

Cluster-Based Resource Allocation for Spectrum-Sharing Femtocell Networks

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Abstract: Femtocells in two-tier femto-macro networks can enhance indoor coverage and improve overall network performance. Macro networks may share spectrum with overlaid femtocells so as to improve spectrum efficiency. However, the deployment of femtocells also brings co-tier and cross-tier interferences, which will significantly degrade system performance. In order to solve this problem efficiently, we propose a distributed scheme to manage wireless resources in this heterogeneous networks. The feasible solution can be obtained by dividing the problem into two sub-problems. First, we propose a femtocells clustering scheme, which uses a mathematical modeling idea based on LINGO, an optimization software that can solve the joint clustering problem for the femtocell access points. The proposed Branch-and-Bound Algorithm and the Simplex Algorithm are used jointly to find the optimal solution by LINGO. The optimality of the proposed clustering algorithm is verified both theoretically and through simulations where the comparison with other algorithms is made. Secondly, a novel algorithm is proposed to allocate sub-channels to the femtocell users (FUEs). Compared with other related schemes, the proposed channel-allocation algorithm can reduce the interference more effectively and achieve higher data-rate fairness among FUEs. Specifically, according to the situation that the FUEs move in the room, the FUE mobility model is proposed to predict the change tendency of path loss values of the FUEs, which can guarantee the mobile service quality and improve system capacity effectively. Finally, power of the femtocell access points is adjusted dynamically through setting the interference threshold to further improve the performance of the system.

Key words: Femtocells; Clustering; Resource Allocation; Branch-and-Bound; the Simplex Algorithm; LINGO;

1. Introduction

Compared with the 3G system, the carrier frequency is increased in the Long Term Evolution (LTE) system, so is the path loss exponent [1-3]. Therefore, the LTE system can't cover the indoor environment well. Research indicates that nearly 90% of the data business and 60% of the voice business occurs indoors, so indoor coverage and communication quality need to be significantly improved [4]. One of the most promising solutions to improve the performance of the LTE system is to deploy small-size cells extensively, which have been widely studied in recent years [5-8]. For example, femtocell can be deployed

in the several scenarios, such as the cognitive radio network [7], the cooperative networks [8]. Compared with the traditional macro cellular networks, the embedded femtocell access points (FAPs) can expand coverage and improve system performance effectively [7-10].

Some exiting studies have been reported on femtocell interference management and resource allocation [11-14]. A graph-based scheme is applied to solve the sub-channel assignment and interference alignment problem in [14]. However, the fairness among users isn't considered. The models of open and closed femtocell services are applied to study the economic aspects of femtocell services with game theoretic models between providers and/or users in [15]. Although part of the system performance is improved, the interference coordination becomes more complex. In the cognitive radio system, a resource allocation algorithm based on game theory is proposed in [16]. Not only is the throughput increased, but the interference is also reduced. However, in these literatures, the clustering optimization is not taken into account.

Many researchers focus on the joint consideration of the clustering optimization problem and the resource allocation problem in the femtocell networks so as to reduce the co-tier interference and improve the system performance in [17-24]. For example, in an orthogonal frequency-division multiple-access (OFDMA) femtocell network serving both quality of service (QoS)-constrained high-priority and best-effort users, a new resource-allocation and admission control algorithm is proposed based on clustering and taking into account QoS requirements in [17]. In order to mitigate inter-femtocell interference (IFI), the disjoint IFI-minimizing clusters are formed by a Max k-Cut clustering algorithm according to the interference graph [18]. In [19], the authors present a new technique for jointly optimizing energy consumption and QoS in heterogeneous cellular networks employing fractional frequency reuse in the macrocell tier, network routing is also applied to maximize the spectrum utilization. According to the clustered femtocell base stations (FBSs) and predicted signal-to-interference-plus-noise ratio (SINR), the power control scheme is applied to femtocell network in the downlink [20]. However, in the above work, the optimal solution is very hard to obtain because of the characteristic of (non-deterministic polynomial (NP)-hard problem. Moreover, the optimal cluster size isn't considered.

Considering joint clustering optimization and resource allocation, a Semi-Definite Programming (SDP) based on random rounding algorithm is proposed in [25] based on CVX, a software package for specifying and solving convex programs. Although the optimal cluster size has been taken into account in this paper, it is not fast enough to obtain the optimal solution for the clustering optimization problem. When the number of the FAPs increases, the algorithm in [25] may not be able to find the optimal solution effectively, and there is no sufficient argument as to whether the clustering optimization problem has an

optimal solution. In addition, their resource allocation algorithm doesn't solve the problem of the average interference effectively. The data-rate fairness issue is not taken into account either. In order to simulate the practical application scene, the mobility of the femtocell users (FUEs) should also be considered.

Considering the above issues, we propose a novel resource allocation algorithm based on clustering for the closed subscriber group FAPs [25]. We formulate the problem of joint clustering optimization and resource allocation, which is a classical NP-hard problem. It is very difficult to solve this problem. Without loss of generality, the problem is divided into two sub-problems, namely the clustering optimization problem and the resource allocation problem. Firstly, Femtocell Gateway (FGW) collects the information about FAPs. According to the clustering optimization algorithm, the FAPs are assigned to different clusters to reduce interference effectively. Then, a FAP is selected as the cluster head in each cluster, which will be responsible for resource allocation in its own cluster. For the above optimization problem, we propose a mathematical modeling idea based on LINGO, which applies the Branch-and-Bound algorithm and the Simplex Algorithm to find the optimal solution. We theoretically prove that the solution obtained by our algorithm is the global optimal solution. The simulation results show that the proposed algorithm can obtain the optimal solution for the clustering optimization problem in an efficient manner. In addition, we put forward a novel algorithm to solve the resource allocation problem. Compared with other allocation algorithms, the proposed algorithm not only reduces interference but also improves the data-rate fairness among FUEs. According to the practical scenarios that the FUEs move in the room, the FUE mobility model is proposed. The tendency of path loss values of the FUEs is predicted based on this model, which will further improve the continuity of all kinds of data services. The data rate requirements of mobile FUEs are met more easily. In order to further enhance FUE data rates and reduce interference between FUEs, a power allocation problem is studied. Finally, power is adjusted dynamically through setting the interference threshold to further improve the performance of the system.

The rest of this paper is organized as follows. In Section 2, the system model is described. In Section 3, the optimization problem for the clustering is formulated. Detailed analysis is given to verify that the found solution by the proposed algorithm is the global optimal solution. In Section 4, a novel resource allocation algorithm is proposed to reduce interference and improve fairness. In Section 5, numerical results for different scenarios and topologies demonstrate the superiority of the proposed schemes. Finally, Section 6 concludes the paper.

2. System model

Fig. 1 shows the topology of two-tier femto-macro networks, in which a large number of FBSs covering a small range are distributed in every room of every layer of big buildings which are in the

coverage of the single overlay macrocell base station (MBS). The macro users (MUEs) served by the outdoor MBS are outdoors and indoors while the FUEs served by FBSs are indoors. The channel propagation conditions between FAPs and their FUEs are perfect. The channel gain includes the path loss (PL) L , the shadow fading L_s and the antenna gain L_a . The total path loss between FAP and its FUE can be expressed as $PL = 15.3 + 37.6 \log d + L + L_s - L_a$. d is the distance between FAP and the FUE.

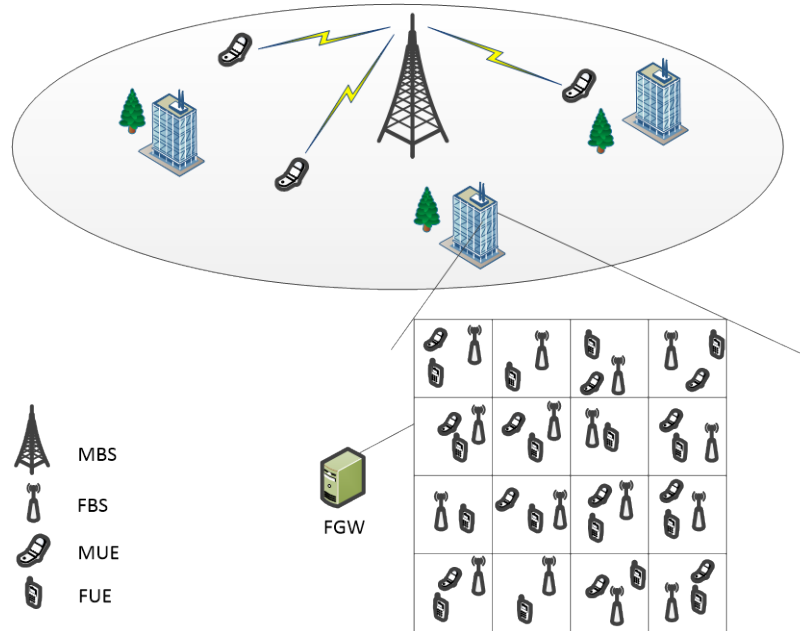


Fig. 1 The model of the femto-macro system

In order to reduce the co-tier interference effectively, FAPs may be grouped into different clusters. However, it is difficult to group FAPs according to their instantaneous channel gains. Therefore, the average channel gain is applied to reduce the complexity. As the distance between the FAP i and the FUE k_i served by the FAP i is quite short, the channel gain between the FAP j and the FUE k_i approximately equals to that between the FAP j and the FAP i on the sub-channel θ (i.e., $g_{j,k_i}^\theta \approx g_{j,i}^\theta$) [25]. Obviously, the channel gain matrix between the FAPs is symmetric, i.e., $g_{j,i}^\theta = g_{i,j}^\theta$. The system parameters are listed in Tab. 1.

