**Vector Perturbation Precoding for MIMO Broadcast Channel with Quantized Channel Feedback**

Peng Lu and Hong-Chuan Yang  
Dept of Elec. & Comp. Engr., University of Victoria,  
Victoria, BC, V8W 3P6, Canada  
E-mail: <penglu, hyang@ece.uvic.ca>

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**Abstract**—In multiple-input multiple-output (MIMO) broadcast channels, the basestation transmits information to multiple noncooperative users simultaneously. With perfect channel state information (CSI) at the basestation, vector perturbation (VP) precoding technique achieves full diversity order. In this paper, we study the symbol error rate (SER) performance of such systems with quantized channel feedback. The channel quantization is modeled by following the rate-distortion theory. Based on minimum mean square error (MMSE) criterion, we study the optimal design of precoding matrix and perturbation vector. Equivalent relations in terms of both MMSE and SER between quantized and perfect channel feedback cases are established, based on which we investigate the relation between feedback load and SER performance.

**Key Words:** MIMO broadcast channels, quantized channel feedback, vector perturbation, diversity order, minimum mean square error, symbol error rate

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**I. INTRODUCTION**

To circumvent the problem of high processing complexity associated with dirty paper coding (DPC) [1], [2], many suboptimal precoding techniques are proposed to mitigate multiuser interference for multiple-input multiple-output (MIMO) broadcast channels. These techniques are categorized into two types: linear and non-linear precoding. The most popular linear precoding techniques include zero forcing (or channel inversion) [3] and minimum mean square error (MMSE) [4]. With lower computational complexity, linear techniques generally suffer from a performance loss comparing to non-linear techniques. Popular non-linear techniques include BLAST [5], vector perturbation (VP) precoding [9], Tomlinson-Harashima (TH) precoding [7], [8] and lattice-basis reduction precoding [10], [11]. Especially, it is proven in [11] that full diversity order equal to the number of transmit antennas is achieved by lattice-basis reduction and VP precoding.

With VP precoding, the transmitter reshapes the data vector with a perturbation vector and generates the transmit vector by multiplying the perturbed data vector with a precoding matrix. By appropriately designing the precoding matrix and perturbation vector, this precoding technique can cancel mutual interference and reduce transmit power. In [9], with zero-forcing and regularized zero-forcing precoding matrix, the authors propose to use the perturbation vector which minimizes the transmit power. In [12]–[14], joint optimal perturbation vector and precoding matrix under MMSE criterion are given.

In this paper, we study the effects of imperfect CSI on the SER performance of VP precoding. We assume that the imperfectness of CSI results from the channel quantization procedure, which is modeled by following the rate-distortion theory. A robust VP precoding scheme is proposed for quantized channel feedback. We establish equivalent relations in terms of MMSE and SER between VP precoding with quantized and perfect channel feedback. Based on the relations, we investigate how feedback load affects the SER performance of VP precoding.

The rest of the paper is organized as follows. Section II consists of system model and introduction to VP precoding. In section III, we describe channel quantization model and VP precoding based on MMSE criterion with quantized channel information. In section IV, we establish equivalent relations between VP precoding with quantized and perfect channel feedback. Some numerical examples are presented in section V. Section VI concludes the paper with some remarks.

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**II. SYSTEM OVERVIEW**

**A. Multiuser MIMO Transmission**

We assume $N_t$ antennas are used at the basestation in a multiuser MIMO system. The basestation transmits information to $N_r$ non-cooperative single-antenna receivers simultaneously, where $N_r \leq N_t$. We denote the channel vector of receiver $i$ as $\mathbf{h}_i = [h_{i1}, \ldots, h_{iN_t}]^T \in \mathbb{C}^{N_t}$, where $h_{ij}$ indicates the channel gain between the $j$th transmit antenna and receiver $i$. The channel gain is assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unitary variance.

