_i(2) \\
    \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & 0 & h_i(L_0)
\end{bmatrix}$$

which is a $L_0$ matrix. The received signal is $y = Q_i x + g$ where all the signals are presented in column vector.

3 Sequential hypothesis test

We use the same Sequential Hypothesis Test (SHT) structure as what Goodman does in order to compare our results with his [2]. The test is based on Sequential observation and updating which is running in a closed-loop. It updates the probabilistic understanding of all the hypotheses after each illumination, and then makes a decision on the next
transmission signal $x^k$. The likelihood function of observation $y^k$ under hypothesis $i$ is

$$p_i^y(y^k) = \frac{1}{(2\pi \sigma_n^2)^{k/2}} \exp\left[-\frac{1}{2\sigma_n^2}(y^k - Q_i x^k)^\top(y^k - Q_i x^k)\right] \tag{3}$$

Since $x^k$ is adaptively optimized for each illumination, $p_i^y(y) = p_i^y(y) = \cdots = p_i^y(y)$ is no longer valid. After $k$th observation, the posterior probability of hypothesis $i$ is

$$P_i^y = \frac{p_i^y(y^k)p_i^{y^{-1}}(y^{k-1}) \cdots p_i^y(y^2)p_i^y(y^1)p_i}{\sum_{j=1}^M p_j^y(y^k)p_j^{y^{-1}}(y^{k-1}) \cdots p_j^y(y^2)p_j^y(y^1)p_j} \tag{4}$$

where $P_i$ is the prior probability for hypothesis $i$.

After the probabilistic understanding of all hypotheses has been updated, the likelihood ratio between each of the hypotheses can be calculated as

$$\Lambda_i^k = \frac{p_i^y(y^k)}{p_j^y(y^k)} = \frac{p_i^y(y^k)p_i^{y^{-1}}(y^{k-1}) \cdots p_i^y(y^2)p_i^y(y^1)p_i}{p_j^y(y^k)p_j^{y^{-1}}(y^{k-1}) \cdots p_j^y(y^2)p_j^y(y^1)p_j} \tag{5}$$

The experiment is terminated and hypothesis $i$ is selected to be true when the condition

$$\Lambda_i^k > \frac{1}{\alpha_{i,j}} \quad \text{for all } j \neq i \tag{6}$$

is met for some $i$ where $\alpha_{i,j}$ is the desired probability of incorrectly selecting hypothesis $j$ given that hypothesis $i$ is true [2]. If the condition is not met for any of the hypothesis, another illumination cycle is repeated.

### 4 Eigensolution waveform design

Goodman has proposed a waveform design technique based on the eigenvector of target autocorrelation matrix [2].

For $M = 2$ case, in order to identify which of the hypothesis is present efficiently, the transmission signal $x^k$ should be the one that separates two probability density functions as far as possible, the one that maximizes the Euclidean distance between the mean values of the PDFs

$$d^2 = (x^k)^\top(Q_i - Q_j)(Q_i - Q_j)x^k. \tag{7}$$

Define the target autocorrelation matrix as

$$\Omega^i = (Q_i - Q_j)^\top(Q_i - Q_j). \tag{8}$$

The solution is the transmission waveform vector $x^i$ that maximize $(x^i)^\top\Omega^i x^i$, which is the eigenvector corresponding to the largest eigenvalue of $\Omega^i$.

When $M > 2$, the autocorrelation matrix is suggested to be in the form

$$\Omega^i = \sum_{i=1}^M \sum_{j=1}^M w_{ij}^i (Q_j - Q_j)^\top(Q_i - Q_i) \tag{9}$$

where $w_{ij}^i = p_i^{y^{-1}}p_j^y$. After $(k-1)$th observation, the posterior probabilities are updated based on the echo signal $y^{k-1}$, and used to calculate the autocorrelation matrix $\Omega^i$. The eigenvector corresponding to the largest eigenvalue of $\Omega^i$ will be the next transmission waveform $x^i$ with constant energy $E = (x^i)^\top x^i$.

