Hysteresis identification and adaptive vibration control for a smart cantilever beam by a piezoelectric actuator

Ting Zhang, Hong Guang Li, Guo Ping Cai

A R T I C L E   I N F O

Article history:
Received 1 March 2013
Received in revised form 28 August 2013
Accepted 28 August 2013
Available online xxx

Keywords:
Adaptive vibration control
Hysteresis
Smart beam
Piezoelectric actuator
Identification

A B S T R A C T

This paper presents some experiments not only for depiction of hysteresis property but also for vibration suppression by designing an adaptive controller to a smart system consisted of a cantilever beam bonded with a piezoelectric actuator. The adaptive controller is composed of an identifier and the minimum variance direct control. With the identifier by employing the recursive extended least square method, an online controlled autoregressive moving average (CARMA) model for the smart beam is proposed to characterize the hysteresis phenomena between the output strain near the fixed end of the cantilever beam and the input voltage applied on the piezoelectric actuator. Based on the CARMA model, an adaptive controller is designed to control the vibration of the nonlinear smart system by using the minimum variance self-tuning direct regulator (MVSTDR). The MVSTDR has a significant advantage of trying to obtain an optimal control results for vibration suppression over the control process. The experiment results demonstrate that it is feasible to suppress vibration by the MVSTDR for the smart beam with the hysteresis property and show that the amplitude reduction quantity of the strain in the frequency spectrum analysis is up to about 83.67% with the adaptive controller at the first natural frequency when the smart beam is subjected to a free vibration.

© 2013 Elsevier B.V. All rights reserved.

1. Introduction

Nonlinear analyses of a smart system, which is consisted of a cantilever beam bonded with a piezoelectric actuator, have drawn much interest of many researchers in recent years [1]. At the same time, the nonlinear control designs on the positioning control and vibration suppression of the smart system have attracted wide attention [2]. The nonlinearity, especially for the hysteresis in the piezoelectric materials [3], may bring about many destructive effects on the precision positioning and vibration control [4]. Therefore, in order to meet the grow requirements of high accuracy positioning systems with its space application [5], the issue on dynamical nonlinear model and active vibration control of a smart beam is a rather challenging task with a controller.

Although many researchers commit themselves to working on the vibration attenuation by dynamically modeling and designing a controller for a smart beam with a sensor and an actuator [6], the analysis studies of hysteresis models with nonlinear controllers are focused on by not many people. For example, an ellipse-based mathematic model characterizing the rate-dependent hysteresis in piezoelectric actuators is developed for the high-accuracy and high-speed tracking control by a hybrid control strategy [7], a finite element formulation for the vibration of piezoceramic laminated plates that takes hysteretic behavior into account is presented using an Ishlinskii model [8], a new methodology with Preisach operators is proposed for the high-precision and robust control by a new integral continuous sliding mode control [9], a classical Preisach model is used to characterize the hysteresis in a piezostack actuator used to move a trailing-edge flap for helicopter vibration control [10], a phenomenological model to characterize the hysteretic behavior of the pre-stressed piezoelectric ceramic stack actuators is put forward by using the Bouc–Wen hysteresis operator to model the hysteretic extension [11], a nonlinear energy-based hysteresis model is developed for a piezoelectric stack actuator [12] and a neural network hysteresis modeling method is proposed for an improved Preisach model of piezoelectric actuators [13]. However, exploring a simplified model describing the hysteretic for the smart system to suppress vibration is still the vital and urgent work.