Table 1 System parameters

Parameters	meaning
F	a set of FAPs
F	the number of FAPs
N	the number of the channels
M	the total number of available channels in a cluster
k_i	the FUE served by the FAP i

PL	the total path loss between FAP and the FUE
$P_{k_i,i}^\theta$	the transmission power between FUE k_i and FAP i on sub-channel θ
$g_{k_i,j}^\theta$	the channel gain between FUE k_i and FAP j on sub-channel θ
d	the distance between FAP and the FUE
L_s	the log-normal shadowing
L_a	the antenna gain of FAPs
Δf	the sub-channel bandwidth

3. Optimization problem for the clustering

Similar to Ref. [25], we also set up an undirected graph $G_1 = (V_1, E_1)$, V_1 is the set of vertices representing the FAPs. E_1 is the set of edges between two vertices. $\omega_{i,j}^+$ and $\omega_{i,j}^-$ are the non-negative link weights. The objective of clustering optimization problem is to group the vertices into sets of similar vertices, where the degree to which they are similar is given by $\omega_{i,j}^+$ and the degree to which they are different is given by $\omega_{i,j}^-$. That is to say, the objective is to find a partition that maximizes the sum of the weights $\omega_{i,j}^+$ of edges inside the sets of partitions and the weights $\omega_{i,j}^-$ of edges between the sets of partitions [25]. If there is serious interference between two FAPs, the link weights $\omega_{i,j}^+$ between two FAPs will be large. Similarly, if the channel gain $g_{i,j}^\theta$ is high between two FAPs, the interference between them will be also very large. We observe that the channel gain and the path loss are inversely proportional, so we have a new idea that sets up $\omega_{i,j}^+ = 1 / PL$.

In order to mitigate the co-tier interference, the FAPs which have the serious interference with each other may be placed into the same cluster. Sub-channels should be assigned orthogonally to these FAPs in the same cluster and reused between any two clusters. When the number of FAPs is increased in a cluster, the reusability of sub-channels and the data rate will decrease, so we should structure the clustering optimization problem reasonably. $\omega_{i,j}^-$ is a penalty term when two FAPs are in different clusters. All in all, we need to find a method to maximize the objective function value in [25]. In this paper, we assume that the value of $\omega_{i,j}^-$ is the same for all FAPs (i.e., $\omega_{i,j}^- = \omega_\varphi^-$). Define $\omega_\varphi^- = \omega_{i,j}^+ + \Delta$, $1 \leq i \leq j \leq F$, $\varphi = \frac{F^2 - F}{2}$, $\Delta > 0$, $\forall \omega_\varphi^-$. The range of ω_φ^- can be expressed as $(\min \omega_{i,j}^+ + \Delta \leq \omega_\varphi^- \leq \max \omega_{i,j}^+ + \Delta)$. As a result, the optimization problem for the clustering is shown as followed:

$$\max \sum_{i \in \mathbf{F}} \sum_{j \in \mathbf{F}} \omega_{i,j}^+ x_{ij} + \omega_{i,j}^- (1 - x_{i,j}) \quad (1)$$

$$C1: x_{i,i} = 1, \forall i \in \mathbf{F} \quad (2)$$

$$C2: x_{i,j} = x_{j,i}, \forall i, j \in \mathbf{F} \quad (3)$$

$$C3: x_{i,j} + x_{j,k} - x_{i,k} \leq 1, \forall i, j, k \in \mathbf{F}, k > i, j \neq i, k \quad (4)$$

$$C4: \sum_{j \in \mathbf{F}} x_{i,j} \leq M, \forall i \in \mathbf{F} \quad (5)$$

$$C5: x_{i,j} \in \{0,1\}, \forall i, j \in \mathbf{F} \quad (6)$$

Where, $x_{i,j}$ is the FAP clustering indication factor. If FAP i and j are in the same cluster, $x_{i,j}=1$, otherwise, $x_{i,j}=0$. $C1$ indicates that any FAP is in the same cluster with itself. $C2$ shows that if FAP i and j are in the same cluster, then FAP j and i will be also in the same cluster. $C3$ indicates that if FAP i and j are in the same cluster and FAP j and k are in the same cluster, then FAP i and k are also in the same cluster. $C4$ states that the number of FAPs is not more than the total number M of available sub-channels in a cluster. $C5$ indicates whether FAP i and j are in the same cluster or not.

In order to prove the solution obtained by the proposed algorithm based on the clustering optimization problem is the global optimal solution, according to Ref. [26-27], we give the following definition:

Definition 1: The vectors that satisfy all the constraint conditions are the feasible solutions. The set consisting of all feasible solutions is called as the feasible set or the feasible region.

Definition 2: The feasible solution that makes the objective function obtain the optimal value is called as the optimal solution of linear programming.

Definition 3: A is a coefficient matrix of constraint variable x . b is a column vector which is composed of the right end of constraints. For the constraints $Ax \leq b$, we assume that the rank of the matrix A is m . P_i ($i=1 \sim n$) is the i -th column vector of the matrix A . B is composed of the m column vector of the A , i.e., $B = (P_{i_1} P_{i_2} \dots P_{i_m})$. If B is a nonsingular matrix, namely $\det B \neq 0$, then B will be not only a base matrix but also the maximum linearly independent subset of the A .

Definition 4: The solution that meets the constraint conditions and in which the non-basic variable is zero is called as the basic solution. The basic solutions with non-negative basic variables are called as basic feasible solutions.

Definition 5: Suppose x_1, x_2, \dots, x_k are the points in the feasible region R . If $\exists \lambda_1, \lambda_2, \dots, \lambda_k$ ($\lambda_i \geq 0, i=1..k, \sum_{i=1}^k \lambda_i = 1$), which makes the equality $x = \sum_{i=1}^k \lambda_i x_i$ be established, x will be called as a convex combination of x_1, x_2, \dots, x_k .

Definition 6: The basic feasible solution that makes the objective function achieve the optimal is called as the basic optimal solution.

Definition 7: We assume that U and V are two points in the feasible region R . If $\forall \lambda \in [0, 1]$, $\exists W(\lambda) = \lambda U + (1-\lambda)V$, which is also in the feasible region R , then the set of feasible region R will be called a convex set.

According to the above definition, we can get the following conclusions:

Corollary 1: According to the definition 3 and 4, if $Ax \leq b$, then the general necessary and sufficient conditions for $x = (x_1, x_2, \dots, x_n)$ to be the basic feasible solution are that the column vector $P_{i1}, P_{i2}, \dots, P_{ik}$ of the matrix A corresponding to the base component $x_{i1}, x_{i2}, \dots, x_{ik}$ of x is linearly independent.

Corollary 2: According to the definition 4, if the unit matrix E is a feasible solution, then they must be basic feasible solutions.

Proof: For the constraint condition $C1$, the main diagonal elements of the unit matrix E are constant. The non-diagonal elements of the E are the basic or the non-basic variables, and their values are zero. Obviously, it also conforms to the definition 4. Therefore, the unit matrix E must be the basic feasible solution.

Based on the above definitions and corollaries, we will propose the following lemmas and proofs.

Lemma 1: The feasible region about the clustering optimization problem is at the border of a convex set.

The optimal solution is $n \times n$ dimensions. Next, we will take an example for the 3×3 dimensions matrix.

Proof: According to the constraints $C1$ and $C2$, we can construct any feasible solution, such as $u = (1, u_1, u_2; u_1, 1, u_3; u_2, u_3, 1)$, $v = (1, v_1, v_2; v_1, 1, v_3; v_2, v_3, 1)$, u and v are the feasible solutions, so they meet the constraints $C3$ and $C4$. Put u and v into the constraints $C3$ and $C4$, there are:

$$\begin{cases} u_2 + u_3 - u_1 \leq 1 \\ v_2 + v_3 - v_1 \leq 1 \end{cases} \quad (7)$$

$$\begin{cases} u_1 + u_2 \leq M - 1 \\ u_1 + u_3 \leq M - 1 \\ u_3 + u_2 \leq M - 1 \\ v_1 + v_2 \leq M - 1 \\ v_1 + v_3 \leq M - 1 \\ v_3 + v_2 \leq M - 1 \end{cases} \quad (8)$$

For the definition 7, there is:

$$\begin{aligned} \mathbf{W}(\lambda) &= \lambda \mathbf{u} + (1 - \lambda) \mathbf{v} \\ &= \begin{pmatrix} 1 & \lambda u_1 + (1 - \lambda) v_1 & \lambda u_2 + (1 - \lambda) v_2 \\ \lambda u_1 + (1 - \lambda) v_1 & 1 & \lambda u_3 + (1 - \lambda) v_3 \\ \lambda u_2 + (1 - \lambda) v_2 & \lambda u_3 + (1 - \lambda) v_3 & 1 \end{pmatrix} \end{aligned} \quad (9)$$

Put $\mathbf{W}(\lambda)$ into the constraint $C3$:

$$\lambda u_2 + (1 - \lambda) v_2 + \lambda u_3 + (1 - \lambda) v_3 - \lambda u_1 - (1 - \lambda) v_1 = \lambda(u_2 + u_3 - u_1) + (1 - \lambda)(v_2 + v_3 - v_1) \quad (10)$$

According to the equation (7), there is:

$$\lambda u_2 + (1 - \lambda) v_2 + \lambda u_3 + (1 - \lambda) v_3 - \lambda u_1 - (1 - \lambda) v_1 \leq \lambda + (1 - \lambda) = 1 \quad (11)$$

Obviously, $\mathbf{W}(\lambda)$ meets the constraint $C3$.