Given a transmit vector $\mathbf{x} = [x_1, \ldots, x_{N_t}]^T \in \mathbb{C}^{N_t}$ over a time slot, the discrete-time complex baseband symbol received by user $i$ is given by

$$y_i = \mathbf{h}_i^T \mathbf{x} + n_i, \quad (1)$$

where $n_i$ is the zero-mean complex Gaussian noise with variance $\sigma_n^2$. With unit transmit power, the SNR is defined as

$$\rho_n = \frac{1}{\sigma_n^2}. \quad (2)$$

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We denote the concatenation of the channel vectors by \( H = [h_1, \ldots, h_{N_t}]^T \), i.e., \( H \in \mathbb{C}^{N_t \times N_r} \), with the \( i \)-th row being receiver \( i \)'s channel vector. The \( N_r \)-dimensional received vector \( y \) is formed by stacking the received signals of all \( N_r \) receivers, i.e., \( y = [y_1, \ldots, y_{N_r}]^T \). Following from (1), the received vector can be expressed as
\[
y = Hx + n, \tag{3}
\]
where \( n = [n_1, \ldots, n_{N_r}]^T \in \mathbb{C}^{N_r} \) is the additive noise vector. Though (3) shares the same form as a point-to-point MIMO system, each receiver only has the knowledge of its own channel vector.

### B. Vector Perturbation Precoding

The **data vector** \( u = [u_1, \ldots, u_{N_r}]^T \) consists of data symbols to be transmitted to the \( N_r \) receivers. The data symbol \( u_i \) is chosen from an \( M \)-ary quadrature amplitude modulation (QAM) constellation \( \mathcal{A} \), where the coordinates of the signal points are odd numbers, i.e., \( \mathcal{A} = \{a_1 + a_Q[a_1, a_Q \in \pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)] \} \). The constellation is bounded by the square region of width \( \sqrt{2} \).

To reduce the power of transmit vector, a **perturbation vector** \( \mathbf{u}' \in \mathbb{C}^{N_r} \) is added to the data vector, resulting in a **perturbed data vector** as \( u + u' \) [9]. Both real and imaginary parts of each element of \( u' \) are integer times of a constant value \( \tau \), which is set as \( \tau = 2\sqrt{2} \) in this paper.

The **transmit vector** \( x \) is formed by multiplying perturbed data vector \( u + u' \) with an \( N_t \times N_r \) precoding matrix \( P \) and then being normalized to unit transmit power as
\[
x = \frac{1}{\sqrt{\|P(u+u')\|^2}} P(u+u'), \tag{4}
\]
where \( P = \|P(u+u')\|^2 \). Here, \( \| \cdot \| \) stands for Euclidean norm. Substituting (4) into (3), the received vector can be written as
\[
y = \frac{1}{\sqrt{\|P\|^2}} HP(u+u') + n. \tag{5}
\]

Assuming that receivers know the scalar \( \sqrt{\|P\|^2} \), receiver \( i \) estimates its data symbol \( u_i \) by performing a modulo operation on the scaled received symbol \( \sqrt{\|P\|} y_i \). This operation translates \( \sqrt{\|P\|} y_i \) into the range of original constellation \( \mathcal{A} \) by adding an integer times of \( \tau \) to its real and imaginary parts separately, which is expressed as
\[
\hat{u}_i = \sqrt{\|P\|} y_i \left( \frac{\Re(\sqrt{\|P\|} y_i)}{\tau} + \frac{1}{2} \right) j + \frac{\Im(\sqrt{\|P\|} y_i)}{\tau} + \frac{1}{2} \left( \Re(\sqrt{\|P\|} y_i) \right), \tag{6}
\]
where \( \lfloor \cdot \rfloor \) means rounding to the nearest smaller integer. The transmitted symbol is then detected as the constellation point in \( \mathcal{A} \) with nearest Euclidean distance.