As for the adopted control law of suppressing vibration, in the past several years, most of professors have worked hard to study with the classical control, the modern control, the adaptive control, the intelligent control, the robust control and the hybrid control of the above several controls. For instance, the classical control is adopted with a proportion–integral–derivative theory with output feedback to control the vibrations of any real life system [14]. In addition, the modern controls are proposed by a modal velocity
feedback control method [15], a modified acceleration feedback law for active vibration control of aerospace structures [16] and a linear quadratic regulator (LQR) controller for achieving vibration suppression of the laminated smart beam [17]. Moreover, the adaptive controls are presented with an adaptive semi-active synchronized switch damping on voltage (SSDV) method [18] and an improved version of SSDV approach [19] to the vibration control of a composite beam, a nonlinear golden section adaptive control (CMNGSAC) algorithm to suppress the vibration of a flexible Cartesian smart material manipulator [2], an adaptive filtered-X least mean square algorithm for non-axial-symmetric vibrations control [20] and an adaptive modified positive position feedback for active vibration control of structure [21]. Furthermore, an adaptive intelligent Neuro-fuzzy control for achieving a high performance piezoelectric vibration absorber [22], the robust control adopted with a sliding mode control (SMC) for suppressing the first two bending modes vibration of beam [23] and the SMC for controlling the attitude motion of a spacecraft [24] are developed. Finally, the hybrid controls are as follow: a robust adaptive sliding mode attitude control for rotation maneuver and vibration suppression of a flexible spacecraft [25], a classical displacement–velocity feedback control combined with robust H2 control for large amplitude vibration control of functionally graded material plates [1], a PD-based hub motion control with a composite of linear and angular velocity feedback controllers [26] and so on. From the above literature review, compared with other control law, it is summarized that the adaptive controls have significant advantages of suppressing vibrations online in the varying environment or complex situations.

Therefore, in this paper, a smart beam with hysteresis property is presented to study vibration suppression by an adaptive controller. In the adaptive control design, an online time-varying model is proposed combining the CARMA model and the recursive extended least square method [27] to describe hysteresis property of the smart cantilevered beam. Meanwhile, with the CARMA model, the adaptive controller is adopted with a minimum variance self-tuning direct regulator (MVSTDTR) [28,29] to suppress nonlinear vibration. Furthermore, the adaptive controller is always trying to pursue an optimal control effect and can make the free vibration of the smart beam attenuating quickly in real time.

The rest of this paper is organized as follow. In Section 2, the hysteresis property of the smart beam consisted of a cantilevered beam, a piezoelectric actuator and a foil gauge is introduced. In Section 3, the minimum variance self-tuning direct regulator for vibration suppression of the smart beam is designed. In Section 4, the experimental verifications of hysteresis identification and vibration suppression by the adaptive regulator are given. Finally some conclusions are given in Section 5.

### 2. Smart beam

A cantilevered beam bonded with a piezoelectric actuator and a foil gauge is shown in Fig. 1. For the purpose of suppressing vibration to the smart beam, a control voltage generated by a designed adaptive controller is applied on the piezoelectric actuator to suppress the disturbance or vibration when the beam is subjected to the external disturbance or vibration in the thickness direction of beam from external environment. Meanwhile the foil gauge near the fixed end of the beam is used for acquiring the strain as feedback transmitted to the controller. Furthermore the geometric parameters of the smart beam are shown in Table 1, which are consisted of the geometric parameters of beam, piezoelectric actuator and others. \( h_b \) and \( h_p \) are thickness and \( \rho_b \) and \( \rho_p \) are density of beam and actuator, respectively. \( E_b \) and \( c_b \) are Young modulus and damping parameter of beam.

However, the strain collected with foil gauge is not linearly related with the actuating voltage applied on the piezoelectric actuator. When a sinusoidal signal voltage at amplitude 150 V and frequency 1.556 Hz is applied on the actuator, the output strain lags behind the actuating voltage in the time domain. This phenomenon is called as hysteresis property. Fig. 2 gives the curve of hysteresis property between the actuating voltage and the output strain. Furthermore, for the nonlinearity, the size and direction of hysteresis loop are related with the frequency and amplitude of actuating voltage. In addition, the nonlinear property may have negative effect on the high-accuracy position control of some structures. In other words, a conventional offline model for the nonlinear smart beam is not fit for suppression control of high precision space structure in changeable environment. Therefore an adaptive controller is introduced to suppress vibration for the nonlinear smart beam with an online model in the next section.