Similarly, bring $\mathbf{W}(\lambda)$ into the constraint $C4$, according to the equation (8), we get:

$$\begin{cases} \lambda u_1 + (1 - \lambda) v_1 + \lambda u_2 + (1 - \lambda) v_2 + 1 = \lambda(u_1 + u_2) + (1 - \lambda)(v_1 + v_2) + 1 \leq M \\ \lambda u_1 + (1 - \lambda) v_1 + \lambda u_3 + (1 - \lambda) v_3 + 1 = \lambda(u_1 + u_3) + (1 - \lambda)(v_1 + v_3) + 1 \leq M \\ \lambda u_3 + (1 - \lambda) v_3 + \lambda u_2 + (1 - \lambda) v_2 + 1 = \lambda(u_3 + u_2) + (1 - \lambda)(v_3 + v_2) + 1 \leq M \end{cases} \quad (12)$$

Clearly, $\mathbf{W}(\lambda)$ meets the constraint $C4$. At the same time, $\mathbf{W}(\lambda)$ meets the constraints $C1$ and $C2$. $C5$ can be converted to $0 \leq x_{i,j} \leq 1$. $\mathbf{W}(\lambda)$ meets the constraint of $0 \leq x_{i,j} \leq 1$. According to the definition 7, we can consider that the set of feasible solutions about the clustering optimization is a convex set, then, $C5 \in (0 \leq x_{i,j} \leq 1)$ is at the limit point, namely, it is at the border. So the lemma is feasible. In the same way, the matrix with $n \times n$ dimension can also be proved to be true.

Lemma 2: If the set of the feasible solutions about the clustering optimization problem is at the border of a convex set, then as long as the set of the feasible solutions is not empty, there must be the optimal solution, which is the limit point.

Proof: We assume that $x_{1,1}x_{1,2}\dots x_{r,r}$ represents all the limit points on the feasible region R . All of the feasible solutions about the clustering optimization problem can be expressed by the basic solutions, i.e.:

$$\forall x \in R = \sum_{i=1}^r \sum_{j=1}^r \lambda_{i,j} x_{i,j}, \lambda_{i,j} \geq 0, \sum_{i=1}^r \sum_{j=1}^r \lambda_{i,j} = 1 \quad (13)$$

The objective function value is:

$$\sum_i \sum_j w_{i,j}^+ x_{i,j} + w_{\phi}^- (1 - x_{i,j}) = s \quad (14)$$

And

$$f^* = \max s = \max \sum_i \sum_j w_{i,j}^+ x_{i,j} + w_{\phi}^- (1 - x_{i,j}), 1 \leq i, j \leq r \quad (15)$$

So we can get:

$$s \leq \sum_{i=1}^r \sum_{j=1}^r \lambda_{i,j} f^* = f^* \quad (16)$$

In summary, the above equations suggest that the objective function value obtained in an arbitrary feasible solution is not more than that in a limit point x_{ϵ} . Therefore, x_{ϵ} is the optimal solution and the lemma is feasible.

Lemma 3: It is the sufficiency and necessity condition that the limit point is the basic feasible solution in the feasible region R .

Proof: Each element can be considered as a variable. Consequently, we convert the objective problem into the standard form of linear programming. We denote $x_{i,j}$ as $x_{1,1}x_{1,2}\dots x_{n,n}$ which corresponds to $x_1x_2\dots x_{n^2}$, according to the corresponding relationship, the objective function can be expressed as the form of $c^T x$, where c and x are column vectors (i.e., $n^2 \times 1$ dimension). Similarly, the constraint condition C3 can be transformed into $Bx \leq 1$. The constraint condition C4 can be transformed into $Cx \leq M$. Finally, we will get the expression:

$$\begin{pmatrix} B \\ C \end{pmatrix} x \leq b \Rightarrow Ax \leq b \quad (17)$$

Necessity: We assume that x is the limit point, according to the C1, $x \neq 0$. According to the corollary 1, if we can prove that the column vector $P_{i1}P_{i2}\dots P_{ik}$ of A corresponding to the base component $x_{i,1}x_{i,2}\dots x_{i,k}$ of x is linearly independent, then the proposed proposition will be established.

Proof: Assume the column vector $P_{i1}P_{i2}...P_{ik}$ is linearly dependent, the following equation holds:

$$\lambda_1 P_{i1} + \lambda_2 P_{i2} + \dots + \lambda_k P_{ik} = 0 \quad (18)$$

For equation (18), there exists at least one non-zero vector with $\lambda_1 \lambda_2 \dots \lambda_k$. We construct a y vector with the dimension of n^2 , in which the component $i_1 i_2 \dots i_k$ corresponds to these parameters $\lambda_1 \lambda_2 \dots \lambda_k$ respectively, the others are zero. As y is not an empty set and the equation $Ay = y_1 P_{i1} + \dots + y_k P_{ik} = 0$ is feasible, we define:

$$\alpha = \min_{1 \leq t \leq k} \left\{ \frac{x_{i,t}}{|y_t|} \mid y_t \neq 0 \right\} \quad (19)$$

Obviously, $\alpha > 0$, for $A(x \pm \alpha y) = Ax \pm \alpha Ay = Ax \leq b$. So we can get two feasible solutions, which can be expressed as $x_1 = x + \alpha y$, $x_2 = x - \alpha y$, respectively. Therefore, $x = \frac{1}{2}x_1 + \frac{1}{2}x_2$, for $\alpha > 0, y \neq 0$.

So, $x \neq x_1 \neq x_2$, we can get x_2 through assigning a value to the parameter, i.e., $\lambda = \frac{1}{2}$. If x can be expressed as a convex combination of the feasible region, which is contradictory to the definition of the pole, therefore, the necessity is proved.

Sufficiency: Let x be a basic feasible solution, x is not zero.

Proof (reduction to absurdity): If x is not only a basic feasible solution but also is not in the pole, then there are two distinct values x_1 and x_2 to meet the equation $x = \lambda x_1 + (1-\lambda)x_2, \lambda \in (0,1)$. They make the equation be established:

$$\begin{cases} Ax_1 = x_{1,1}P_{i1} + x_{1,2}P_{i2} + \dots + x_{1,k}P_{ik} = 0 \\ Ax_2 = x_{2,1}P_{i1} + x_{2,2}P_{i2} + \dots + x_{2,k}P_{ik} = 0 \end{cases} \quad (20)$$

As x_1 and x_2 are the different feasible solutions, for the equation:

$$Ax_1 - Ax_2 = (x_{1,1} - x_{2,1})P_{i1} + \dots + (x_{1,k} - x_{2,k})P_{ik} = 0 \quad (21)$$

There is at least one non-zero coefficient, if $P_{i1}P_{i2}...P_{ik}$ is linearly dependent, it will be contradictory to the fact that $P_{i1}P_{i2}...P_{ik}$ is linearly independent, so the fundamental solution must be in the pole. Thus, lemma 3 is accurate.

Lemma 4: We assume that the rank of the constraint coefficient matrix A is m , the column vectors of the A are Non-zero vectors. If the feasible solutions exist, then these solutions must be fundamental solutions (or the pole values).

Proof: Similarly, we convert the clustering optimization problem into the standardized linear

programming form by the above method. Firstly, we assume $\mathbf{x} = (x_1 \ x_2 \dots x_{n^2})^T$ is the feasible solution which satisfies the constraint conditions $\mathbf{Ax} \leq \mathbf{b}$. In the same way, for the constraint condition $C1$, \mathbf{x} has a non-zero element, the number of the positive components of \mathbf{x} is k . They are $x_1 > 0, \dots, x_k > 0$, respectively, the rest components of \mathbf{x} are zero. These positive components corresponding to the column vector of the \mathbf{A} are $\mathbf{P}_1 \dots \mathbf{P}_k$, respectively. According to the corollary 1, we need to prove they are linearly independent.

Assume these vectors $\mathbf{P}_1 \dots \mathbf{P}_k$ are linearly dependent, i.e., at least one parameter of $\lambda_1 \ \lambda_2 \dots \lambda_k$ isn't zero, so, the equation $\lambda_1 \mathbf{P}_1 + \lambda_2 \mathbf{P}_2 + \dots + \lambda_k \mathbf{P}_k = \mathbf{0}$ is established. Firstly, we imagine there is at least one parameter $\lambda_i > 0, 1 \leq i \leq k$, then, we can construct a column vector $\boldsymbol{\lambda} = (\lambda_1 \dots \lambda_k, 0 \dots 0)^T$ with the dimension of n^2 , therefore, the equation $\mathbf{A}\boldsymbol{\lambda} = \lambda_1 \mathbf{P}_1 + \dots + \lambda_k \mathbf{P}_k = \mathbf{0}$ is established. $\exists \lambda_i > 0$, we define α as

$$\alpha = \min_{1 \leq i \leq k} \left\{ \frac{x_i}{|\lambda_i|} \mid \lambda_i \neq 0 \right\}. \text{ Then } \mathbf{x} - \alpha \boldsymbol{\lambda} \geq 0, \mathbf{A}(\mathbf{x} - \alpha \boldsymbol{\lambda}) = \mathbf{Ax} \leq \mathbf{b}. \text{ Clearly, } \mathbf{x} - \alpha \boldsymbol{\lambda} \text{ is a feasible solution.}$$

Now, the l -th component of the feasible solution $\mathbf{x} - \alpha \boldsymbol{\lambda}$ can be expressed as $x_l - \alpha \lambda_l = 0$, then, the new feasible solution can be shown as $\mathbf{x}_\alpha = \mathbf{x} - \alpha \boldsymbol{\lambda} = (x_1 - \alpha \lambda_1 \dots x_2 - \alpha \lambda_2, 0, x_{i+1} - \alpha \lambda_{i+1} \dots x_k - \alpha \lambda_k, 0 \dots 0)$. Compared with the solution \mathbf{x} , the feasible solution \mathbf{x}_α is the lack of a positive component \mathbf{P}_l . The new column vector obtained by removing \mathbf{P}_l is expressed as $\mathbf{P}_1 \ \mathbf{P}_2 \dots \mathbf{P}_{k-1}$. If the new column vectors are linearly independent, the lemma will be true. Otherwise, we need to continue to find the new feasible solution by the above method until the column vector is transformed into a unit matrix. According to the corollary 2, it is clear that the unit matrix is basic feasible solution, so the lemma 4 can be proved.