### C. Optimal Design with MMSE

From (5), we define a **deviation vector**, which measures the distortion of the scaled received vector \( \sqrt{\|P\|} y \) from perturbed data vector \( u + u' \), as
\[
d = \sqrt{\|P\|} y - (u + u') = (HP - I_{N_r})(u + u') + \sqrt{\|P\|} n, \tag{7}
\]
where \( I_{N_r} \) is a identity matrix of size \( N_r \times N_r \). Given data vector \( u \) and channel realization \( H \), the MSE as a function of \( u' \) and \( P \) is expressed as
\[
e(u', P) = \mathbb{E}_n(||d||^2 | H, u) \tag{8}
\]
\[
e = (\|HP - I_{N_r}\| (u + u')|^2 + N_r \sigma_n^2),
\]
where \( \mathbb{E}_n(\cdot) \) means taking the expectation over \( n \). The optimal precoding matrix that minimizes (8) is found to be [13]
\[
P_{o, mmse} = H^H (HH^H + N_r \sigma_n^2 I_{N_r}^{-1}). \tag{9}
\]
The optimal perturbation vector can be obtained as
\[
u_{o, mmse} = \arg \min_{\mathbf{u}'} N_r \sigma_n^2 (u + u')^H, \tag{10}
\]
\[
(HH^H + N_r \sigma_n^2 I_{N_r}^{-1})(u + u'),
\]
and the achieved MMSE with \( u' = u_{o, mmse} \) is equal to
\[
e_{o, mmse} = N_r \sigma_n^2 (u + u_{o, mmse})^H, \tag{11}
\]
\[
(HH^H + N_r \sigma_n^2 I_{N_r}^{-1})(u + u_{o, mmse}).
\]

### III. Vector Perturbation With Quantized Channel Feedback

#### A. Channel Quantization Model

In practical systems, receiver \( i \) can only feed back limited amount of information on its channel vector to the basestation, based on which the basestation tries to reconstruct \( \hat{h}_i \). The reconstructed channel vector at the basestation is called **quantized channel vector** and is denoted as \( \hat{h}_i = [\hat{h}_{i1}, \ldots, \hat{h}_{iN_t}]^T \in \mathbb{C}^{N_t} \). The average squared-error distortion between \( h_i \) and \( \hat{h}_i \) is defined as
\[
D = \mathbb{E}_{\hat{h}_i, h_i}(\|h_i - \hat{h}_i\|^2). \tag{12}
\]
Since feedback information aims at reducing the uncertainty of \( h_i \) at the basestation, we define **feedback load** as the amount of reduced uncertainty of \( h_i \) when using \( \hat{h}_i \) to represent it, which is equal to \( T(h_i, \hat{h}_i) \), the mutual information between \( h_i \) and \( \hat{h}_i \).

Given a constraint \( D \) on the average distortion, the lower bound of feedback load, denoted as \( R(D) \), can be expressed as
\[
R(D) = \min_{f_{h_i|\hat{h}_i}(\hat{h}_i|h_i)}: \mathbb{E}_{\hat{h}_i, h_i}(\|h_i - \hat{h}_i\|^2) \leq D \tag{13}
\]
where \( f_{h_i|\hat{h}_i}(\hat{h}_i|h_i) \) denotes the conditional PDF of \( h_i \) given \( \hat{h}_i \). Following the rate distortion theory, \( R(D) \) is given by [15]
\[
R(D) = N_t \log_2 \frac{N_r}{D} \tag{14}
\]
The equality is achieved only when the following two conditions are met
\[
1) N_t \text{ elements of } h_i \text{ are quantized independently with individual distortion constraint of } D/N_t. \text{ For convenience,}
\]
\[
1) \text{The results of } h_{ij} \text{ and } \Delta h_{ij} \text{ following Gaussian distribution may not hold for practical quantization schemes. We adopt the rate distortion theory here to study the minimum requirement on feedback load and for the mathematical tractability.}
\]
we let \( \sigma_q^2 = D/N_t \) and term it as quantization noise power.

2) \( h_{ij} \) and \( \Delta h_{ij} = h_{ij} - \hat{h}_{ij} \) are independent random variables following independent zero-mean complex Gaussian distribution with variance \( 1 - \sigma_q^2 \) and \( \sigma_q^2 \), respectively.