### 5 Adaptive single-tone waveform design

Goodman’s method which is called the Eigensolution gets an excellent result in simulation [2]. But the assumption of $x$ as an arbitrary waveform is not commonly enough for radar transmitters, since most of the transmitters can only emit signals with constant envelope.

Let $x$ be a single-tone waveform which is widely used in current radar systems. The objective of the method is to choose an appropriate frequency for $x$ in order to make more efficient observations, to make a decision among all the hypotheses with fewer transmissions.

The proposed method also starts with $M = 2$ case. As we can see in Goodman’s method, the main idea of the optimization is to maximize the Euclidean distance between the mean values of two probability density functions. That is

$$u_i^1 = Q_i x^i, \quad u_i^2 = Q_i x^j. \tag{10}$$

$$d^2(u_i^1, u_i^2) = \|u_i^1 - u_i^2\|_2 = (u_i^1 - u_i^2)(u_i^1 - u_i^2)^\top \tag{11}$$

$$= \|x^i - L_i\|^2 \tag{12}$$

where $u_i^1$ is the mean value of probability density function under hypothesis $i$.

We reuse convolution operator to denote $u_i^1$ as

$$u_i^1 = x^i \ast h_i \tag{13}$$

where the length of $x^i$ is $L_x$, the length of $h_i$ is $L_{h_i}$, and the length of $u_i^1$ is $L_x + L_{h_i} - 1$.

Some zeros are attached at the end of $x^i$ and $h_i$, so as to replace linear convolution $*$ by circular convolution $\otimes$.

$$\begin{bmatrix} u_i^1(1) \\ u_i^1(2) \\ \vdots \\ u_i^1(L_x - 1) \\ u_i^1(L_x) \end{bmatrix} = \begin{bmatrix} x^i(1) \\ x^i(2) \\ \vdots \\ x^i(L_x) \end{bmatrix} \otimes \begin{bmatrix} h_i(0) \\ \vdots \\ h_i(L_{h_i}) \end{bmatrix} \tag{14}$$

For any $N \geq L_x$, the N-point Discrete Fourier Transform (DFT) of the signal is

$$\text{DFT}(u_i^1, N) = \text{DFT}(x^i, N) \ast \text{DFT}(h_i, N). \tag{15}$$

where $\ast$ denotes array multiplication, and zeros are attached to fit the length of DFT. Let $H = \text{DFT}(h_i, N)$, $X^i = \text{DFT}(x^i, N)$, and $U_i = \text{DFT}(u_i^1, N)$. Equation (15) can be write as

$$U_i = X^i \ast H \tag{16}$$

Since Discrete Fourier Transform keeps Euclidean distance between two vectors, the objective function can be written as

$$d_i^2(u_i^1, u_i^2) = \frac{1}{N} d_i^2(U_i^1, U_i^2). \tag{17}$$

Together with equation (16), our goal is to maximize
\[ d^i_k(u^i_k, u^j_l) = \frac{1}{N^2} d^i_k(X^i \cdot H^i_k, X^j \cdot H^j_l) \] (18)

Assume \( x^i \) is a single-tone waveform with digital frequency \( \frac{2\pi}{N} \), the spectrum \( X^i \) is thus approximately an impulse function

\[
X^i(r) \approx \begin{cases} 
\sqrt{\frac{\pi}{2}} & r = l, N - l \\
0 & \text{else} 
\end{cases} \quad \text{for } 0 < l < N/2 \tag{19}
\]

\[
X^i(r) = \begin{cases} 
\sqrt{\frac{\pi}{2}} & r = l \\
0 & \text{else} 
\end{cases} \quad \text{for } l = 0 \text{ or } l = N/2 \tag{20}
\]

where \( E \) is the energy of the transmission signal. From eq. (18-20), we get the Euclidean distance between the mean values of two PDFs for a single-tone waveform transmission as

\[
d^i_k(u^i_k, u^j_l) = E[|H^i_k(l) - H^j_l(l)|^2] \tag{21}
\]

where \( 0 \leq l \leq N/2 \).

