Rate-perceptual-distortion optimization (RpDO) based picture coding

- Issues and Challenges

Hong Ren Wu  
School of Elec. & Comp. Engineering  
RMIT University  
Melbourne, Australia  
e-mail: henry.wu@rmit.edu.au

Weisi Lin  
School of Computer Engineering  
Nanyang Technological University  
Singapore  
e-mail: wslin@ntu.edu.sg

King Ngai Ngan  
Department of Electronic Engineering  
Chinese University of Hong Kong  
Hong Kong, P.R. China  
e-mail: knngan@ee.cuhk.edu.hk

Abstract—Perceptual image and video coders have been reported extensively in the literature with a recent survey listing sixteen representative PVCs (perceptual video coders) for single-view video coding, ten of which conform to H.263, MPEG-4 or H.264/AVC international standard. Successful approaches to perceptual picture coding are classified as perceptual predictive coding, perceptual quantization, rate-perceptual-distortion optimization (RpDO) and perception-based pre-, loop- and post-filtering. This contribution analyzes and examines the performance and tradeoffs of representative perceptual as well as traditional waveform based picture coding methods in terms of their effectiveness in quality constrained coding and RpDO criterion in the context of visual quality controlled or regulated visual communication, broadcasting, entertainment and consumer electronic products, systems and services or applications. Fundamental flaws of performance benchmarking using a rate-driven coding strategy are interrogated for visual quality assured services or user experience. It is revealed that while existing perceptual distortion measures (PDMs) successfully grade distortions corresponding to HVS (human visual system) perception, their quantitative correspondence, or lack of it, to discernible levels by human visual perception in terms of just-not-noticeable-difference (JNND), just-noticeable-difference level 1 (JND1), just-noticeable-difference level 2 (JND2), etc. has not been thoroughly investigated nor established, and this motivates the related initial work reported in this paper. Issues and constraints of existing picture coding standards are discussed in the context of developing a future picture coding standard framework for constant visual picture quality systems and services.

Keywords—picture coding; rate-perceptual-distortion optimization; just-noticeable-difference; perceptual distortion measure; perceptual picture quality assessment

I. INTRODUCTION

Human visual perception based coding of visual signals has been reported extensively for provision of user-centric quality assured visual communication, broadcasting and entertainment services and applications where a designated constant visual quality is imposed as the constraint in either perceptually lossless coding design whose performance is theoretically lower-bounded by perceptual entropy or perceptually lossy coding design which is governed by rate-perceptual-distortion optimization (RpDO) theory [1].

Two observations have been made known since the very beginning of visual signal coding and compression research based on Shannon’s entropy theory [2] and rate-distortion theory [3,4], i.e., first, when taking account of visual picture quality as perceived by human visual system (HVS), significantly high compression ratios than what is achievable by information lossless (or entropy) coding are possible where the compressed images are visually comparable to [5] or indistinguishable from their originals [6]; second, constant bitrate coder design and constant distortion (or “quality”) coder design can be formulated when coding distortion is either inevitable or impractical due to various constraints [7]. What has been less clear, so it seems though, due to various historical reasons and the current dominance of bitrate driven coding approach insofar as standardization activities and existing products and systems are concerned [8] is the difference between how effective a digital picture coding approach/method/solution is measured using the aforementioned two alternative coding design principles. For a constant bitrate coder to lay a claim to its effectiveness in compression performance, it has to hold a fixed bitrate and then to maximize picture quality, whereas for a constant quality coder to be effective, it must, first and foremost, be able to hold a given picture quality and, then, to do so at the lowest possible bitrate.