Since receivers quantize their channel vectors separately, the quantized channel vectors and quantization noise vectors of different receivers are independent. The quantized channel matrix \( \hat{H} \in \mathbb{C}^{N_r \times N_t} \) is formed as \( \hat{H} = [\hat{h}_{1}, \ldots, \hat{h}_{N_r}]^T \). The quantization noise matrix \( \Delta H \in \mathbb{C}^{N_r \times N_t} \) is defined as the difference between \( H \) and \( \hat{H} \).

\[
\Delta H = H - \hat{H}. \tag{15}
\]

With the same quantization noise power constraints for all receivers, the signal to quantization noise power ratio (SQR) is equal to

\[
\rho_q = \frac{1}{\sigma_q^2}. \tag{16}
\]

### B. MMSE-based Design

Assuming the basestation has the following partial CSI: quantized channel matrix \( \hat{H} \), quantization noise power \( \sigma_q^2 \), and additive noise power \( \sigma_n^2 \), we consider the joint design of precoding matrix and perturbation vector associated with VP precoding based on MMSE criterion.

With arbitrary precoding matrix \( \hat{P} \) and perturbation vector \( \hat{u}' \), the received signal vector can be written as

\[
y = \frac{1}{\sqrt{\hat{P}^T}} H \hat{P}(\mathbf{u} + \mathbf{u}') + \mathbf{n}, \tag{17}
\]

where \( \hat{P}_1 = \|\hat{P}(\mathbf{u} + \mathbf{u}')\|^2 \). Substituting (15) into (17), we have

\[
y = \frac{1}{\sqrt{\hat{P}_1}} \left( (\mathbf{u} + \mathbf{u}') + (H \hat{P} - I_{N_r})(\mathbf{u} + \mathbf{u}') + \Delta H \hat{P}(\mathbf{u} + \mathbf{u}') \right) + \mathbf{n}. \tag{18}
\]

The deviation vector is obtained as

\[
\hat{d} = \sqrt{\hat{P}_1} y - (\mathbf{u} + \mathbf{u}') = (H \hat{P} - I_{N_r})(\mathbf{u} + \mathbf{u}') + \Delta H \hat{P}(\mathbf{u} + \mathbf{u}') + \sqrt{\hat{P}_1} \mathbf{n}. \tag{19}
\]

Given data vector \( \mathbf{u} \) and quantized channel matrix \( \hat{H} \), the MSE as a function of \( \hat{P} \) and \( \mathbf{u}' \) is obtained by taking expectation taken over \( \mathbf{n} \) and \( H \) as

\[
e(\hat{P}, \mathbf{u}') = E_{\mathbf{n}} E_{\Delta H} \|\hat{d}\|^2 |\mathbf{u}, \hat{H} \|
= \|(H \hat{P} - I_{N_r})(\mathbf{u} + \mathbf{u}')\|^2 + N_r \hat{P}_1(\sigma_q^2 + \sigma_n^2). \tag{20}
\]

A comparison between (8) and (20) shows that optimization problems of MMSE VP precoding with quantized and perfect channel feedback share a similar form. It follows that the optimal precoding matrix is given by [16]

\[
\hat{P}_{o, \text{mmse}} = \hat{H}^H (\hat{H} \hat{H}^H + N_r(\sigma_q^2 + \sigma_n^2)I_{N_r})^{-1}. \tag{21}
\]

The optimal perturbation vector can be found with

\[
\hat{u}'_{o, \text{mmse}} = \arg\min_{\mathbf{u}} N_r(\sigma_q^2 + \sigma_n^2)(\mathbf{u} + \mathbf{u}')^H \times (\hat{H} \hat{H}^H + N_r(\sigma_q^2 + \sigma_n^2)I_{N_r})^{-1} (\mathbf{u} + \mathbf{u}'). \tag{22}
\]

Substituting \( \hat{u}'_{o, \text{mmse}} \) and \( \hat{P}_{o, \text{mmse}} \) into (20) leads to the MMSE as

\[
\hat{e}_{o, \text{mmse}} = N_r(\sigma_q^2 + \sigma_n^2)(\mathbf{u} + \mathbf{u}')^H \times (\hat{H} \hat{H}^H + N_r(\sigma_q^2 + \sigma_n^2)I_{N_r})^{-1} (\mathbf{u} + \mathbf{u}'). \tag{23}
\]

### IV. ANALYSIS

In this section, we first establish equivalent relations between quantized and perfect channel feedback cases in terms of MMSE and SER. Based on the equivalent relations, we then develop a feedback scaling rule to ensure full diversity order for VP precoding with quantized channel feedback.

