Analytical Investigation of the Performance of Packet-level FEC Techniques in VoIP Communications

LEOPOLDO ESTRADA[1], DENI TORRES[1], HOMERO TORAL[1,2]
[1]Department of Electrical Engineering
CINVESTAV IPN
GUADALAJARA, JALISCO, MEXICO
[2]Department of Postgraduate
ITSCH
LAS CHOAPAS, VERACRUZ, MEXICO
lestrada@gdl.cinvestav.mx, www.gdl.cinvestav.mx

Abstract: - In this work, the distributions of the number of received and lost packets, respectively named gap and burst, of a VoIP communication are modeled with discrete finite-state Markov chains. Through a study of measurements from monitored VoIP calls, it is shown that these models can adequately represent the geometric-type decay of these distributions. Two-state model performs well for homogeneous losses but for non-homogeneous losses four-state model fits better. An analysis of the performance of a packet-level FEC scheme, based on n-packet redundancy, is developed. The perceived packet loss rate that results of applying this correction scheme is quantified. The studied measurements show that 1-packet redundancy is generally sufficient to improve the quality of the communication to an acceptable level.

Key-Words: - VoIP, packet loss rate, burst length distribution, n-packet FEC.

1 Introduction

Internet became the point of convergence of information and media transmission. Data, voice, video, etc., are transmitted through the same communication channel. The service provided by the Internet is named “best effort”, which means that the devices between links generally do not differentiate between the types of traffic and there is neither resource reservation nor prioritization. Congestion due to the high demand of network resources is the cause of the impairment of its quality of service, which consists of delay problems, i.e., the delay and its variation (delay *jitter*) are higher, and packet loss. The a*utomatic repeat request* (ARQ) technique, the correction scheme of the *transmission control protocol* (TCP), is used to eliminate (or reduce) packet losses, but it is not suitable for many real-time and near real-time applications, which have tighter delay tolerance. Then, other types of error correction techniques, adequate for these applications, are needed, e.g. *multiple packet transmission* (MPT) or *forward error correction* (FEC), to assure certain quality of service.

In this work, modeling of packet loss of a VoIP communication through a wide area network (WAN) is developed. Discrete finite-state Markov chains are used to represent how these losses occur in the communication channel, which consists of a sequence of routers connected by links through which the packets traverse.

Consecutive packet receipts and losses are named gaps and bursts, respectively. Due to the time-correlated occupancy of the network, packet losses commonly occur in bursts such that their lengths follow a geometric-type distribution, as well as gaps [1] [2].

At small time scales, i.e. a few seconds or minutes, a two-state Markov chain can reproduce this phenomenon, but a non-homogeneous behavior becomes noticeable at larger scales and, in this case, the two-state Markov chain is insufficient, thus a more general model is necessary. The four-state Markov chain seems to capture or simulate better this widely known non-homogeneous behavior of the characteristics of network traffic. The four-state model approach allows us to represent and simulate those periods with low and high *packet loss rate* (PLR) that alternate in sequence according to certain probability.

MPT consists of sending copies of packets when high losses occur, these copies must be equally spaced in the time interval they are sent in order to maximize the probability of receipt [3]. Although this technique has the advantage that it is very easy to implement, it is not convenient because of the high bandwidth requirement.

The N-packet FEC technique consists of sending information about packet n along with later packets, i.e., with packets $n+1$, $n+2$, …, $n+N$, in order to reconstruct packet n in the case it is lost. With this correction scheme, the last N packets of a burst can be recovered and then, the perceived PLR of the end user is lower than the PLR due to the network. The amount of redundancy generally is defined as a function of the PLR [4], e.g., it is not efficient to send redundant information if there are no missing packets. This correction scheme, which is performed at packet level, is the scope of the analysis presented in this work. Codification schemes of the redundant information on later packets are described in [5].

2 Contributions

The contributions of this work are summarized as follows:

1. A statistical description of the two-state and four-state Markov chains, assuming that it is time-homogeneous (i.e., the probabilities of transition between states are constant) is presented.
2. An analytical description of the performance of n-packet FEC scheme is given, i.e., the perceived PLR as a function of the network loss rate, the burst length distribution and the level of redundancy.
3. A set of measurements, which consists of monitored VoIP calls from which loss sequences are obtained, is studied in order to verify the proposed models.