Lemma 5: We denote the basic optimal solution as \mathbf{x} , its check number with respect to non-basic variable is less than zero, and the variable x_δ is a non-basic variable with a negative check number. There is $x_\delta = 0$ in all the optimal solutions. Namely, the non-basic variables of the optimal solutions are zero.

Proof:

Table 2 The simplex tableau based on LINGO:

		x_1	...	x_r	...	x_m	x_{m+1}	...	x_{m+k}	...	x_{n^2}
\mathbf{z}	z_0	0	...	0	...	0	c_{m+1}	...	c_{m+k}	...	c_{n^2}
x_1	b_1	1	...	0	...	0	a_{1m+1}	...	a_{1m+k}	...	a_{1n^2}
.
.

$$\begin{array}{cccccccccccc}
 x_r & b_r & 0 & \dots & 1 & \dots & 0 & a_{rm+1} & \dots & a_{rm+k} & \dots & a_{rn^2} \\
 \cdot & \cdot & \cdot & & \cdot & & \cdot & \cdot & & \cdot & & \cdot \\
 \hline
 x_{n^2} & b_m & 0 & \dots & 0 & \dots & 1 & a_{mm+1} & \dots & a_{mm+k} & \dots & a_{mn^2}
 \end{array}$$

Where, $b_i \geq 0, i = 1 \dots m, c_j \leq 0, j = m + 1, \dots n^2, \mathbf{x} = (b_1 \dots b_m 0 \dots 0)^T$.

Assume there are $c_j \leq 0, j = m + 1, \dots n^2, c_\delta < 0, m + 1 \leq \delta \leq n^2$ in the simplex tableau, if there is a feasible solution expressed as $\mathbf{x} = (x_1 \dots x_\delta \dots x_{n^2})^T \geq 0$, at least one element will be more than zero (i.e., $x_\delta > 0$), then, the objective function value of the clustering optimization problem is

$$z = z_0 - \sum_{i=m+1}^{n^2} c_i x_i \geq z_0 - c_\delta x_\delta > z_0.$$

Obviously, x is not the optimal solution, it is incompatible with the lemma, so the original proposition is true.

According to the literatures [26-28], we draw into the following theorems:

Theorem 1: In the case of the optimal solution, if the check number of the non-basic variables x_i is less than zero, the number of the optimal solutions will be the only one. If at least one check number of the non-basic variables x_i is zero, the others are less than zero, a necessary and sufficient condition for the only optimal solution is that the column vectors corresponding to all non-basic variables in which the check number is zero don't include the positive component in the simplex tableau.

The steps of obtaining the check number are expressed as below:

Firstly, we need to find the maximum linearly dependent subset of the coefficient matrix A . Next, we need to make a judgment between the basic variables and the non-basic variables. The rules of determining the loop is as follows:

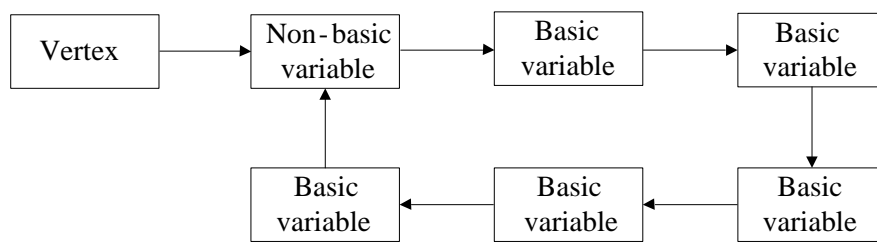


Fig. 2 The flow chart of the check number

We denote the non-basic variable as digital one (i.e., 1), these basic variables are digital two, three, etc. by analogy along the direction of the arrow.

e.g.: In order to get the check number of the non-basic variable x_{11} , the loop can be expressed as below:

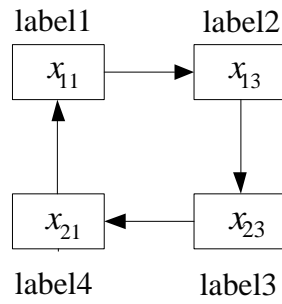


Fig. 3 The flow chart of an example

The check number of the non-basic variable x_{11} is the sum of these coefficients corresponding to the odd label variables in the objective function minus the sum of these coefficients corresponding to the even label variables in the objective function. In the LINGO platform, the check number and the simplex tableau are updated automatically by the simplex algorithm.

In order to solve the above optimization problem, we propose a novel strategy based on the LINGO platform [29-30]. It can find the optimal solution through the branch-and-bound algorithm, which modifies the branch direction by the simplex algorithm to avoid falling into a local optimum and greatly improve the operation efficiency. The chart of the proposed strategy based on LINGO platform is expressed as follows:

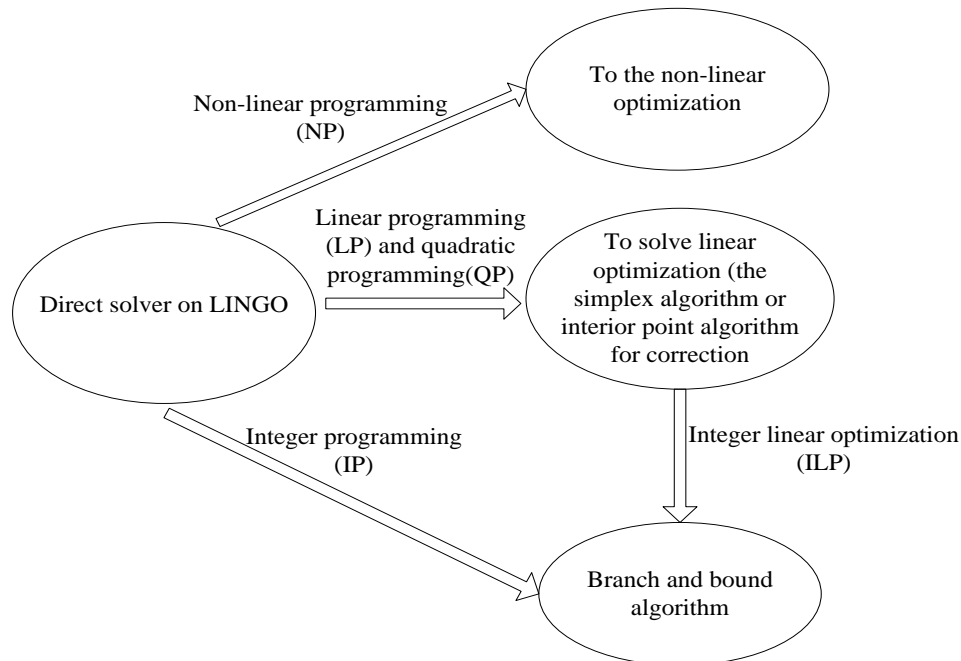


Fig. 4 The framework of solving the clustering optimization problem

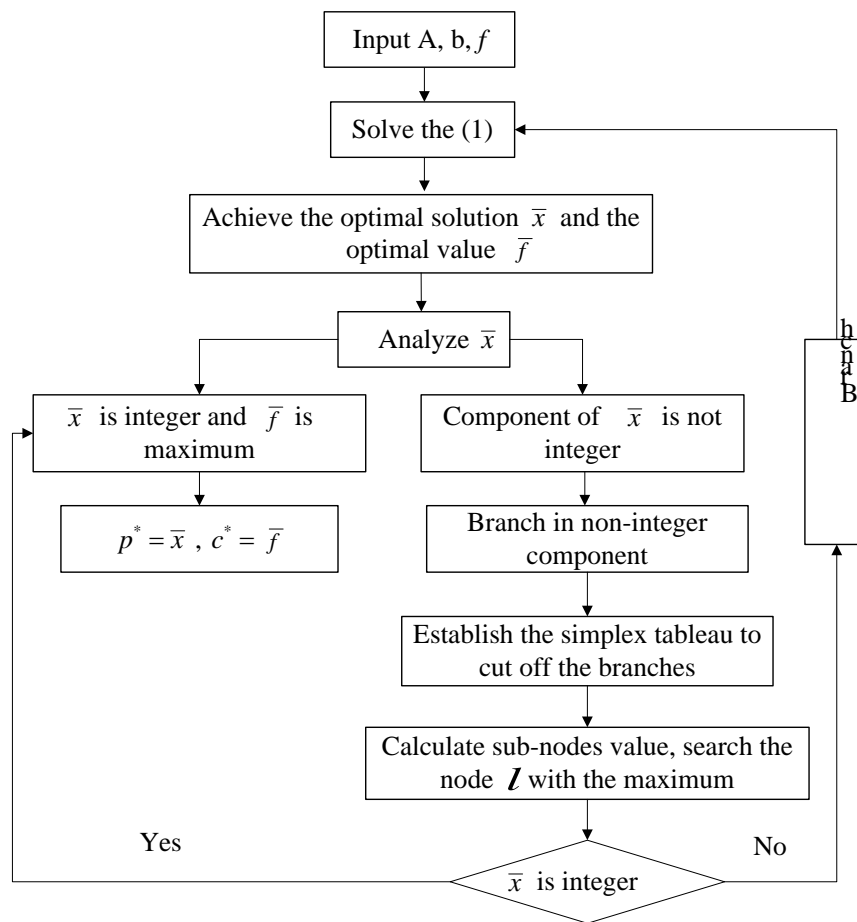


Fig. 5 The flow chart of the proposed clustering optimization algorithm(Algorithm 1)

The specific steps of the proposed clustering optimization algorithm (named as Algorithm 1 in this paper) can be described as follows:

Step 1) Transform the above optimization problem into a LINGO model;

Step 2) Apply the direct solution procedure to process equality constraint of the model;

Step 3) Recognize the type of the model on the LINGO (it is integer linear programming in this paper);

Step 4) Call the procedure of the Branch-and-Bound algorithm and the simplex algorithm, the procedure is described as follows:

Firstly, we denote the current maximum objective function value as c^* , the branch layer as p , the function value as \bar{f} in the Fig. 5, the work-piece order corresponding to c^* as p^* , the current node as $p1$, i.e., the sequence corresponding to the current node which needs to be branched.