### Table 1

<table>
<thead>
<tr>
<th>Beam parameters</th>
<th>( l_b = 0.903 \text{ m} )</th>
<th>( w_b = 0.035 \text{ m} )</th>
<th>( h_b = 0.0015 \text{ m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_b = 2.7 \times 10^3 \text{ kg/m}^3 )</td>
<td>( E_b = 70 \text{ GPa} )</td>
<td>( c_b = 0.005 )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Piezoelectric parameters and others</th>
<th>( l_p = 0.057 \text{ m} )</th>
<th>( w_p = 0.02 \text{ m} )</th>
<th>( h_p = 0.0005 \text{ m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_p = 7.6 \times 10^3 \text{ kg/m}^3 )</td>
<td>( x_p = 0.086 \text{ m} )</td>
<td>( x_p = 0.02 \text{ m} )</td>
<td></td>
</tr>
</tbody>
</table>

### Fig. 1

A cantilevered beam bonded with a piezoelectric actuator.

### Fig. 2

Hysteresis property.
3. Design of minimum variance self-tuning controller

In this study, we adopt the minimum variance self-tuning direct regulator (MVSTDRE) to suppress vibration [28,29], as shown in Fig. 3. The identifier of the adaptive controller is proposed with a recursive extended least square method (RELSM) to describe the hysteresis property. With the identifier, an online varying-time model is constructed, and the process parameters of the varying-time model are transmitted to the control designer. Finally, when the designer is always running incessantly, the control voltage is calculated by minimum variance direct control law to actuating the piezoelectric actuator for attenuating the vibration amplitude of the beam in shorter time.

A robust identification method depends on its rapidity and stability, and a good identification method can decrease oscillate at the beginning of identification process and can make smart beam actuating accurately for suppressing vibration. In the paper, the RELSM [30] is merged in the MVSTDRE. In addition, the RELSM is simple and is easy to realize in C language.

First with the RELSM, the identification mathematical formula of the smart beam is adopted with CARMA model in Eq. (1):

\[
A(z^{-1})y(k) = B(z^{-1})u(k - d) + C(z^{-1})\xi(k),
\]

where \(u(k)\) and \(y(k)\) are the input and output of the system, respectively, \(d\) is pure time delay, \(\xi(k)\) is the random disturbance, \(k\) is sample point and

\[
\begin{align*}
A(z^{-1}) &= 1 + a_1z^{-1} + a_2z^{-2} + \cdots + a_{n_a}z^{-n_a} \\
B(z^{-1}) &= b_0 + b_1z^{-1} + b_2z^{-2} + \cdots + b_{n_b}z^{-n_b} \quad (b_0 \neq 0) \\
C(z^{-1}) &= 1 + c_1z^{-1} + c_2z^{-2} + \cdots + c_{n_c}z^{-n_c}
\end{align*}
\]

Then the CARMA model is written as least square form in Eq. (2):

\[
y(k) = \psi^T(k)\vartheta + \xi(k),
\]

where

\[
\psi(k) = [-y(k - 1), \ldots, -y(k - n_a), u(k - d), \ldots, u(k - d - n_b), \xi(k - 1), \ldots, \xi(k - n_c)]^T \in \mathbb{R}^{(n_a + n_b + n_c) \times 1}
\]

Assuming the estimated parameters vector of \(\vartheta\) as \(\hat{\vartheta}\), the estimated output \(\hat{y}\) at sample point \(k\) is expressed as follows

\[
\hat{y}(k) = \psi^T(k)\hat{\vartheta},
\]

where \(\hat{\vartheta} = [\hat{a}_1, \ldots, \hat{a}_{n_a}, \hat{b}_0, \ldots, \hat{b}_{n_b}, \hat{c}_1, \ldots, \hat{c}_{n_c}]^T\).