### A. MMSE Equivalence between Quantized and Perfect Channel Feedback Cases

**Lemma 1:** The MMSE of VP precoding with quantized channel feedback with quantization noise power \( \sigma_q^2 \) and additive noise power \( \sigma_n^2 \) is equal to that of perfect channel feedback with additive noise power \( \sigma_n^2 + \frac{\sigma_q^2}{1-\sigma_q^2} \).

**Proof:** The proof of lemma 1 consists of two steps. In step 1, we show for a given \( \hat{H} \) with quantization noise power \( \sigma_q^2 \) and additive noise power \( \sigma_n^2 \) (case 1), there exists a corresponding \( \tilde{H} \) with additive noise power \( \sigma_n^2 + \frac{\sigma_q^2}{1-\sigma_q^2} \) (case 2) which leads to the same MMSE. From channel quantization model, we know that \( \hat{H} \) has the same distribution as \( H \) except its elements having a reduced variance of \( 1-\sigma_q^2 \). Define a new random matrix as

\[
\tilde{H} = \frac{1}{\sqrt{1-\sigma_q^2}} \hat{H}, \tag{24}
\]

which has the same distribution as \( H \). Eq. (22) can be rewritten in terms of \( \tilde{H} \) as

\[
\hat{u}'_{o, \text{mmse}} = \arg\min_{\mathbf{u}} N_r(\sigma_q^2 + \sigma_n^2)(\mathbf{u} + \mathbf{u}')^H \times \left( \tilde{H} \tilde{H}^H + N_r(\sigma_q^2 + \sigma_n^2)I_{N_r} \right)^{-1} (\mathbf{u} + \mathbf{u}'). \tag{25}
\]

Solution of Eq. (25) is the same as that of perfect channel feedback with channel matrix \( \tilde{H} \), data vector \( \mathbf{u} \) and additive noise power \( \sigma_n^2 + \frac{\sigma_q^2}{1-\sigma_q^2} \). Therefore, after taking expectation of the square error over the distribution of both additive and quantization noise for case 1 while only over additive noise for case 2, the resulting MMSE will be identical.

In step 2, the resulting MMSE of step 1 is further averaged over the distribution of \( \tilde{H} \) and \( H \) (or \( \hat{H} \)). Here, \( \tilde{H} \) is related to \( H \) as given by equation (24). Since \( \tilde{H} \) is simply a scaled version of \( H \) as given by Eq. (24), the resulting MMSE will remain the same after averaging over the distribution of \( \tilde{H} \) and \( H \). In this sense, there exists an MMSE equivalence between MMSE VP with the quantized and perfect channel feedback.

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B. SER Equivalence between Quantized and Perfect Feedback

Treating all receivers equally, the SER of a multiuser MIMO system is defined as the average SER of all receivers. Let \( e(\sigma_n^2, \sigma_q^2) \) denote the SER of VP precoding with quantization noise power \( \sigma_q^2 \) and additive noise power \( \sigma_n^2 \). Note that \( \sigma_q^2 = 0 \) corresponds to the SER with perfect channel feedback.

**Lemma 2:** The SER of VP precoding with quantized channel feedback with quantization noise power \( \sigma_q^2 \) and additive noise power \( \sigma_n^2 \) is equal to that of perfect channel feedback with additive noise power \( \frac{\sigma_n^2 + \sigma_q^2}{1 - \sigma_q^2} \), namely

\[
e(\sigma_n^2, \sigma_q^2) = c \left( \frac{\sigma_n^2 + \sigma_q^2}{1 - \sigma_q^2}, 0 \right) .
\]

**Proof:** Firstly, given \( H \) and \( u \), we show that deviation vectors of the following two cases follow the same distribution. The first case is quantized channel feedback with quantization noise power \( \sigma_q^2 \) and additive noise power \( \sigma_n^2 \). The second case is perfect channel feedback with channel matrix \( \tilde{H} \) and additive noise power \( \frac{\sigma_n^2 + \sigma_q^2}{1 - \sigma_q^2} \). Here, \( \tilde{H} \) is related to \( H \) as given by (24).