3 Finite-state Markov Chains

3.1 Matrix Representation of the Steady-state

Let $S = S_1, S_2, \ldots, S_m$ be the m states of an m-state Markov chain and let p_{ij} be the probability of the chain to pass from the state S_i to the state S_j, i.e., $p_{ij} = P(X_i = x_i | X_{i-1} = x_{i-1})$. Having the Markov property means that, given the present state, future states are independent of the past states, i.e., $P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \ldots) = P(X_{n+1} = x_{n+1} | X_n = x_n)$. The Markov chains used in this work also are time-homogeneous, which means that the probabilities of transition between states are constant over time, i.e., $P(X_{n+1} = x_{n+1} | X_n = x_n) = P(X_n = x_n | X_{n-1} = x_{n-1})$.

All states communicate (are reachable from) each other, which makes the chain irreducible. Also, the chain is aperiodic, i.e., state S_i can be reached from itself in any number of steps ($n = 1,2,3, \ldots$).

The probabilities of transitions between states can be represented by a *transition matrix*. The elements of the one-step $m \times m$ transition matrix $\underline{T}$ are $T_{ij} = p_{ij}$. To obtain the n-step transition matrix it is necessary to multiply the matrix itself n times [6], i.e.,

$$\underline{T}_n = \underline{T}^n \quad (1)$$

As the number of steps (n) increases, the probability of transition to the state S_i depends less of the initial state. i.e., as n tends to ∞, the matrix $\underline{T}_n$ converges to a matrix with the next form:

$$\underline{T}_\infty = \lim_{n\to\infty} \underline{T}_n = \begin{bmatrix} s_1 & s_2 & \cdots & s_m \\ s_1 & s_2 & \cdots & s_m \\ \vdots & \vdots & \ddots & \vdots \\ s_1 & s_2 & \cdots & s_m \end{bmatrix} \quad (2)$$

such that

$$s_1 + s_2 + \cdots + s_m = 1 \quad (3)$$

In (2) and (3), s_i represents the named *steady* probability of state S_i. The steady-state transition matrix $\underline{T}_\infty$ can be obtained then by solving (3) and (4) [7]:

$$\overline{S}\underline{T} = \overline{S} \quad (4)$$

where $\overline{S} = [s_1 \quad s_2 \quad \ldots \quad s_m]$.

Assuming that the chain is irreducible and aperiodic, the matrix $\underline{T}_\infty$ is well defined and unique.

3.1.1 Two-state Markov Chain

The two-state Markov chain is shown in Fig. 1. State S_1 represents packet loss and S_2, packet receipt. Two substitutions ($p_{11} = 1 - p_{12}$ and $p_{22} = 1 - p_{21}$) are made in order to represent the chain with the lowest number of parameters. The steady-state probability of the chain to be in the state S_1, namely the PLR, is given by (5) [4]:

$$s_1 = \frac{p_{21}}{p_{12} + p_{21}} \quad (5)$$

and clearly $s_2 = 1 - s_1$.

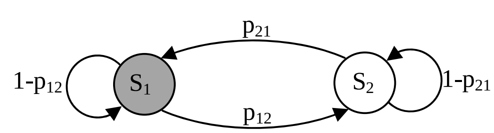

Fig. 1: Two-state Markov chain. White and shady circles represent correct and erroneous states, respectively.

The burst and gap length distributions ($f_b(k)$ and $f_g(k)$, respectively) can be expressed in terms of p_{12} and p_{21}, as expressed by (6) and (7):

$$f_b(k) = p_{12}(1 - p_{12})^{k-1} \quad (6)$$

$$f_g(k) = p_{21}(1 - p_{21})^{k-1} \quad (7)$$

which have also respective means $E\{f_b(k)\} = 1/p_{12}$

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and $E\{f_g(k)\} = 1/p_{21}$. It is easy to proof (6), as $\sum_{k=1}^{\infty} f_b(k) = 1$ and $f_b(k+1) = f_b(k) \cdot (1 - p_{21})$; and similarly for (7).