Then the error between the actual output and estimated output is \(\varepsilon(k)\) and can be written as

\[
\varepsilon(k) = y(k) - \hat{y}(k) = y(k) - \psi^T(k)\hat{\vartheta}.
\]

In \(N\)th observation, we get the derivative (equal to zero) with respect to the sum \(\delta\) of error \(\varepsilon\) in Eq. (4), and it can be expressed as

\[
\frac{\partial S}{\partial \vartheta} = \frac{1}{\sum_{k=0}^{N} \varepsilon(k)} = \frac{1}{\psi^T k \psi + \Phi^T \Phi \hat{\vartheta}}
\]

where

\[
\begin{align*}
\Phi &= \psi \hat{\vartheta}, \\
\varepsilon(k) &= y(k) - \psi^T(k)\hat{\vartheta}, \\
\hat{\vartheta} &= \Phi^T Y_k
\end{align*}
\]

Assuming the \(P\) written as

\[
P(k) = (\Phi_k^T \Phi_k)^{-1} \Phi_k^T Y_k,
\]

where \(\Phi_k = (\Phi_{k-1}, k, \psi^T(k)) \in \mathbb{R}^{k \times (n_a + n_b + n_c) + 1}, Y_k = (y_{k-1}, y(k)) \in \mathbb{R}^{k \times 1}.
\]

Assuming the \(P\) written as

\[
P(k) = (\Phi_{k-1}^T \Phi_{k-1})^{-1} \Phi_{k-1}^T Y_{k-1} = [P^{-1}(k) + \psi(k)\psi^T(k)]^{-1},
\]

then the inverse matrix of \(P\) is derived as

\[
P^{-1}(k) = P^{-1}(k - 1) + \psi(k)\psi^T(k).
\]

Due to Eq. (6), the estimated vector \(\hat{\vartheta}(k - 1)\) is expressed as

\[
\hat{\vartheta}(k - 1) = (\Phi_{k-1}^T \Phi_{k-1})^{-1} \Phi_{k-1}^T Y_{k-1} = [P^{-1}(k - 1) + \psi(k)\psi^T(k)]^{-1} 
\]

Owing to Eqs. (8) and (9), then \(\Phi_{k-1}^T Y_{k-1}\) is obtained as

\[
\Phi_{k-1}^T Y_{k-1} = P^{-1}(k - 1)\hat{\vartheta}(k - 1) = [P^{-1}(k - 1) - \psi(k)\psi^T(k)]\hat{\vartheta}(k - 1).
\]

From Eqs. (6) and (7), the estimated parameters vector \(\hat{\vartheta}(k)\) is written as

\[
\hat{\vartheta}(k) = P(k)\Phi_k^T Y_k = P(k)\Phi_{k-1}^T Y_{k-1} + \psi(k)\psi^T(k).
\]

Substituting Eq. (11) into Eq. (11), then \(\hat{\vartheta}(k)\) is given as

\[
\hat{\vartheta}(k) = \hat{\vartheta}(k - 1) + K(k)[y(k) - \psi^T(k)\hat{\vartheta}(k - 1)],
\]

where

\[
K(k) = P(k)\psi(k).
\]
From the Lemma 1 in the Appendix and Eq. (7), the $P(k)$ is expressed as

$$P(k) = P(k - 1) - P(k - 1) \psi(k) \times [1 + \psi^T(k)P(k - 1)\psi(k)]^{-1} \psi^T(k)P(k - 1).$$

(14)

Substituting Eq. (14) into Eq. (13), $K(k)$ is derived as

$$K(k) = P(k - 1)\psi(k) - \frac{P(k - 1)\psi(k)\psi^T(k)P(k - 1)\psi(k)}{1 + \psi^T(k)P(k - 1)\psi(k)}.$$

(15)

From Eqs. (14) and (15), then $P(k)$ is obtained as

$$P(k) = [1 - K(k)\psi^T(k)]P(k - 1).$$

(16)

Because the $\xi(k)$ in $P(k)$ is not observable, the estimated value $\hat{\xi}(k)$ replaces the $\xi(k)$ in Eq. (17).

$$\hat{\xi}(k) = \xi(k).$$

(17)