Compared with constant bitrate coding, the development of constant picture quality coding design has been sporadic and slow for reasons more than one, including fixed bandwidth transmission dictated by the circuit-switched communication networks [9] and television broadcasting bandwidth allocations [10], lack of widely accepted and understood quantitative perception based quality measures [11-13] in place of the time-honored MSE (mean squared error) for rate-distortion optimization (R-DO) using contemporary picture coding frameworks [7], incomplete knowledge of human vision and HVS modelling in relation to picture coding [1], and perhaps...
not so small a degree of complacency with following the so far very successful bitrate-driven coding design approach by which each more recent picture coding standard demonstrates a significant gain over its predecessors in terms of, e.g., the PSNR (peak signal-to-noise ratio) with appreciable visual quality improvement [8], regardless the fact that the HVS does not compute the PSNR (or the MSE) [14].

Surrounding issues associated with R-DO based picture coding techniques and designs, this paper analyzes the performance and tradeoffs of representative perceptual as well as traditional waveform based picture coding methods in terms of their effectiveness in quality constrained coding and R-DO criterion in the context of visual quality controlled or regulated visual communication, broadcasting, entertainment and consumer electronic products, systems and services or applications. It examines the premise of performance benchmarking and evaluation using a rate-driven coding strategy in the context of visual quality assured services or user experience. It interrogates whether existing perceptual distortion measures (PDMs) can predict discernible levels by human visual perception in terms of, e.g., just-not-noticeable-difference (JNND), just-noticeable-difference level 1 (JND1), JND level 2 (JND2), etc., consistently, while they have been reported to grade distortions successfully in correspondence with HVS perception [15–18]. Issues and constraints of existing picture coding standards are discussed in the context of developing a future picture coding standard framework for constant visual picture quality systems and services.

In the rest of this paper, Section II discusses useful concepts and measures for picture coders. Section III presents issues related to constant quality coding, since coding quality (rather than bitrate) is what users perceive, while Section IV describes perceptual manipulations which are either standard compliant or beyond the current compliance. The last section highlights the main points being covered and possibilities of future exploration.

II. QUALITY ASSESSMENT FOR PICTURE CODER DESIGN

A. Quality Assessment for Picture Coding and Transmission

Picture quality or quality of experience (QoE) assessment is not just for its own sake, but linked closely to and serves visual signal compression and transmission where R-D theory is applied for product, system and service quality optimization [12,13,19,20]. In constant bitrate coder design and visual services, a perceptual picture measure ranks visual quality of picture coders, visual communication systems or services, and it concerns with who provides the best picture quality for a designated bitrate or bandwidth. In constant visual quality coder design and services, however, a perceptual distortion measure must be able to detect JNND as the reference quality level at which the compressed picture is indistinguishable from the original, and predict consistently discernible levels by human visual perception in terms of JND1, JND2, etc. How this set of perceptually discernible levels of distortions/quality is mapped into practical scales of various applications, e.g., perceptual utility of pictures (UoP) [21], depends very much on the application in question and requires participation of targeted human observers who have necessary domain knowledge of intended applications, e.g., radiologists and radiographers in medical diagnostic imaging [6].

B. R-DO versus R-DO

From R-D optimization perspective, it is widely understood that use of raw mathematical distortion measures, such as the MSE, do not guarantee visual superiority since the HVS does not compute the MSE [1,14]. In R-DO optimization where perceptual distortion or utility measure matters, setting a rate constraint, Rc, as shown in Fig. 1 is redundant from perceptual distortion controlled coding viewpoint. The perceptual bitrate constraint, Rc, makes more sense which delivers a picture quality comparable to JND1. In comparison, Rc is neither sufficient to guarantee a distortion level at JNND nor necessary to achieve, e.g., JND1 in Fig. 1. By the same token, setting a distortion constraint, Dc, is not effective in holding a designated visual quality appreciable to the HVS since it cannot guarantee a picture distortion level at JND, nor is it necessary to deliver picture quality at JND. As the entropy defines the lower bound of the bitrate required for information lossless picture coding, the perceptual entropy sets the minimum bitrate required for perceptually lossless picture coding [1]. Similarly, in UoP regulated picture coding in terms of a utility measure, utility entropy can be defined as the minimum bitrate to reconstruct a picture required to achieve complete feature recognition equivalent to the perceptually lossless picture including the original as illustrated in Fig. 1.