3.1.2 Four-state Markov Chain

The four-state Markov chain is shown in Fig. 2. Missing arrows indicate zero probability. States S_1 and S_3 (shady circles) represent packet losses (erroneous); S_2 and S_4 (white circles), packet receipt (correct).

Six parameters $(p_{21}, p_{12}, p_{43}, p_{34}, p_{23}, p_{32} \in (0,1))$ are necessary to define all the transition probabilities. Without loss of generality, probabilities of transitions between correct states, as well as transitions between erroneous ones, have been set to zero.

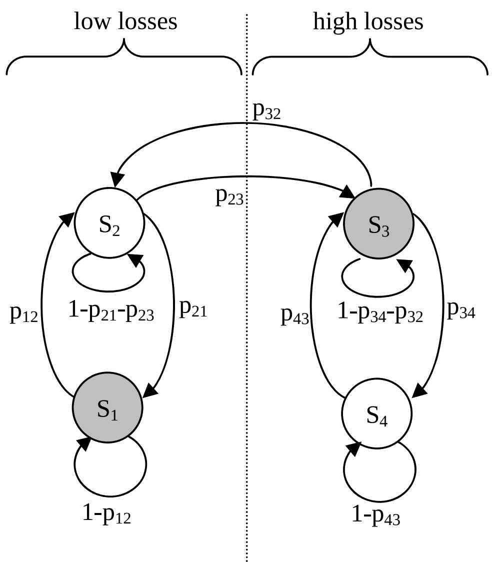

Fig. 2: Four-state Markov chain. Only two types of transitions between different states are allowed: from correct to erroneous and from erroneous to correct.

The four steady-state probabilities of this chain are:

$$s_1 = \frac{1}{1 + \frac{p_{12}}{p_{21}} + \frac{p_{12}p_{23}}{p_{21}P_{32}} + \frac{p_{12}p_{23}p_{34}}{p_{21}p_{32}p_{43}}} \quad (8)$$

$$s_2 = \frac{1}{1 + \frac{p_{21}}{p_{12}} + \frac{p_{23}}{p_{32}} + \frac{p_{23}p_{34}}{p_{32}p_{43}}} \quad (9)$$

$$s_3 = \frac{1}{1 + \frac{p_{34}}{p_{43}} + \frac{p_{32}}{p_{23}} + \frac{p_{21}p_{32}}{p_{12}p_{23}}} \quad (10)$$

$$s_4 = \frac{1}{1 + \frac{p_{43}}{p_{34}} + \frac{p_{32}p_{43}}{p_{23}p_{34}} + \frac{p_{21}p_{32}p_{43}}{p_{12}p_{23}p_{34}}} \quad (11)$$

The probability of the chain to be either in S_1 or in S_3, that corresponds to PLR, is then:

$$r = s_1 + s_3 \quad (12)$$

The average burst length $(\bar{b})$ is calculated as the quotient of the probability of loss and the probability of transition from a lossless state to a loss state (13), that is:

$$\bar{b} = \frac{s_1 + s_3}{s_2(p_{21} + p_{23}) + s_4(p_{43})} \quad (13)$$

Similarly, the average gap length is:

$$\bar{g} = \frac{s_2 + s_4}{s_1(p_{12}) + s_3(p_{34} + p_{32})} \quad (14)$$

Note that the transitions from error state to correct state and vice versa have equal probability, i.e. $s_2(p_{21} + p_{23}) + s_4(p_{43}) = s_1(p_{12}) + s_3(p_{34} + p_{32})$.

The distribution of the burst length can be derived the following the next procedure:

Let $f_b(k)$ denote the probability that the burst length is k; $C_1(k)$, the probability that the burst length is k or greater and the k^{th} transmission is from state S_1 and $C_3(b)$, the probability that the burst length is k or greater and k^{th} transmission is from state S_1 and $C_b(k)$, the probability that the burst length is k or greater such that $C_b(k) = C_1(k) + C_3(k)$ and $f_b(k) = C_b(k) - C_b(k+1)$. Clearly $C_b(k) = \sum_{i=k}^{\infty} f_b(i)$. Also, as transitions between states S_1 and S_3 have zero probability, $C_1(k+1) = C_1(k)(1 - p_{12}) = C_1(1)(1 - p_{12})^k$ and $C_3(k+1) = C_3(k)(1 - p_{34} - p_{32}) = C_3(1)(1 - p_{34} - p_{32})^k$. Then to calculate $f_b(k)$ it is necessary to obtain $C_1(1)$ and $C_3(1)$, whose respective values are $C_1(1) = s_2 p_{21}/[s_2(p_{21} + p_{23}) + s_4 p_{43}]$ and $C_3(1) = (s_2 p_{23} + s_4 p_{43})/[s_2(p_{21} + p_{23}) + s_4 p_{43}]$.