1) *Initialization:* Let $p = 0$, $p1 = A$, (i.e., empty set), $c^* = \infty$, where the current node $p1$ is the root node.

- 2) *Calculate sub-node*: Each sub-node can be obtained from the branch of the current node, then, we calculate the lower bound l of each sub-node. Finally, these sub-nodes are sorted from small to large by the lower bound value. Update p (let $p = p + 1$).
- 3) *Establish the simplex tableau*: Take each sub-node as the initial value to establish the simplex tableau. It is proved that the optimal solution is in the boundary and the check number corresponding to the non-basic variables is the non-positive. So it is essential to cut off the branches which deviate from the direction of the boundary and this condition which the check number is the non-positive. Update p .
- 4) *Update parameter*: When all the current nodes are detected, we need to update p , let $p = p - 1$. The next process will skip to step 7, otherwise, we will mark the node b_1 with the maximum lower bound value as Q in each sub-node of the current layer (there, it is the p -th layer). The work-piece p_2 of the p -th position corresponding to Q is added at the end of the node p_1 . Let the current node equal to Q . Then, the next process will skip to step 5.
- 5) *Loop 1*: The above steps suggest that the current node has the maximum lower bound value in the co-tier nodes which have the same originating layer. If the lower bound value of the current node is not less than c^* , it is not necessary to search the current node and the co-tier nodes which have the same originating layer. Therefore, the process which probes the originating nodes of the current node is over. Update p (i.e., $p = p - 1$). The last one work-piece of p_1 is removed. Finally, please skip to step 7, otherwise, skip to step 6.
- 6) *Loop 2*: Suppose $p = n$, we can get a suboptimal order and make $p^* = p_1$. c^* is the lower bound value in the current node. We update p as $p = p - 1$. Please skip to step 7 after removing the last one work-piece of p_1 . Otherwise, skip to step 2.
- 7) *Achieve optimal solution*: If $p \neq 0$, please skip to step 4 after removing the last one work-piece of p_1 . Otherwise, the whole process ends, so c^* is the optimal value of the objective function and p^* is the optimal solution.

The above process is completed based on the LINGO platform.

Next we will give a theoretical analysis about the solution obtained by the proposed clustering optimization Algorithm 1.

First, according to the proofs from lemma 1, 2 and 4, we conclude that the proposed clustering optimization problem must have an optimal solution.

Secondly, the solution obtained by the proposed Algorithm 1 meets the lemma 5 and the define 5, so it is the basic feasible solution.

Then, according to the lemma 3, it is known that the solution is also the limit point.

Finally, according to the lemma 4 and theorem 1, it is verified theoretically that the clustering optimization solution is the global optimal solution.

4. The problem of resource allocation

One of the key technologies in femtocell networks is the channel allocation technology, which can improve the FUEs experience effectively. In most exiting researches, the location of FUE is fixed. However, in practical applications, the random movements of FUEs in the room will change the distance between FUE and FBS, which can affect the channel gain, and it may be reduce the quality of the mobile service and system capacity of femtocell networks. In another word, the FBS still uses the location information I_1 of the last time slot of a FUE in current time slot, however, the location information may have changed in current time slot. For example, when the path loss PL_2 of the new FUE location information I_2 is greater than the path loss PL_1 of the original location information I_1 , if a sub-channel is being allocated to this FUE according to the path loss PL_1 , then the communication may be interrupted. Therefore, sub-channel allocation based on the real-time channel gain is still a problem today. This paper proposes a new idea, which can predict the future location information of the FUE and compute PL_2 according to the Gaussian distribution model. When $PL_2 > PL_1$, update $PL_1 = PL_2$. If $PL_2 < PL_1$, the channel condition of the new location will be changed better. In order to ensure real-time communication, PL_1 still takes the original value. The channel gain is calculated by PL_1 to allocate sub-channels so as to ensure the communication quality of the mobile FUEs.

Next, we will introduce the resources allocation and the FUEs mobile model.

(1) Sub-channels allocation problem is converted into a maximization data rate problem, the following questions are constructed:

$$\max_{\Gamma_{k_i,i}^\theta} \sum_{i \in c_i} \sum_{\theta=1}^N \Gamma_{k_i,i}^\theta \Delta f \log_2(1 + P_{k_i,i}^\theta \gamma_{k_i,i}^\theta) \quad (22)$$

$$C1: \sum_{\theta=1}^N \Gamma_{k_i,i}^\theta \Delta f \log_2(1 + P_{k_i,i}^\theta \gamma_{k_i,i}^\theta) \geq \phi_{k_i}^{\min}, \forall i \quad (23)$$

$$C2: \sum_{\theta=1}^N \Gamma_{k_i,i}^\theta P_{k_i,i}^\theta \leq p_i^{\max}, \forall i \quad (24)$$

$$C3: \Gamma_{k_i,i}^\theta P_{k_i,i}^\theta g_{k_\omega,i}^\theta \leq \zeta_{k_\omega}^\theta, \forall \theta, i \quad (25)$$

$$C4: \Gamma_{k_i,i}^\theta P_{k_i,i}^\theta g_{k_j,i}^\theta \leq \zeta_{k_j}^\theta, \forall \theta, i \in \mathcal{C}_l, j \notin \mathcal{C}_l \quad (26)$$

$$C5: \sum_{i \in \mathcal{C}_l} \Gamma_{k_i,i}^\theta = 1, \forall \theta \quad (27)$$

where, $\Gamma_{k_i,i}^\theta \in \{0,1\}$ is an indicator that takes the value of 1 if sub-channel θ is allocated to the link between FUE k_i and FAP i , 0 otherwise. $P_{k_i,i}^\theta$ is the transmission power of the FAP i on the sub-channel n . $\gamma_{k_i,i}^\theta$ is the SINR of the FUE k_i in the FAP i on the sub-channel θ . It can be defined as $\gamma_{k_i,i}^\theta = \frac{g_{k_i,i}^\theta}{\sum_{j \neq i, j \in \mathcal{F}} P_{k_j,j}^\theta g_{k_i,i}^\theta + P_{k_\omega,\omega}^\theta g_{k_i,\omega}^\theta + N_0}$. $P_{M_\omega,M}^\theta$ is the transmission power of the MUE M_ω served by MBS M . N_0 is the noise power. ϕ_i is the threshold of the minimum data rate requirement. C2 represents that the total transmission power is not more than the maximum transmission power p_i^{\max} . C3 suggests that the interference between the MUE and FAP is not more than the interference threshold $\zeta_{k_\omega}^\theta$. C4 suggests that the interference between the FUE and FAP is not more than the interference threshold $\zeta_{k_j}^\theta$, \mathcal{C}_l is the l -th FAP cluster obtained by Algorithm 1. C5 says that these sub-channels are allocated orthogonally in the same cluster.

The above problem about the resource allocation can be simplified as $k_i(\theta) = \underset{i}{\operatorname{argmax}} \left(\frac{P_{k_i,i}^\theta \gamma_{k_i,i}^\theta}{g_{k_u,i}^\theta} \right)$,

where k_u is either a neighboring MUE or an FUE whichever has higher channel gain to the target femtocell i [25], $k_i(\theta)$ denotes that sub-channel θ is assigned to FUE k_i . Namely, the sub-channel is allocated to the FUE with the maximal value of $\frac{P_{k_i,i}^\theta \gamma_{k_i,i}^\theta}{g_{k_u,i}^\theta}$ in each cluster.

(2) The Gaussian distribution is adopted to simulate the moving model of FUEs, first of all, the position of FUE in the future is defined as:

$$(x_t, y_t) = \begin{cases} x_{t-1} + v_t \Delta t \cos(\alpha_t) + n_{t-1} \\ y_{t-1} + v_t \Delta t \sin(\alpha_t) + n_{t-1} \end{cases} \quad (28)$$

Among them, (x_{t-1}, y_{t-1}) is the previous position of FUE, v_t is the moving speed of FUE at the time t , Δt is the transition time interval, $\alpha_t \in [0, 2\pi]$ is the moving direction of FUE at the time t , n_{t-1} is the gaussian noise.

The speed, direction and distance between FUE and FBS are shown as follows:

$$v_t = N(v_{t-1}, \beta \Delta t) \quad (29)$$

$$\alpha_t = N(\alpha_{t-1}, 2\pi - a \tan(\sqrt{v_t}/2) \Delta t) \quad (30)$$

$$d = \sqrt{x_t^2 + y_t^2} \quad (31)$$

where v_{t-1} and α_{t-1} are the previous speed and direction of FUE respectively. $N(\mu, \tau)$ is Gaussian distribution, μ as average, τ as the standard variance. β is the mobile acceleration for FUE.

However, in [25], the resource allocation algorithm doesn't solve the problem of the average interference effectively. The authors don't take into account the data-rate fairness and the mobility of FUEs. In this paper, not only do we solve the average interference effectively, but we also consider data-rate fairness and the FUEs mobility. The proposed sub-channels allocation algorithm can be expressed as follows:

We give every FAP a serial number, then put them into the different clusters by the Algorithm 1. We denote the total number of sub-channels as N and the total number of FUEs as fue . We set up the allocation instruction matrix $T = \text{zeros}(N, fue)$ and the interference instruction matrix $G = (fue, fue, N)$. Initializes the location of FUE x_{t-1}, y_{t-1} and the direction α_{t-1} . Calculate the path loss PL_1 of the current position. According to the formula (28-31), this paper predicts the future position of FUE, and the corresponding path loss PL_2 . If $PL_2 > PL_1$, update $PL_1 = PL_2$.