C. Scales for Perceptual Quality/Distortion Measurement

Constant perceptual quality coder design and visual signal transmission services rely on perceptual distortion or quality measures, which are able to assess levels of quality discernible to human viewers, for R-DO to uphold an agreed or a designated visual quality acceptable to the users at the minimum bitrate. Goodness of these perceptual measures are
appraised and validated by subjective test data as the ground truth [15-18]. Absolute category rating (ACR) has been widely used in subjective picture quality evaluations [18,22] whose data have been often assumed as the ground truth and used to evaluate or validate perceptual distortion or quality metrics [15-18]. However, it is not clear whether this ground truth so acquired and claimed using the existing ACR or similar schemes is sufficiently adequate, accurate or suitable for design of constant quality or quality regulated picture coders and performance evaluations. For example, there is no guarantee that a score of “excellent” in a five- [22] (or eleven- [23]) level scale for rating overall picture quality when the actual scores marked by viewers do not (and they rarely do [8]) achieve the full mark corresponds to the JNND level which can be used to guide perceptually lossless picture coding. Nor a score out of 100 necessarily commits itself to a discernible level of quality or distortion comparable to that perceived by the HVS, which is able to uphold a constant visual quality in perceptual picture coding based on the RpD-DO.

Furthermore, human perception and judgment in a psychophysical measurement task perform usually better in comparison tasks than casting an absolute rating [24]. To address the issue regarding unreliability and fluctuations associated with subjective test data using absolute rating schemes due to contextual effects [25] and varying experience and expectations of observers, a distortion detection strategy is considered compared with the ACR to ascertain JND levels [1] or VDUs (visual distortion units) [26] or distinguishable utility levels pertaining to a designated application [21] in correspondence to constant picture coding approach and design philosophy. As shown in [26], the relative threshold elevation for a VDU varies from one VDU to the next as function of spatial frequency and orientation which does not appear to be linear.

III. EFFECTIVENESS FOR CONSTANT QUALITY CODING

As discussed previously, to determine whether a constant quality coder is effective, the first criterion to apply is that it must demonstrate its ability to compress any given picture at a designated visual quality, and the second is to do so at lower bitrates than what its alternatives can offer. This is crucial to evaluation of constant quality picture coding performance, since the variation of picture quality with time greatly affects the user experience [27-29].

A. Current Mindset for Performance Evaluations

Coding performance evaluations, including those where perception based measures are considered, have been by and large dominated by a mindset based on bitrate-driven design approach. The winner was declared if for the same given bitrate, it demonstrated a superior picture quality measured by either the time honored measures such as the PSNR or the MSE [8] or quantitative perceptual distortion/quality measures [15-18] or subjective assessments [22,23]. When the visual quality was considered, the focus was still on which coder achieved significant bitrate savings at a comparable visual quality, instead of whether it was able to deliver designated picture quality levels discernible to human viewers at lower rates. In other words, performance evaluations based on the current mindset are able to assess effectiveness of bitrate-driven coder design, nonetheless do not address the key issue regarding effectiveness of (visual) quality-driven coder design.

Take the performance evaluation of H.265/HEVC (high efficiency video coding) recently reported in [8] for example. The superior effectiveness of H.265/HEVC as a bitrate-driven coder has been amply demonstrated over all its predecessors in terms of delivering a designated bitrate at a significantly better quality using either the PSNR or visual quality. If, however, visual quality assured service at a designated quality level is set as the performance criterion, according to a performance shown in Fig. 2 of [8], there is an up to 3 dB difference by the HEVC MP (main profile) coder for two different test sequences for a given bitrate of 2 Mbps, even if the PSNR is accepted as an appropriate fidelity measure. Alternatively, given a PSNR value, say in this example 37 dB, it is most likely representing different levels of visual picture quality for these two videos coded at about 2 Mbps and 512 kbps respectively. In any event, effectiveness of all coders under this comparative study as a quality-driven coder has not been considered, demonstrated or established.