Therefore the estimated value $\hat{\psi}(k)$ takes the place of $\psi(k)$ as follows:

$$\hat{\psi}(k) = [-y(k - 1), \ldots, -y(k - d - n_q), u(k - d), \ldots, u(k - d - n_b),$$

$$\hat{\xi}(k - 1), \ldots, \hat{\xi}(k - n_c)]^T \in \mathbb{R}^{n_q + n_b + n_c + 1}.$$

Finally, because of Eqs. (12), (15) and (16), the recursive extended least square identification law is obtained as

$$\hat{\theta}(k) = \hat{\theta}(k - 1) + K(k)[y(k) - \hat{\psi}^T(k)\hat{\theta}(k - 1)]$$

$$K(k) = \frac{P(k - 1)\hat{\psi}(k)}{1 + \hat{\psi}^T(k)P(k - 1)\hat{\psi}(k)}.$$

(18)

Based on the recursive extended least square identification method, the minimum variance self-tuning direct regulator is designed in this section. First, the CARMA in Eq. (1) is employed for the system model, and the predicted output in sample point $k + d$ is assumed as $\hat{y}(k + d|k)$ with the regulator. In addition, the predicted error is given as

$$\hat{y}(k + d|k) = y(k + d) - \hat{y}(k + d|k).$$

(19)

Then, minimizing the variance $(\hat{y}^2(k + d|k))$ and assuming $y^*(k + d|k)$ as the optimal output of system in sample point $k + d$ as shown in Fig. 4, the control strategy $u$ is gained in Eq. (20):

$$C(z^{-1})y^*(k + d|k) = G(z^{-1})y(k) + F(z^{-1})u(k).$$

(20)

Therefore, in the paper, the regulator is designed for the purpose of vibration reduction. Therefore $y^*$ is set to zero signal. So the control voltage is obtained from Eq. (25) as

$$u(k) = \frac{1}{f_0} \left[ - \sum_{i=1}^{n_q} f_i u(k - i) - y^*(k + d) + \sum_{i=1}^{n_c} c_i y^*(k + d - i) - \sum_{i=0}^{n_q} \hat{g}_i y(k - i) \right].$$

(25)

4. Experimental verifications

Some experiments must be performed if the theoretical design wants to be accepted by many people. From the experimental setup
photograph in Fig. 5, an aluminum cantilever beam bonded with a piezoelectric actuator is presented. Moreover, near the fixed end, a bonded foil gage is used to acquire the strain of smart beam. Then the acquired strain through strain amplifier is transmitted to a DSP demo board based on TMS320F2812. Next, the DSP processor mainly is used to process strain data and generate the actuating voltage signal for the power amplifier to actuate the cantilevered beam. The computer loads the control program (C language) to the DSP processor and saves the collected experiment data. The iron weight (about 20 kg) on the ground is used for calibrating the initial constant displacement (about 10 cm) on the tip of beam in the thickness direction at the beginning of the vibration suppression experiments. The experimental results conclude two parts: identification results for the hysteresis property and vibration suppression results for the smart beam.

4.1. Experimental identifications for hysteresis property

First, some experimental results are presented to testify the identifying capacity of the estimator for hysteresis property of the smart beam with the online identification method. When the control block in Fig. 3 is an open loop and the actuating voltage is directly applied with a sinusoidal signal \((150 \times \sin(2\pi \times 1.556t))\) in Fig. 6(a), the actual collected output strain by the foil gauge and identifying strain in time domain are solid line and dotted curve in Fig. 6(b), respectively. Moreover, Fig. 7 presents the relations between the actuating voltage in Fig. 6(a) and the two strain curves in Fig. 6(b) in a period when the output strain response is steady in resonant frequency. For the hysteresis nonlinearity, the process parameters \((f_0, f_1, g_0, g_1, c_1)\) of the online model are varying-time in the whole experiments in Fig. 8 when the smart beam is set as a second-order system. Finally, from Fig. 6(b), Figs. 7 and 8, the identifying data can be fit well with the actual collect data, and the hysteresis identification results indicate that the hysteresis phenomenon can be identified by the online CARMA with the varying-time parameters.