Sub-channels are assigned orthogonally in the same cluster and reused among different clusters. Firstly, we allocate all of the sub-channels to each FUE in a cluster. Secondly, we assume that FAPs send signals by the average transmission power (P_i^{\max} / F). The average allocated power will be reallocated

after the sub-channels are allocated. We choose the sub-channels for the FUEs to make $\frac{P_{k_i,i}^\theta \gamma_{k_i,i}^\theta}{g_{k_u,i}^\theta}$

maximum in each cluster, then update $T(\theta, k_i) = 1$. From the former clusters, we find out the FUE j who uses the same sub-channel with the current cluster. We detect the FUE successively to judge whether its channel quality is beyond the interference threshold by $\Gamma_{k_i,i}^\theta P_{k_i,i}^\theta g_{k_j,i}^\theta \geq \zeta_{k_j}^\theta, \forall \theta, i \in C_j, j \notin C_i$ in current cluster, if it does, we will make $G(k_i, k_j, \theta) = G(k_j, k_i, \theta) = 1$, and continue to detect whether the achieved data rate ϕ_{k_i} of FUE i meets the minimum data rate requirement $\phi_{k_i}^{\min}$ or not. If one of two constraints

cannot be met, we will allocate the sub-channel n to another FUE to make $\frac{P_{k_i,i}^\theta \gamma_{k_i,i}^\theta}{g_{k_u,i}^\theta}$ maximum in the

current cluster, then we repeat it until all the constraints are met. If all constraints are not met for all FUEs in the cluster, we will remove n , update T and G .

In order to guarantee the fairness among FUEs, we will adjust the sub-channel allocation result. There may be the FUEs who are not assigned a sub-channel. So, according to the indication matrix T of channel assignment, we can detect whether a current FUE has been assigned a sub-channel or not. If the FUE k_i is not assigned a sub-channel, we will find out the set N' of sub-channels on which the current

FUE k_i has interference with other FUEs by the instruction matrix G . We find out a sub-channel θ' from the set N' to make the data rate of the FUE k_i maximum. If the sub-channel θ' is allocated to another FUE k_j in the current cluster ψ_k and the number of the sub-channels allocated to the FUE k_j is more than one, we will delete the sub-channel θ' from the set of sub-channels allocated to the FUE k_j , and allocate θ' to k_i . If there is only one sub-channel for the FUE k_j , we will give up θ' , and exclude θ' from the set N' . Repeat it until all of FUEs are assigned. Finally, update T .

To reduce the interference among clusters further, to begin from the first FUE in the current cluster, we find out the FUE k_j from the former several clusters who uses the same sub-channel with the current FUE k_i . We need to detect whether the interference is less than the interference threshold between the current FUE k_i and the FUE k_j . If it does not, we will distinguish whether the number $N1$ of sub-channels allocated to the FUE k_j is more than one by T . If it does, we will delete the interference sub-channel from the k_j . Finally, we repeat it until all the FUEs are tested in current cluster.

Algorithm 2: The proposed sub-channels allocation algorithm

1. **Initialize** $T = \text{zeros}(N, \text{fue})$, $G = (\text{fue}, \text{fue}, N) = 0$, x_{t-1} , y_{t-1} and α_{t-1} .
2. Calculate PL_1 . Predict PL_2 . If $PL_2 > PL_1$, update $PL_1 = PL_2$.
3. **for** $\theta = 1 : N$
4. $\theta \rightarrow k_i$, s.t. $\max \frac{P_{k_i,i}^\theta \gamma_{k_i,i}^\theta}{g_{k_u,i}^\theta}$, $T(\theta, k_i) = 1$
5. **end for**
6. **while** (j)
7. **if** $\Gamma_{k_i,i}^\theta P_{k_i,i}^\theta g_{k_j,i}^\theta \geq \zeta_{k_j}$, $\forall \theta, i \in C_j, j \notin C_j$ or $\phi_{k_i} < \phi_{k_i}^{\min}$
8. Make $G(k_i, k_j, \theta) = G(k_j, k_i, \theta) = 1$
9. $\theta \rightarrow$ the other FUEs to make $\frac{P_{k_i,i}^\theta \gamma_{k_i,i}^\theta}{g_{k_u,i}^\theta}$ maximum
10. **end if**
11. **end while**
12. **if** θ can't meet all of the FUEs in the cluster, remove θ
13. Update T and G
14. **end if**
15. **while** (k_i)
16. Find out the set N' by G
17. $\theta' \in N' \rightarrow k_i$ to make the data rate maximum
18. **if** $\theta' \in k_j, k_j \neq k_i, k_j \in \psi_k$ and the number of the sub-channels of the FUE k_j is more than 1

5. Simulation results

We assume that there are 16 rooms for one layer in a building, which are uniformly distributed in two scenarios with the range of $40\text{m} \times 40\text{m}$ (1600m^2) or $70\text{m} \times 70\text{m}$ (4900m^2), so the area of each room is 100m^2 or 306m^2 . We implement the algorithms many times (1000-60000 times, see Table 4) under various topologies and scenarios, and the algorithms run iteratively each time until the optimal solution is found or it reaches the maximum iterations number (30000 in this paper). ω_ϕ^- is a random value within the feasible range. The simulation results show no matter what value of ω_ϕ^- is, the proposed algorithm can always find out the global optimal solution efficiently compared with those existing algorithms so as to determine the maximum value of the objective function.

The average data rate is defined as $\frac{\sum_{i=1}^F \sum_{\theta=1}^N \Gamma_{k_i,i}^\theta \Delta f \log_2(1 + P_{k_i,i}^\theta \gamma_{k_i,i}^\theta)}{F}$. The average interference at

FUE k_f on the sub-channel n is defined as $\frac{\sum_{i=1}^F (\sum_{\theta=1}^N (\sum_{j=1, j \neq i}^F \Gamma_{k_j,j}^\theta \Gamma_{k_i,i}^\theta g_{k_i,j}^\theta P_{k_j,j}^\theta))}{NF}$. The data rate

fairness is defined as $FF = \frac{(\sum_{k_i=1}^K \phi_{k_i})^2}{K \sum_{k_i=1}^K (\phi_{k_i})^2}$, where ϕ_{k_i} denotes the data rate of the FUE k_i . K is the total

number of FUEs.

The others simulation parameters are shown in Tab. 3:

Table 3 The simulation parameters

Parameters	Value
Carrier frequency	2.0GHz
Available sub-channels	4
Sub-channel bandwidth, Δf	180KHz
Number of FUEs per femtocell	1-2
Distance between FUE and FAP, d	3m
Macrocell radius	500m
Distance between indoor building and MBS	100m
Standard deviation of shadowing between macrocell and indoor UE	4dB
Outdoor wall loss	20dB
Floor loss	18.3dB
Noise power density	-174dBm/Hz
Macrocell transmission power	20W
FAP maximum transmission power, P_i^{\max}	30mW
Minimum data rate requirement, ϕ_i	100bps

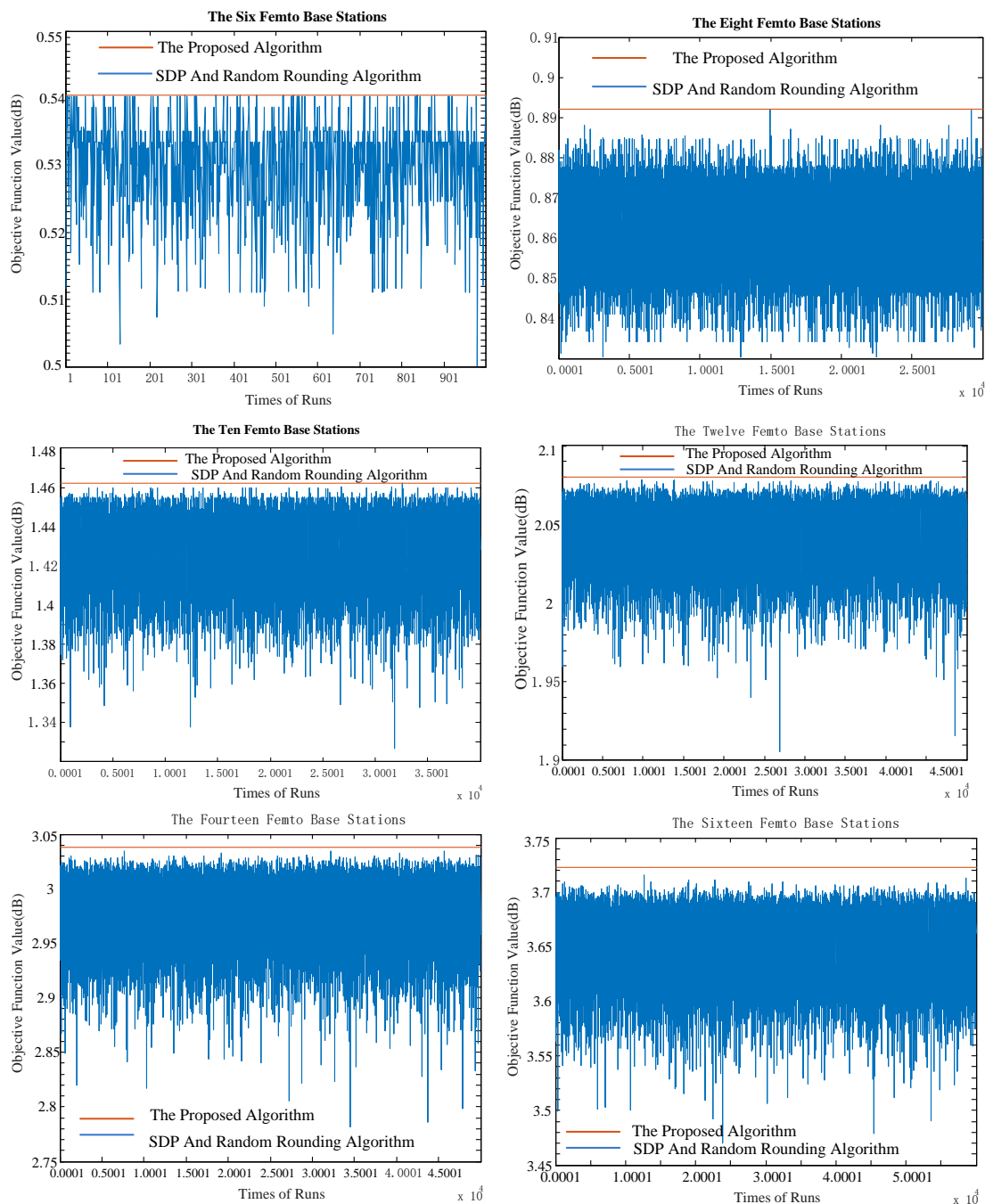


Fig. 6 Objective function value vs. the number of counting

Fig 6 shows each objective function value in 1000 times runs under the case of 6, 8, 10, 12, 14, 16 FAPs. The global optimal solution can always be successfully found each time under all the cases by the proposed algorithm (red points) but limited times by the SDP clustering algorithm in [25] (blue points) when the numbers of FAPs are 6, 8 10. However, the SDP clustering algorithm cannot find the global optimal solution when the numbers of FAPs are 12, 14, 16. That is because the proposed algorithm simplifies the optimal constraint items and revises the direction of searching for the optimal solution. So it can accurately narrow the scope of the solution space so as to find the optimal solution easily. However,