A counterexample is given in [6] that effectiveness of a JPEG 2000 bit-stream compliant perceptually lossless image coder was demonstrated in terms of its ability to hold visual distortions at or below JNND level whereas a JPEG-LS near-lossless coder (with d=2, i.e., the maximum pixel difference between the compressed and the original images less than or equal to two) failed to do so in double blind subjective evaluations. In this sense, the HVS based approaches are most likely the means to deliver effective, efficient compression and transmission strategies for visual signals in terms of RpD criterion.

B. Perceptual Distortion Measures for RpD-DO

A fundamental principle for quality-driven picture coding design is RpD optimization, where perceptual distortion measure (PDM) plays a key role in aligning steps/units of distortions, hopefully, consistently with discernible levels by human visual perception in terms of, e.g., JNND, JND1, JND2, and so on [1,26]. Using RpD-DO based approach to constant quality picture coding design, the designated visual quality level is achieved by controlling perceptual distortions estimated by the PDM at a corresponding (constant) level while minimizing the required coding bitrate. It then begs the question whether the existing perceptual distortion measures can consistently predict discernible levels by the HVS, while they have been reported to grade distortions reasonably successfully in correspondence with HVS perception [15-18].

To explore this issue, a preliminary experimental investigation was conducted as follows. Images were generated using an open source JPEG 2000 coder [51] at various (increasingly higher) compression ratios for a total 81 variations for each of eleven test images. This provides a range of test pictures that capture the transition points between JND levels. JNDn image is determined relative to the JNDb-n) image, except for JND1 which was relative to the reference image, such that JND2 is relative to JND1 and JND3 to JND2. Perceptual distortion or quality measures were computed for sets of images at JND1,
Figure 2. Sample images. (a) original Bikes image; (b) processed Bikes with JND$_1$; (c) processed Bikes with JND$_2$; (d) processed Bikes with JND$_3$; (e) the difference between (a) and (b) with an offset of 128 and contrast enhancement which assist in visualization in pdf format or photo printing form; (f) the difference between (a) and (b) with an offset of 128; (g) the difference between (a) and (c) with an offset of 128; (h) the difference between (a) and (d) with an offset of 128.

JND$_1$ and JND$_3$, respectively. The 95% CI (confidence interval) was used to identify the upper and lower bounds relative to the mean (and standard deviation of the data) in which 95% of the responses resided in (based on a sparse data set with eleven images per JND level). If the variation is such that most of the responses from a metric (i.e., >50%) do sit outside the 95% CI range, then one may be inclined to conclude that the behavior of that metric is inconsistent, i.e., the metric is ineffective. In Table I, preliminary data were collected for a well-known early perceptual distortion metric based on the DCT (discrete cosine transform) decomposition, using DCTune 2.0 in error calculation mode [30]. Two observations deserve immediate attention. First, for images at the same JND level, the metric produced a range of values. Second, apart from JND$_1$, acceptance rates were lower than 50%, indicating that the metric is ineffective as a measure to predict discernible visual quality levels. Similar results were obtained for a number of other perceptual distortion or quality measures [31-36]. In other words, modeling JND$_n$ is still a challenge when $n>1$.

Fig.2 shows a representative test image, Bikes, with its coded versions at JND$_1$, JND$_2$ and JND$_3$ levels, respectively. The difference images with an offset of 128 between the reference and coded images at the first three JND levels are also shown in Fig.2 to appreciate where noticeable distortions between each distortion levels are observed on a reference monitor (e.g., a Sony BVM-L231 23" trimaster LCD reference monitor was used in this case).