The Euclidean distance \( d^i_k(u^i_k, u^j_l) \) reaches maximum when the frequency of the transmission signal points to the largest difference between the spectrums of two hypotheses.

\[
l^k = \arg \max_l (|H^i_k(l) - H^j_l(l)|^2) \tag{22}
\]

When \( M > 2 \), the distance among all the hypotheses can be defined as

\[
d^i_k(u^i_k, u^j_l, \ldots, u^M_M) = E \sum_{i=1}^{M-1} \sum_{j=1}^{M} \omega_{i,j}^k |H^i_k(l) - H^j_l(l)|^2 \tag{23}
\]

where \( \omega_{i,j} = P_{i-1}^{k-1} \), which depends on the posterior probability of each hypothesis after \( k-1 \)th observation. It is the weighted sum of the Euclidean distances between all pairs of hypotheses. The algorithm of adaptively optimize the transmission frequency \( \frac{2\pi}{N} l^k \) for \( x^i \) is as follow:

1) Calculate the spectrum of every hypothesis’s impulse response using N-point DFT which \( N \geq L_z \), and get \( H^i_k \).

2) Calculate equation (23) with \( H^i_k \) and the posterior probability \( P_{i-1}^{k-1} \) of step \( k-1 \) for all integers \( l \) which \( 0 \leq l \leq N/2 \).

3) The \( l \) maximizes \( d^i_k(u^i_k, u^j_l, \ldots, u^M_M) \) is the transmission frequency for step \( k \)

\[
l^k = \arg \max_l (\sum_{i=1}^{M-1} \sum_{j=1}^{M} \omega_{i,j}^k |H^i_k(l) - H^j_l(l)|^2) \tag{24}
\]

and \( x^i \) will be a single-tone waveform at frequency \( \frac{2\pi}{N} l^k \).

4) After received echo signal \( y^i \), updates all the probabilistic understanding of the environments using equation (3) and (4).

5) Calculate the likelihood ratio between each of the hypotheses using equation (5) and check the results with equation (6). If the sequential test is still going, back to procedure 2). Otherwise, the sequential test has a result.

Comparing with the Eigensolution, this method provides almost the same performance with highly reduced computational cost in sequential transmissions.

6 Results

We simulate several waveform design methods using the same Sequential Hypothesis Test structure which the specified error rate is \( \alpha_{i,j} = 0.01 \) for all the Hypotheses. And the prior probability \( P_i \) is set to be \( 1/M \) for every Hypothesis. 5000 sets of \( M = 4 \) impulse responses are generated from a flat PSD with length of \( L_z = 31 \). The length of transmission signal is also set as \( L_z = 31 \). The average power of AWGN at the receiver is \( \sigma^2 = 1 \).

Fig. 1 shows the average number of iterations required for each waveform design approach as a function of energy units allocated to a single illumination. The impulse waveform is defined as \( x^i = [\sqrt{E} 0 \ldots 0]^t \) which is a constant waveform. And the Eigensolution is the method proposed by Goodman based on the eigenvector of target autocorrelation matrix. Eigensolution (non-adaptive) calculates \( x^i \) only with equal prior probabilities without change when the hypothesis probabilities are updated. Single-Tone is our method that restricts the transmission signal \( x^i \) to be a single-tone waveform.

As we can see, both Eigensolution and Single-Tone show great performance with obviously fewer illuminations to reach decisions than two methods with constant transmission waveforms.

![Fig.1. Average illuminations to reach a decision versus energy per illumination](image-url)
Fig. 2 shows the detailed evolution of an experiment. Four hypotheses were assumed and hypothesis 1 was the present target. The upper figure depicts the probabilities of each hypothesis varying with the number of illuminations while the lower one shows the waveform frequencies of each illumination. The experiment started with the probability of 25% for each hypothesis, and ended with a decision on hypothesis 1.

7 Conclusions

We have proposed and simulated a new waveform design method for target recognition in cognitive radar. The waveform is restricted to be single-tone with constant envelope which is commonly used in practical transmitters. A same structure of sequential hypothesis test has been used for all methods. Our method performs almost the same as the method using arbitrary waveforms dose. Further, our method reduces the computational cost in sequential transmissions.

References