As the minimum burst length is 1, $C_b(1) = C_1(1) + C_3(1) = 1$. Then, the distribution of the burst length is:

$$f_b(k) = C_1(1)Q_1(k) + C_3(1)Q_3(k) \quad (15)$$

where $Q_1(k) = (1 - p_{12})^{k-1} - (1 - p_{12})^k = p_{12}(1 - p_{12})^{k-1}$ and $Q_3(k) = (1 - p_{34} - p_{32})^{k-1} - (1 - p_{34} - p_{32})^k = (p_{34} + p_{32})(1 - p_{34} - p_{32})^{k-1}$. As expressed by (15), $f_b(k)$ is the sum of two geometric series with respective rates $1 - p_{12}$ and $1 - p_{34} - p_{32}$; this implies that $f_b(k)$ is a decreasing function of k, i.e., bursts of greater length have lower probabilities than shorter ones.

A similar procedure can be followed to obtain the gap length distribution $(f_g(k))$, which is:

$$f_g(k) = C_2(1)Q_2(k) + C_4(1)Q_4(k) \quad (16)$$

where $C_2(1) = (s_1 p_{12} + s_3 p_{32})/[s_1 p_{12} + s_3(p_{32} + p_{34})]$, $C_4(1) = (s_3 p_{34})/[s_1 p_{12} + s_3(p_{32} + p_{34})]$, $Q_2(k) = (1 - p_{21} - p_{23})^{k-1} - (1 - p_{21} - p_{23})^k = (p_{21} + p_{23})(1 - p_{21} - p_{23})^{k-1}$ and $Q_4(k) = (1 - p_{43})^{k-1} - (1 - p_{43})^k = p_{43}(1 - p_{43})^{k-1}$. Also note that $C_2(1) + C_4(1) = 1$.

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Note that, although the resulting equations correspond to the four-state model of Fig. 2, this procedure shows the key for the generalized method for any finite-state Markov chain, which is to find first the cumulative density function (CDF), i.e., $C_b(k)$ and $C_g(k)$.

4 Performance Evaluation of the N-packet FEC Technique

N-packet FEC consists of that packet $n+1$ contains information about packet n, so that if packet n is lost, it can be approximately reconstructed from the redundant information. Packet n cannot be reconstructed if there is no redundant information, i.e. when packet $n+1$ is also lost. The 1-packet FEC technique performance can be described as: it reduces the size of a burst of length k to $k-1$. The perceived PLR ($r_1{}'$) is proportional to the perceived average burst length, which in this case decreases by 1 (packet), then it is equal to:

$$r_1{}' = \frac{(\bar{b}-1)r}{\bar{b}} \tag{17}$$

where $\bar{b}$, the average burst length, is $\bar{b} = \sum_{k=1}^{\infty} k f_b(k)$ and $f_b(k)$ is the burst length distribution.

If the redundancy level extends to N packets, i.e. packet n has information about $n+1$, $n+2$, …, $n+N$ packets, the length of all bursts decreases from k to $\max(0, k-N)$ packets, then the new burst length distribution $f_b^{'}(k)$ is:

$$f_b^{'}(k) = \begin{cases} \sum_{i=1}^{N} f_b(i); & k=0 \\ f_b(k+N); & k>0 \end{cases} \tag{18}$$

Note that (18) considers bursts of zero length. The interpretation of this is as follows: bursts do really occur in the network but, as they are corrected by a N-packet FEC technique, they are diminished (when $k>N$) or eliminated (when $k \le N$) in the receiver. Then, $f_b^{'}(k)$ is the new burst length distribution and its mean can be calculated as:

$$\bar{b}' = \sum_{k=0}^{\infty} \left[k f_b^{'}(k)\right] \tag{19}$$

$$\bar{b}' = \bar{b} - N + \sum_{k=1}^{N} (N-k) f_b(k) \tag{20}$$

Consequently, the perceived PLR is:

$$r_N' = \frac{\left[\bar{b} - N + \sum_{k=1}^{N-1} (N-k) f_b(k)\right] r}{\bar{b}} \tag{21}$$

which is a generalized form of (17).