IV. PERCEPTUAL CODING AND STANDARD FRAMEWORK

Perceptual image and video coders have been reported extensively [1] with a recent survey listing sixteen representative PVCs (perceptual video coders) for single-view video coding, ten of which conform to H.264/AVC international standard [37]; the reader may refer to [1] and [37] for an overview. In this section, issues will be firstly highlighted that have addressed or enabled perceptual coding in standard frameworks. An analysis of how perceptual coding approaches are implemented and/or embedded into existing picture coding framework or standards may point to constructive ways forward to development of perception-based new picture coding framework, algorithms and/or standards of high quality of experience for visual communication, broadcasting, entertainment and consumer electronic products, systems and services or applications.

A. Embedding Perceptual Coding Techniques

All existing picture coding standards have already embedded basic, well-known perceptual considerations, e.g., sparse sampling of chrominance components, frequency-dependent quantization matrices, and zigzag scan of transform matrixes. The 95% CI was used to identify the upper and lower bounds relative to the mean (and standard deviation of the data) in which 95% of the responses resided in (based on a sparse data set with eleven images per JND level). If the variation is such that most of the responses from a metric (i.e., >50%) do sit outside the 95% CI range, then one may be inclined to conclude that the behavior of that metric is inconsistent, i.e., the metric is ineffective. In Table I, preliminary data were collected for a well-known early perceptual distortion metric based on the DCT (discrete cosine transform) decomposition, using DCTune 2.0 in error calculation mode [30]. Two observations deserve immediate attention. First, for images at the same JND level, the metric produced a range of values. Second, apart from JND$_1$, acceptance rates were lower than 50%, indicating that the metric is ineffective as a measure to predict discernible visual quality levels. Similar results were obtained for a number of other perceptual distortion or quality measures [31-36]. In other words, modeling JND$_n$ is still a challenge when $n>1$.

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Table I. Perceptual Distortion Metric Outputs When Evaluating Images at First Three JND Levels

<table>
<thead>
<tr>
<th>Images</th>
<th>Metric Values</th>
<th>DC Tune Metric [30]</th>
<th>CI Acceptance a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>JND$_1$</td>
<td>JND$_2$</td>
<td>JND$_3$</td>
</tr>
<tr>
<td>Bikes</td>
<td>6.795</td>
<td>7.208</td>
<td>10.056</td>
</tr>
<tr>
<td>Buildings</td>
<td>3.286</td>
<td>4.901</td>
<td>6.338</td>
</tr>
<tr>
<td>Frontbuilding</td>
<td>2.691</td>
<td>4.968</td>
<td>5.661</td>
</tr>
<tr>
<td>Girl</td>
<td>2.505</td>
<td>3.728</td>
<td>4.569</td>
</tr>
<tr>
<td>House</td>
<td>2.737</td>
<td>4.016</td>
<td>5.302</td>
</tr>
<tr>
<td>Lighthouse</td>
<td>1.706</td>
<td>2.510</td>
<td>3.180</td>
</tr>
<tr>
<td>Paintedhouse</td>
<td>2.970</td>
<td>5.436</td>
<td>5.990</td>
</tr>
<tr>
<td>Sailing1</td>
<td>2.871</td>
<td>5.257</td>
<td>5.748</td>
</tr>
<tr>
<td>Stream</td>
<td>5.552</td>
<td>6.844</td>
<td>10.169</td>
</tr>
<tr>
<td>Rapids</td>
<td>4.155</td>
<td>6.802</td>
<td>8.452</td>
</tr>
<tr>
<td>Womanhat</td>
<td>1.972</td>
<td>3.249</td>
<td>4.062</td>
</tr>
<tr>
<td>Mean (µ)</td>
<td>3.384</td>
<td>4.993</td>
<td>6.339</td>
</tr>
<tr>
<td>s.d. (σ)</td>
<td>1.545</td>
<td>3.893</td>
<td>2.299</td>
</tr>
</tbody>
</table>

Acceptance % 63.64% 45.45% 45.45%

a. Acceptance (1)/Rejection (0)
coefficients. In fact, perceptual modeling and processing can play crucial roles far beyond standards, such as pre-processing, an encoder and post-processing since a standard only specifies the decoder. Successful approaches to perceptual picture coding are classified in [1] as perceptual predictive coding, perceptual quantization, R₂DO and perception-based pre-, loop- and post-filtering.