In the experiments, the different amplitudes and different frequencies of applied actuating voltage have significant influence on the hysteresis characteristic. As for the hysteresis property at different amplitudes and different frequencies of actuating voltage, some experimental results are presented to verify the effectiveness of describing hysteresis phenomena in the online CARMA model. Here, we focus on the hysteresis property between the input actuating voltage applied on the piezoelectric actuator and the output strain of the smart beam.

However the amplitude value of applied voltage cannot exceed the maximum limited value (150 V) of the power supply equipment. So when we study the influences on the hysteresis phenomenon caused by different amplitudes of voltage, the amplitudes of input voltage are set to \(15 \times n\) \((n = 10, 9, 8 \text{ and } 7)\). At the same time, when we work over the influences on the hysteresis property by the different frequencies of input voltage, the frequencies are set between the right and left sides of primary natural frequency (1.556 Hz) because the collected strain is not considerable and the hysteresis effect is not obvious below frequency 1 Hz.

The identifications results of hysteresis property in different amplitudes and different frequencies of the actuating voltage for the smart beam are shown in Figs. 9 and 10, respectively. The
actual data are collected with the foil gauge while the identifying data is calculated from the online identification law. Fig. 9 gives the hysteresis effect when the voltage at different amplitudes is applied on the piezoelectric actuator. Obviously, the hysteresis losses are 2.0628 V mm/m, 1.8218 V mm/m, 1.5364 V mm/m and 1.3356 V mm/m at \( n = 10, 9, 8 \) and 7, respectively. At the same time, Fig. 9 presents the hysteresis curves at frequency=1.5 Hz, 1.55 Hz, 1.6 Hz and 1.7 Hz, their hysteresis losses are 1.0195 V mm/m, 2.9173 V mm/m, 1.3148 V mm/m and 0.5755 V mm/m, respectively.

In short, in Figs. 9 and 10, compared with the actual data collected by the foil gauge, the identification method (RELSM) can characterize well the hysteresis property. Furthermore, the size of hysteresis loop and the hysteresis loss are affected by not only the amplitude but also the frequency of input voltage applied on the piezoelectric actuator. More importantly, the CAMAR model can accurately track the actual data with its time-varying parameters. Therefore, the online time-varying CARMA model can be used to describe the hysteresis characteristic.

4.2. Experimental verification of adaptive vibration suppression

Experimental results of vibration suppression are attained when control block is a closed loop. The control voltage is calculated through the minimum variance self-tuning direct control law with the feedback strain. To illustrate the adaptive control effectiveness,
Fig. 11 presents the experiment results of strain suppression without control and with adaptive control. In contrast with the output strain response without control, the amplitude attenuation of strain with control is damping to a small value after the time is about 10 s. Fig. 12 shows the control voltage from adaptive controller. The maximum amplitude of control voltage is more than 150 V before the time is about 2.5 s. However the value of more than 150 V is outputted with 150 V by the power apply equipment because of its maximum limited voltage 150 V.

Fig. 13 presents the spectral analysis of strain without control and with control of Fig. 11. From the frequency response of strain, the peaks at the primary nature frequency without control and with control are 0.001696 mm/m at frequency 1.563 Hz and 0.000276 mm/m at frequency 1.526 Hz, respectively. The results denote that the amplitude suppression quantity of strain with adaptive control is about 83.67%. Moreover, the frequency 1.526 Hz of peak with control is less than the frequency 1.563 Hz of peak without control. It is proved that the damping of smart beam is becoming larger with the adaptive regulator. Therefore, as is illustrated that it is feasible to suppress the free vibration by the MVSTDR for the smart beam with hysteresis property through the practical experiments verifications.