Note that (21) expresses the perceived PLR of the receiver without considering other sources of losses, e.g., additional perceived losses occur if packets are delayed more than certain threshold (e.g., de-jitter buffer size). In this case, although the packet arrived to the receiver, it is dropped and consequently, lost. Furthermore, bit-level errors that may be present in received but corrupted packets are an important source of errors, especially for wireless communications.

5 Modeling from a Loss Sequence

Let us define the loss sequence as follows:

$$Y_k = \begin{cases} 0; & \text{if packet } k \text{ is received} \\ 1; & \text{if packet } k \text{ is lost} \end{cases} \tag{22}$$

From the loss sequence, the probabilities of transitions were also estimated using the following algorithms:

5.1 Two-state Parameters Estimation

The estimations of p_{12} and p_{21} are: $p_{12} = t_{c\to e}/n_1$ and $p_{21} = t_{e\to c}/n_0$, where $t_{c\to e}$ and $t_{e\to c}$ are the respective number of transitions from correct states to error states and from error states to correct states, and n_0 and n_1 are the respective number of received and lost packets.

5.2 Four-state Parameters Estimation

In this case the values of the sequence Y_t are divided into regions of two types: the first with lower loss rate (whose first and last values are zeros) and the second with higher loss rate (whose first and last values are ones) than certain threshold, e.g. 1%. Then, from the first region, p_{12} and p_{21} are estimated the same way than in a two-state model. Similarly, p_{43} and p_{34} are estimated from the second region. Finally, let $t_{1st\to 2nd}$ be the number of transitions from the first region to the second; $t_{2nd\to 1st}$, the number of transitions from the second to the first; n_{1st}, the number of received packets in the first region (zeros) and n_{2nd}, the number of lost packets in the second region (ones), then $p_{23} = t_{1st\to 2nd}/n_{1st}$ and $p_{32} = t_{2nd\to 1st}/n_{2nd}$.

6 Characterization of Measured Data Traces

The measurements studied in this work are those corresponding to Sets 3 and 4, described in [8]. There are 48 data traces. Each one of these represents a 1-hour VoIP call. As each one of these calls was monitored at one endpoint, its respective series of sequence numbers (of received packets) was captured and, from this, the loss sequence (Y_k) was obtained. Also the respective PLR and respective gap and burst length distributions ($f_g(k)$ and $f_b(k)$) were estimated.

From the estimated probabilities of transitions, the gap and burst length distributions of both two-state and four-state models, defined by (6) and (7) for two-state model and (15) and (16) for the four-state model are obtained. The square root of the mean squared error between the respective distributions obtained from measured and theoretical models is also calculated.

6.1 Results

The burst and gap length distributions for one of the captured traces, obtained from a VoIP call with codec G.711 and packet inter-departure time of $20ms$, are shown respectively in Fig. 3 and Fig. 4.

In Fig. 3 it is shown that the burst length decays rapidly, e.g., to zero probability for burst of length lower than 5 packets. It is also observed that both two-state and four-state models can characterize this decay.

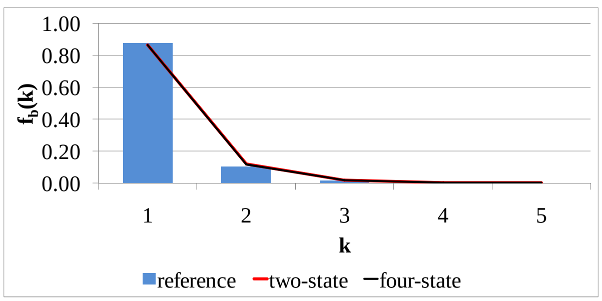

Fig. 3: Burst length distribution of one of the loss sequences.