1) **Perceptual predictive coding:** It has been implemented either as pre-filtering or loop-filtering.

   a) **Pre-filtering:** It is implemented outside of prescribed coding systems or standards and performs visual filtering to deliver perceptually lossless coding [6] or bandlimits visual signals to an acceptable picture quality, removing picture acquisition noise, before standard compression applied for a given transmission or storage media. A JND model can facilitate pre-processing to improve video compressibility by reduce signal variance without perceptual loss [38].

   b) **Loop-filtering:** JND thresholding or throttling of MC (motion compensated prediction) residual images where residuals below JND are skipped saving bits and computation complexity, while those above JND go through the normal residual encoding process, without affecting the decoding process [39,40].

2) **Perceptual quantization:** Different techniques have been reported over the years.

   a) **Visual weight before quantization:** For transform based coding, coefficients are weighted to be visually uniform before standard quantization process is applied [41].

   b) **JND profile based quantization:** It chooses the smallest quantization step size to be less than twice the JND threshold for a given coefficient assuming a transform or subband based coding strategy for perceptually lossless coding and larger step sizes by scaling for higher compression gains. It requires significant side information or an estimation technique at decoder to predict JND profile used by encoder [42,43].

   c) **Global quantization matrix:** It uses the JND map derive a single global quantization matrix to be used for all image blocks.

   d) **Perceptual quantization with visual attention:** The use of foveation and visual-attention models allow perceptually significant visual information to be coded with more bits [44,45].

3) **R₂DO:** Perceptual rate control and R₂DO are also possible within the framework of current standards [46–48].

4) **Post-processing:** There have been initial attempts [1,49] to incorporate perceptual factors, and there is a call for more investigation due to its importance to the end-user experience.

B. **Waveform Coding to Perception Based Picture Coding**

Without exhausting all issues regarding if and what needs to be done, if at all, a few immediate and obvious questions are highlighted in an effort to advance pure waveform based coding to perception based coding for visual signal compression and transmission or storage services and applications.

Mathematical framework currently adopted by existing picture coding standards is based on block based complete transforms, e.g., the DCT, integer DCT or discrete wavelet transforms which has no or little control over aliasing when quantization is applied for compression or refined directional information. Use of small block sizes may provide better adaptivity to encoding of varying spatial details, but make it difficult for any meaningful visual masking model to be useful due to diminishing visual angle at normal viewing distance. Temporal fluctuation artifacts [50] and temporal pumping artifacts [51] associated with implementations of contemporary video coding standards require effective temporal models and their applications to adaptive or perceptual quantization strategies and/or R₂DO over, e.g., a GoP (group of pictures). The issue regarding HVS models for encoding of inter-frame and/or inter-view prediction error (or residual) image coding requires further investigation [1]. As discussed in Section III.B, design and assessment of perceptual distortion or quality measures which correspond to discernible distortion or quality levels by the HVS are required for visual quality-driven picture coding using the R₂DO design principle.

V. **Concluding Remarks**

This paper examines issues associated with constant visual quality picture coding from an R₂DO perspective with provision of user-centric quality assured visual communication, broadcasting and entertainment services and applications in sight. Interpretation of effectiveness of picture coding is underscored for visual quality-driven coder design, and importance of perception-based distortion or quality measure is re-examined for R₂DO based quality-driven coder design. In general, there is a lack of systematic exploration in constant quality coding, as well as considerations for different JND levels and utility factors. In addition, issues associated with embedding perceptual coding techniques into existing picture coding standards are discussed which may assist with development of perceptual picture coding framework for future standards.

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