Fig. 14 shows the process parameters \( f, g, c \) of vibration suppression with the MVSTDR for the smart beam in the experiments. The process parameters \( f_0, f_1, g_0, g_1 \) and \( c_1 \) converge to the values 0.1591, 0.1023, 0.1784, –0.0288 and 0.0215, respectively. Fig. 15 shows the hysteresis identification relations between control voltage in Fig. 12 and strain in Fig. 11. Furthermore the red dotted line is identification data and the solid line is collected curve with foil gauge. It is clear that the hysteresis loop of the strain is becoming increasingly smaller when the free vibration is suppressed with the MVSTDR.

In conclusion, by the practical experimental verifications for the smart beam, not only the hysteresis property can be identified well with the recursive extended least square method, but also the amplitude reductions of strain in spectral analysis are 83.67% with the minimum variance self-tuning direct regulator. Above all, the validity of vibration suppression with the adaptive regulator is testified effectively.

5. Conclusions

In this paper, a smart beam with hysteresis property is presented for vibration suppression with the minimum variance self-tuning direct regulator. An online time-varying model is constructed to characterize the hysteresis property by a CAMAR model based on the recursive extended least square method. Furthermore the CAMAR model is used to represent well the hysteresis effect in the smart beam by practical experiments. Compared with the other control law, the MVSTDR is an online law with an identifier and has an advantage of pursuing the optimum control effect. In addition, the suppression amounts of strain for the smart beam are 83.67% with the MVSTDR by experiment verifications. In short, it is testified that the adaptive regulator MVSTDR can effectively attenuate vibration for the smart beam.

Acknowledgement

This work was funded by the National Natural Science Foundation of China, under grant no. 11372176 and no. 10972137.
Appendix.

Lemma 1. Assume $A$, $(A+BC)$ and $(I+CA^{-1}B)$ are nonsingular matrices, then

$$(A+BC)^{-1} = A^{-1} - A^{-1}B(I+CA^{-1}B)^{-1}CA^{-1}.$$  

Proof:

$$(A+BC)A^{-1} - A^{-1}B(I+CA^{-1}B)^{-1}CA^{-1}\nonumber = I + BCA^{-1} - B(I+CA^{-1}B)CA^{-1} - BCA^{-1}B(I+CA^{-1}B)^{-1}CA^{-1}\nonumber = I + BCA^{-1} - B(I+CA^{-1}B)(I+CA^{-1}B)^{-1}CA^{-1}\nonumber = I.$$  

End.

Then setting $A = P^{-1}(k-1)$, $B = \phi(k)$ and $B = \phi^T(k)$, from Eq. (7) in Section 3, Eq. (14) in Section 3.1 is attained by Lemma 1.

References


Biographies

Ting Zhang received her B.S. degree in Mechatronic Engineering from East China Jiao Tong University in 2008 and M.S. degree in Mechatronic Engineering from Northeastern University in 2010, China. She is currently a Ph.D. candidate at the Institute of Vibration, Shock and Noise, School of Mechanical Engineering from Shanghai Jiao Tong University, in China working on dynamically modeling and adaptive vibration control of smart materials applied on the space structures, which are her main interests.

Hong Guang Li received his B.S. in Engineering Mechanics, 1993, and M.S. in Computational Mechanics, 1996 from Dalian University of Technology, China. He received his Ph.D. in School of Mechanical Engineering and Automation from Northeastern University, 1999. He is the Professor at Institute of Vibration, Shock and Noise, School of Mechanical Engineering, Shanghai Jiao Tong University, China. In recent years, the research work focuses on nonlinear vibration theory and its application, mechanical vibration, the finite element analysis in mechanics systems, signal processing and fault diagnosis.

Guo Ping Cai received his B.S. degree in Mining Machinery from Taiyuan Heavy Machinery Institute in 1987 and M. S. degree in Mechanical Manufacturing from Henan University of Science and Technology in 1993, China. He received his Ph.D. in Engineering Mechanics, 2000, from Xi’an Jiao Tong University, China. He is the Professor at Department of Engineering Mechanics, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, China. In recent years, the research work focuses on dynamics and control of flexible structural systems, dynamics and control of time-delay systems, and dynamics and control of structures.