The gap length distribution decays slower than the burst length distribution. There exist gaps of tens and hundreds of packets with non-negligible probability and, in this case, the less flexible one-parameter formula of the two-state model cannot fit the measured distribution, in contrast with the four-state model, which fits it adequately.

The SMSE for burst length distribution of both two-state and four-state model is quite similar (less than 0.01) for most traces, as seen in Fig. 5. But there is a remarkable difference between both models in the gap distribution. In Fig. 6 it can be observed that the SMSE four-state model fits remarkably better the gap distribution for most traces (its maximum SMSE is 0.002).

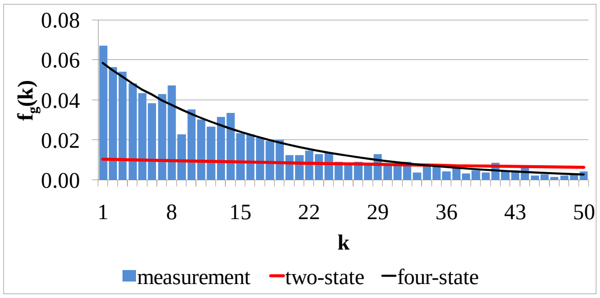

Fig. 4: Gap length distribution of one of the loss sequences.

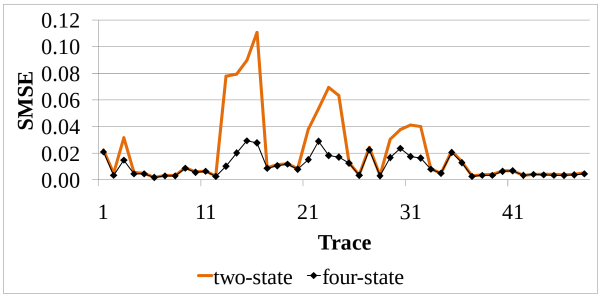

Fig. 5: SMSE of two-state and four-state burst length distribution.

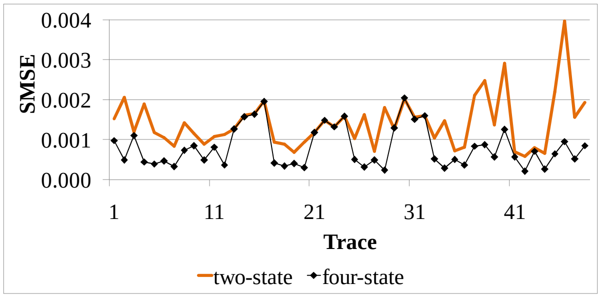

Fig. 6: SMSE of two-state and four-state gap length distribution.

Fig. 7 shows the PLR of the 48 studied data traces, which is calculated as the quotient of the number of lost packets and the number of sent packets. Also, by applying (21), the perceived PLR after a N-packet FEC is estimated for $N = 1$, 2 and 3.

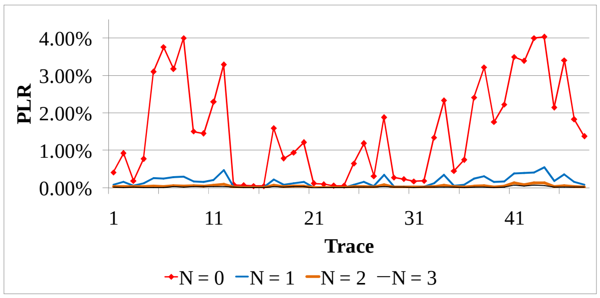

Fig. 7: Perceived PLR for redundancy of $N = 0,1,2,3$ packets.

To determine how the performance is improved when increasing the level of redundancy (N), the relative gain is calculated, which defined as:

$$\Delta_{r'}(N) = \frac{-(r'_N - r'_{N-1})}{r}; N > 0 \qquad (23)$$

Fig. 8 shows the relative gain for the studied traces for the redundancy levels $N = 1$, 2 and 3. The major relative gain (approximately 80%) is obtained by adding redundancy of one packet, i.e., for $N = 1$. In this case the perceived PLR decreases below 0.55% for all studied traces, which is acceptable for VoIP calls. Although PLR constraints can be lower than 0.1% for Internet backbone routers or public telephony systems, a less strict limitation applies for VoIP providers and user local's ISP networks, where losses up to 1% are considered undetectable [9].