For case 1, the deviation vector can be obtained from (19) by replacing \( \hat{P} \) with the optimal precoding matrix \( \hat{P}_{o,\text{mse}} = \tilde{H}^H (\tilde{H} \tilde{H}^H + N_r(\sigma_n^2 + \sigma_q^2)I_{N_r})^{-1} \) as

\[
d_1 = (\tilde{H} \hat{P}_{o,\text{mse}} - I_{N_r})(u + \hat{u})
\]

\[
+ \Delta \hat{P}_{o,\text{mse}}(u + \hat{u}) + \|\hat{P}_{o,\text{mse}}(u + \hat{u})\|\mathbf{n}_2 .
\]

Since case 2 uses (25) to search for perturbation vector, the result remain the same as that of case 1 and is still denoted as \( \hat{u}' \). Substituting \( \hat{P}_{o,\text{mse}} = \tilde{H}^H (\tilde{H} \tilde{H}^H + N_r(\sigma_n^2 + \sigma_q^2)I_{N_r})^{-1} \) into (7) yields the deviation vector of case 2 as

\[
d_2 = (\tilde{H} \hat{P}_{o,\text{mse}} - I_{N_r})(u + \hat{u}) + \|\hat{P}_{o,\text{mse}}(u + \hat{u})\|\mathbf{n}_2 .
\]

Using (24), we have \( \hat{P}_{o,\text{mse}} = \frac{1}{\sigma_q^2 + \sigma_n^2} P_{o,\text{mse}} \). It follows that part 1 of (27) and (28) are equal.

For convenience, we set \( c = \|\hat{P}_{o,\text{mse}}(u + \hat{u})\|^2 = \frac{1}{\sigma_q^2 + \sigma_n^2} \|P_{o,\text{mse}}(u + \hat{u})\|^2 \). Since elements of \( \Delta \hat{H} \) and \( \mathbf{n}_1 \) are i.i.d. Gaussian random variables with variance \( \sigma_q^2 \) and \( \sigma_n^2 \), respectively, it can be shown that elements of part 2 of (27) are i.i.d. Gaussian random variables with variance \( c(\sigma_q^2 + \sigma_n^2) \). Similarly, noting that elements of \( \mathbf{n}_2 \) have variance \( \sigma_q^2 + \sigma_n^2 \), part 2 of (28) can be proven to have the same distribution as that of (27).

Since deviation vector determines the number of wrongly detected symbols, case 1 and case 2 achieve the same SER. Furthermore, because the mapping between \( \tilde{H} \) and \( H \) is one to one, the achieved SER after averaging over the distribution of \( \tilde{H} \) and \( H \) will also be equal.

Table I gives the simulation results of the SER performance of VP precoding with quantized channel feedback and its equivalent perfect channel feedback, whose data is in the parenthesis. The close match between the two sets of data verifies the equivalent relation given by (26).

<table>
<thead>
<tr>
<th>Table I</th>
</tr>
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<tbody>
<tr>
<td>SER for different SNR and SQRT with VP precoding</td>
</tr>
<tr>
<td>( N_t = N_r = M = 4, 10^5 ) simulations</td>
</tr>
<tr>
<td>SNR:SNR</td>
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<tr>
<td>---------</td>
</tr>
<tr>
<td>5 dB</td>
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<tr>
<td>10 dB</td>
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<tr>
<td>20 dB</td>
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</table>

C. Feedback Load and SER Performance

As an application of the equivalent relations, we investigate the tradeoff between feedback load and SER performance. For VP precoding with perfect channel feedback, it is proved in [11] that full diversity order of \( N_t \) is achieved, namely

\[
\lim_{\rho \to \infty} -\log(e(\sigma_n^2, 0)) = \lim_{\sigma_q^2 \to 0} \frac{\log(e(\sigma_n^2, 0))}{\log(\sigma_q^2)} = N_t .
\]

With quantized channel feedback, how much feedback load is required to obtain the same preceding diversity order is of particular interest. The following lemma answer this question.