the principle of the SDP clustering algorithm based on the CVX platform is that the constraint of the original problem is flabby, so the scope of the solution space is expanded. When the upper bound of the original problem is determined, the system will generate a group of random numbers obeying the normal distribution so as to search the optimal solution by the Random Rounding algorithm, which easily falls into a local optimum value because of the number randomly generated. In addition, the latitude of the Random Rounding algorithm increases exponentially with the increase of the number of FAPs. Therefore, with the increase of the number of FAPs, it is hard to obtain the global optimal solution. In this paper, no matter how many the number of FAPs is, the proposed algorithm will find the global optimal solution efficiently.

Table 4 Comparison of the efficiency of the algorithms

The number of FAPs in 40×40m	SDP Clustering Algorithm in [25]				the Proposed Clustering Algorithm		
	The iterations for the optimal solution first obtained	The required time (second) when running a time	Whether to find out the optimal solution	Total run times /the times of the obtained optimal solution	The iterations of the optimal solution first obtained in 1000 times	The required time (second) when running a time	Whether to find out the optimal solution in each times
6	78	2.489	yes	1000/9	2	0.045	yes
8	15000	43.418	yes	30000/1	7	0.050	yes
10	32496	64.545	yes	40000/1	5	0.049	yes
12	0	229.636	no	50000/0	8	0.056	yes
14	0	262.019	no	50000/0	7	0.047	yes
16	0	366.51	no	60000/0	8	0.058	yes

As shown in Tab. 4, when the numbers of the FAPs are 6, 8 and 10, the global optimal solution can be obtained by the SDP clustering algorithm, but the efficiency is lower than that of the proposed algorithm. For example, when there are 6 FAPs in the scope of 40m×40m, in 1000 times runs, the times of the global optimal solution obtained by the SDP clustering algorithms is 9. However, the proposed algorithm can always find the optimal solution successfully under all the cases. Also, the global optimal solution can be first obtained at the 78-*th* iteration by the SDP clustering algorithms while at the 2-*nd* iteration by the proposed algorithm. When the number of the FAPs is 8, the global optimal solution is only obtained once (at the 15000-*th* iteration) in 30000 times runs. Similarly, a global optimal solution is

obtained once (at the 32496-th iteration) in 40000 times runs. However, the proposed algorithm first obtains the global optimal solution at the 7-th and 5-th iteration respectively under the above two cases. Also, Tab. 4 shows that the time that the global optimal solution is obtained by the proposed algorithm is much less than that by the SDP clustering algorithm, so the proposed algorithm is more effective.

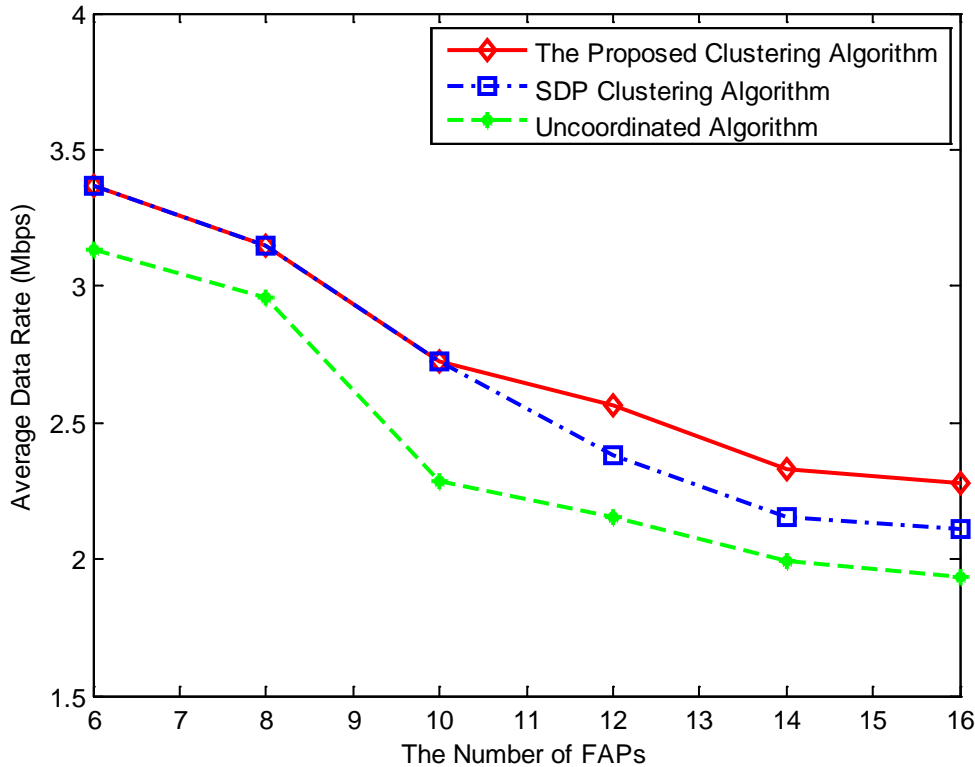


Fig. 7 Average data rate vs. the number of FAPs

Fig. 8

Fig. 7 shows the simulation scenario is that the FAPs are uniformly distributed in the scope of $40 \times 40 m^2$, the interference threshold is $\zeta_k^n = 10^{-11}$. We apply the Algorithm 3 of Ref. [8] to allocate the resources. Because both the SDP clustering algorithm and the proposed algorithm can find the global optimal solution when the numbers of FAPs are 6-10, the average data rate of them is the same. However, the average data rate obtained by the proposed algorithm is higher than the SDP clustering algorithm with the increase of the number of FAPs. That is because SDP clustering algorithm can not find the global optimal solution when the scope of the solution space is expanded. In addition, under the condition of the uncoordinated scheme in [25], the average data rate is the lowest because of the existence of more interference. Also, when the number of FAPs increases, the average data rate will reduce, because the interferences among the adjacent FAPs are increased.

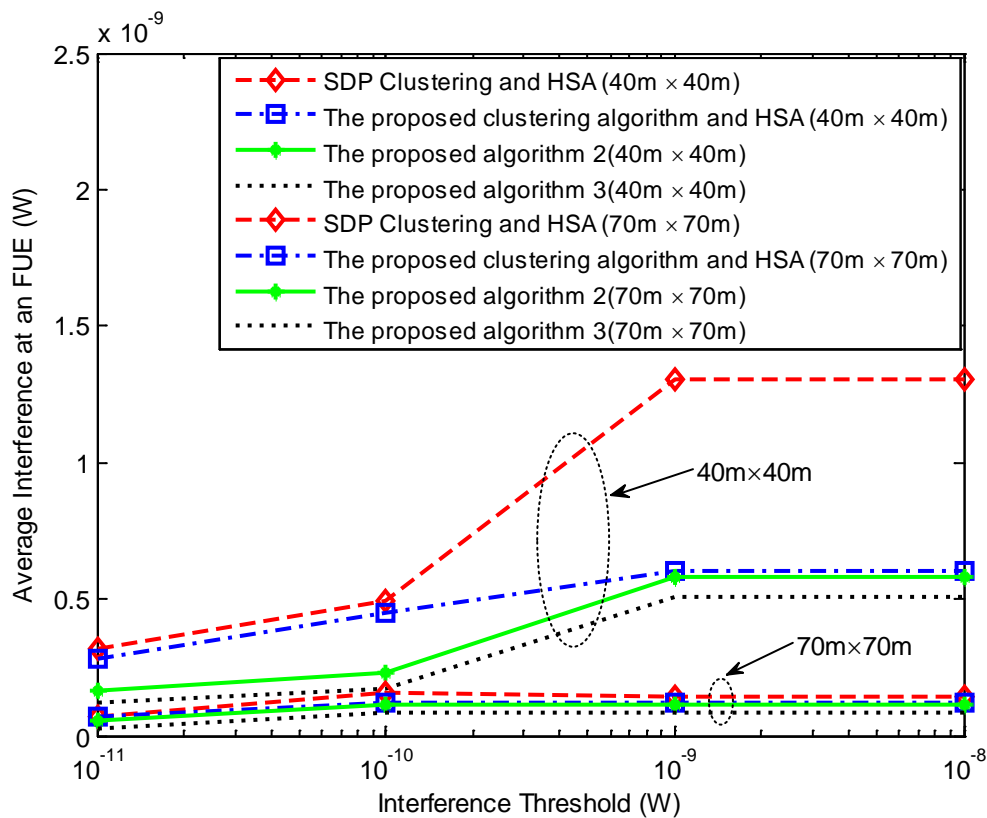


Fig. 9 Average interference at an FUE vs. interference threshold

Fig. 8 show the comparison results of several algorithms under two scenarios of $40 \times 40 m^2$ and $70 \times 70 m^2$, including the proposed clustering algorithm and HSA (Heuristic Sub-channel Allocation algorithm in [25]), SDP Clustering and HSA, the proposed Algorithm 2 (the proposed clustering algorithm and resource allocation algorithm), and the proposed Algorithm 3 (the proposed clustering algorithm, resource allocation algorithm and power allocation algorithm). It's shown that the average interferences at an FUE achieved by the proposed algorithms are lower than that by the SDP clustering algorithm and HSA algorithm, which indicates that the proposed clustering algorithm, resource allocation algorithm and power allocation algorithm all can reduce the interference from inter-FAPs and intra-FAPs more effectively than the SDP clustering algorithm and HSA algorithm. It is worth mentioning that the average interferences obtained by the proposed algorithms are much lower than that by the SDP clustering algorithm and HSA algorithm. In addition, Fig. 8 shows that the larger the distributed range is, the smaller the value of the interference between FUEs will be in the case of the same number of FAPs. That is because the distance between two FUEs increases so that the path loss increases. Also, the average interference of FUE increases with the increase of the interference threshold. However, when the interference threshold is increased to the threshold value, the average interference of FUE will not change. That is because the