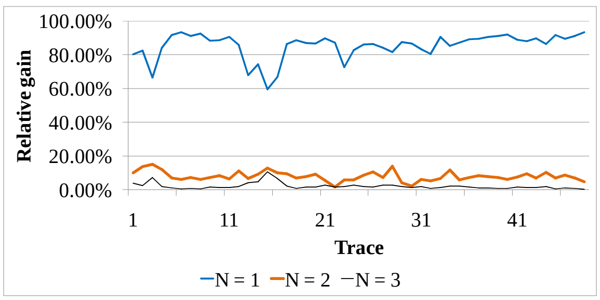

Fig. 8: Relative gain of the perceived PLR.

7 Conclusions

In this work, modeling and characterization of packet loss for a VoIP communication is presented. Formulae for theoretical gap and burst length distributions, as a function of the probabilities of transitions for both models, are given.

The strategy used to obtain the gap and burst length distributions of four-state model presented in Section 3.1.2 exemplifies the generalized methodology for a m-state Markov chain model, which consists of finding first their respective CDF, i.e., $C_b(k)$ and $C_g(k)$.

Algorithms for reconstructions, i.e., estimation of the probabilities of transitions between states for two-state and four-state models, are also described.

It is shown through an evaluation based on SMSE that both two-state (at least for most cases) and four-state models can capture the geometric-type decay of the distribution of the burst length, but the two-state model fails when modeling the gap length distribution when non-homogeneous losses are present. I.e., the gap length distribution is the sum of two geometrical series, as defined by (16), not only one, as defined by (7).

An analysis of the N-packet FEC scheme is also presented. The expected perceived PLR obtained with this correction scheme is quantified, as expresses (21). This resulting general formula applies for the N-packet FEC scheme, regardless of the shape of the burst length distribution.

Through the study of the measurements and the computation of the perceived PLR and relative gain, it is shown that 1-packet FEC is generally sufficient to improve the quality of the communication to an acceptable level, e.g., where the PLR is lower than 1%.

References:

[1] H. Schulzrinne, J. F. Kurose, D. F. Towsley. Loss Correlation for Queues with Bursty Input Streams, *Proc. IEEE ICC '92, Chicago II*, 1992, pp.219-224.

[2] J. R. Yee, E. J. Weldon, Evaluation of the Performance of Error Correcting Codes on a Gilbert Channel, *IEEE transactions on Communications*, Vol. 43 No. 8, 1995.

[3] J. Bolot, S. Fosse-Parisis, D. Towsley. Adaptive FEC-based Error Control for Interactive Audio in the Internet, *Proceedings of IEEE INFOCOM*, 1998, pp. 1453-1460.

[4] H. Lee, H. Lee. A Packet Loss Recovery Scheme Based on the Gap Statistics, *C. Kim (Ed.): ICOIN 2005, LNCS 3391, Springer-Verlag Berlin Heidelberg*, 2005, pp. 627-634.

[5] N. Erdöl, C. Castellucia, Ali Zilouchian. Recovery of Missing Speech Packets Using the Short-time Energy and Zero-crossing Measurement, *IEEE Transactions on Speech and Audio Processing*, Vol. 1 No. 3, July 1993.

[6] W. Ching, M. K. Ng., *Markov Chains: Models, Algorithms and Applications*, Springer, 2006.

[7] G. G. Yin, Q. Zhang, *Discrete-Time Markov Models, Two-Time-Scale Methods and Applications*, Springer, 2005.

[8] H. Toral, D. Torres, C. Hernandez, L. Estrada. Self-similarity, Packet Loss, Jitter, and Packet Size: Empirical Relationships for VoIP, *18th International Conference on Electronics, Communications and Computers, CONIELECOMP*, March 2008.

[9] K. Maheswari, M. Punithavalli, A Survey of Packet Loss in VoIP, *International Journal of Computational Intelligence Research*, Vol. 5, No. 1, 2009, pp. 57–66.