**Lemma 3:** In order to achieve the same diversity order as perfect channel feedback, VP precoding with quantized channel feedback has to increase each user’s feedback load by at least \( 3.32N_t \) bits for every 10 dB increase in SNR.

**Proof:** Let feedback load increases with SNR at the rate such that

\[
\sigma_q^2 = k(\sigma_n^2)^\alpha + o(\sigma_n^2),
\]

where \( \alpha \in \mathbb{R} \) is the lowest power within the polynomial, namely

\[
\lim_{\sigma_q^2 \to 0} \frac{o(\sigma_q^2)}{(\sigma_q^2)^\alpha} = 0 .
\]

From (29), we can express the SER with perfect channel feedback as

\[
\log(e(\sigma_n^2, 0)) = N_t \log(\sigma_n^2) + o(\log(\sigma_n^2)) ,
\]

where

\[
\lim_{\sigma_q^2 \to 0} \frac{o(\log(\sigma_q^2))}{\log(\sigma_q^2)} = 0 .
\]

Using (26), (32), and (30), we have

\[
\log(e(\sigma_n^2, \sigma_q^2)) = N_t \log\left(\frac{\sigma_n^2 + k(\sigma_n^2)^\alpha + o(\sigma_n^2)}{1 - k(\sigma_n^2)^\alpha - o(\sigma_n^2)}\right) + o\left(\log\left(\frac{\sigma_n^2 + k(\sigma_n^2)^\alpha + o(\sigma_n^2)}{1 - k(\sigma_n^2)^\alpha - o(\sigma_n^2)}\right)\right) .
\]

The diversity order can be determined as

\[
\lim_{\sigma_q^2 \to 0} \frac{\log(e(\sigma_n^2, \sigma_q^2))}{\log(\sigma_n^2)} = \left\{ \begin{array}{ll}
N_t & \alpha \geq 1 \\
\alpha N_t & 0 < \alpha < 1 .
\end{array} \right.
\]
Therefore, we have to keep $\sigma_n^2 = k(\sigma_n^2)$ in order to achieve full diversity order. Combined with (14), we obtain a simple relation between feedback load and SNR:

$$R(D) = N_t \left( \log_2(\rho) - \log_2(k) \right),$$

which reveals that we need an extra $3.32 N_t$ bits per user for every 10 dB increase in SNR to achieve full diversity order.

Interestingly, the same scaling law was observed in [17], [18] to prevent the sum rate of MIMO broadcast channels with limited feedback becoming interference-limited.

V. Numerical Examples

In Fig. 1, we plot the SER of VP precoding as a function of SNR for different feedback schemes. The values $N_t = N_r = 4$, $M = 4$ are used. With a fixed feedback load corresponding to $\text{SQR}=17$ dB, the SER performance becomes quantization-noise-limited at high SNR region. Moreover, its SER floor is shown to be approximately the SER of perfect channel feedback with $\text{SNR}=17$ dB. When the feedback load is adjusted in a way such that $\sigma_n^2 = \sigma_n^2$, the SER is shown to decrease at the same rate as perfect channel feedback at high SNR region. Therefore, full diversity order is achieved with this feedback scheme. As expected, the performance gap between quantized channel feedback with $\sigma_n^2 = \sigma_n^2$ and perfect channel feedback is about 3 dB at high SNR. On the other hand, SER performance of feedback schemes, whose quantization noise power decrease at rates $(\sigma_n^2)^{2/3}$ and $(\sigma_n^2)^{2}$, can not achieve the same diversity order.

VI. Concluding Remarks

In this paper, we study the VP precoding with quantized channel feedback. Based on the MMSE criterion, we consider a joint optimal design of precoding matrix and perturbation vector. We model channel vector quantization following the rate-distortion criterion, based on which we establish an equivalent relation between the SER of VP precoding with quantized and perfect channel feedback. This equivalent relation can be conveniently used to evaluate the SER performance of VP precoding with quantized channel feedback relative to that of perfect channel feedback. Applying the equivalent relation, we investigate the tradeoff between SER performance and feedback load. We show that feedback load per user should scale at $N_t \log_2(\rho)$ to achieve full diversity order.

REFERENCES