larger the interference threshold is, the more the number of the valid sub-channel is, and the larger the average interference is. When the interference threshold is increased to the threshold value, the interference between the FUEs is less than the interference threshold, so, we don't need to consider the interference. When the interference threshold continues to be improved, the allocation of the sub-channels will not be changed. In addition, it is obvious that the interference threshold value is $10^{-9}(W)$ in the range of $40 \times 40 m^2$ and $10^{-10}(W)$ in the range of $70 \times 70 m^2$. It shows that the larger the distribution range of the FAPs is, the smaller the interference threshold value will be.

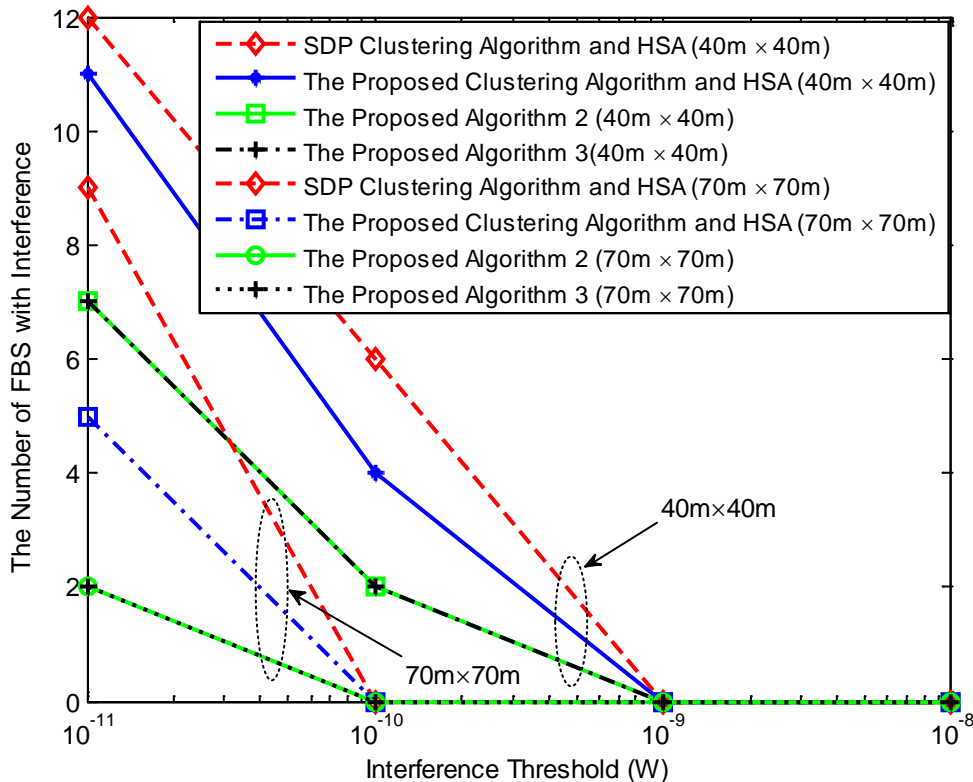


Fig. 10 The number of FBS with interference vs. interference threshold

Similarly, Fig.9 shows that the number of the FAPs which cause interference by the proposed clustering algorithm and HSA algorithm is also less than that by the SDP clustering algorithm and HSA algorithm. So it also further validates that the solution obtained by the proposed clustering algorithm is the optimal solution. Also, the number of the FAPs with interference by the proposed resource allocation algorithm is also less than that by the HSA algorithm. It indicates that the proposed resource allocation algorithm can reduce the interference effectively and enhance the experience of the FUEs. Furthermore, Fig. 9 shows that the lower the interference threshold is, the greater the number of the FAPs with interference will be. If interference threshold is larger than the threshold value, the number of the FAPs

with interference will be zero. We can also see that, the wider the distributed range of the FAPs is, the fewer the number of FAPs with interference will be in the same algorithm.

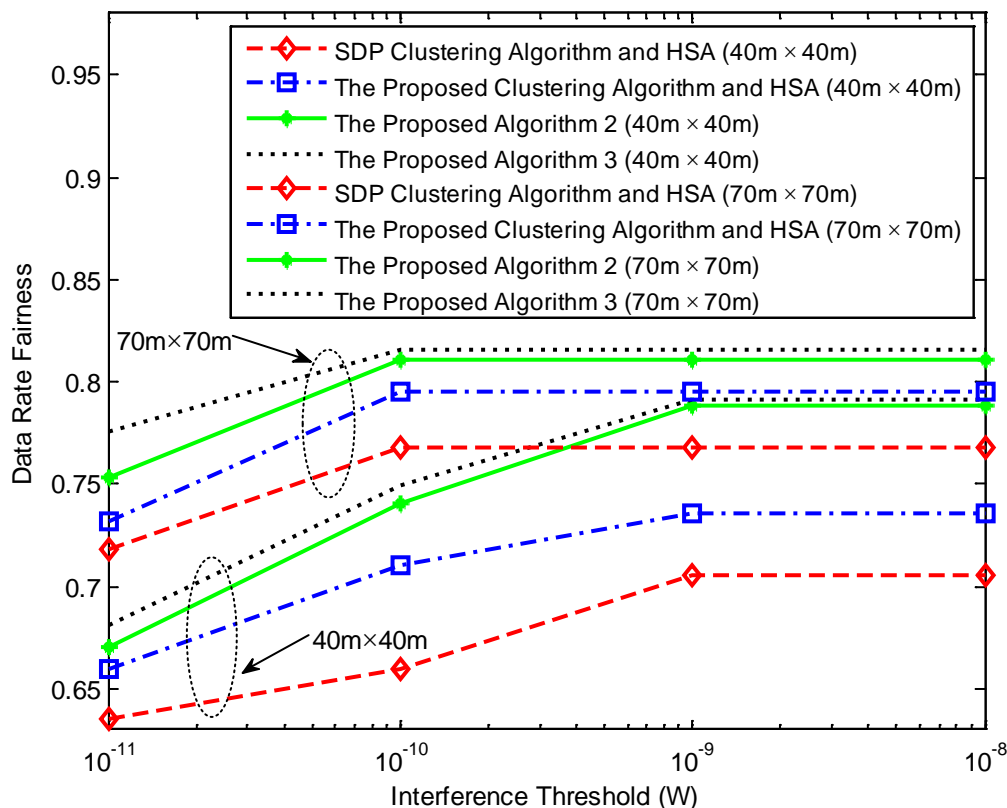


Fig. 11 Data rate fairness vs. interference threshold

Fig. 10 shows that the data rate fairness of the FUE achieved by the proposed clustering algorithm is higher than that by the SDP clustering algorithm with the increase of the interference threshold, when the two algorithms both use the HSA algorithm to allocate sub-channels in the same simulation scenario. It further explains the proposed clustering algorithm is superior to the SDP clustering algorithm. Simultaneously, the simulation results further confirm the above theoretical proof that the optimal clustering solution can be found by the proposed clustering algorithm. The data rate fairness of the FUE achieved by the proposed Algorithm 2 is always higher than that by the HSA algorithm, when they both use the proposed clustering algorithm to make the FAPs into clusters. It shows the proposed sub-channels allocation algorithm not only can reduce the interference of FUE, but also take into account the fairness of FUE. Similarly, Compared with the proposed Algorithm 2, the data rate fairness of the FUE by the proposed Algorithm 3 is improved further. In addition, the values of the data rate fairness increase with the increase of the interference threshold but holding constant after reaching the interference threshold value. The reason for this phenomenon is that the number of the FUEs with interference decreases when the

interference threshold increases, so the number of the available sub-channels increases. As a result, the opportunity that each FUE gets the sub-channel increases.

6. Conclusion

This paper investigates the interference management and resource allocation problems for tow-tier heterogeneous networks. First, we propose a mathematical modeling idea based on LINGO which can efficiently solve the joint clustering problem for the FAPs. The Branch-and-Bound algorithm and the simplex algorithm are used synthetically to find the optimal solution by LINGO. In addition, not only does this paper theoretically prove that the solution obtained by the proposed clustering algorithm is the global optimal solution, but the simulation results have also showed that the proposed algorithm can obtain the optimal solution compared with other algorithms based on the same clustering optimization problems and improve the efficiency of searching for the optimal solution. Secondly, a FAP is selected as a cluster head that is responsible for resource allocation among the FAPs in that cluster. Finally, we put forward a novel strategy for resource allocation. Compared with other related schemes, the proposed resource allocation algorithm can achieve lower interference between FUEs and higher data-rate fairness.

7. Acknowledgments

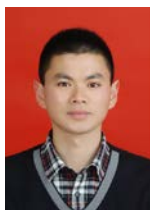
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