Torque Vectoring with a feedback and feed forward controller - applied to a through the road hybrid electric vehicle

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Abstract

This paper concentrates on the torque commands for electric propulsion motors in a through the road hybrid electric vehicle. By using a linear quadratic gaussian controller, a flat feed forward controller and a linear desired value generator the lateral vehicle dynamics are influenced. Understeering, oversteering, agility and cornering speed can be optimized by proper controller design. A 14 degree of freedom vehicle model with a Dugoff tire model is used to simulate the vehicle behaviour. The simulation results show improved vehicle dynamics and increased handling for the driver compared to an equal distributed torque command.

1. Introduction

Today more and more vehicles are produced as hybrid electric vehicles (HEV). On possible solution to implement an electric drivetrain into a internal combustion engine (ICE) based vehicle is the so called Through the Road (TtR) hybrid electric vehicle. In a TtR-HEV [1] the existing drive line will not be changed. Only the axle with no drive will be equipped with the electric components. These components include an electric energy storage - mostly a lithium-ion battery - and an electric machine controlled by an inverter.

For the electric drive there exist two design possibilities. In the first case there is one electric motor which is controlled by one inverter and mounted somewhere in the chassis. This system applies the motor torque to a final drive which routes the torque to the wheels. For the second design two electric motors with two inverters are necessary. These motors can be located in the chassis or inside the driving wheels as hub motors.

With two motors the size of the machines can be smaller and a final drive is not needed. With two independent motors it is possible to apply positive and negative torque individually to each wheel. This means that it is possible to accelerate and brake the wheels independently. The torque generation of electric machines is very quick and accurate for accelerating and braking the driving wheels [2]. With the possibility of controlling the wheels individually the question arises: How to distribute the torque between the two motors? To answer this question a torque vectoring controller is developed.

In this paper a control strategy for torque vectoring will be presented and simulated for a through the road hybrid electric vehicle. The control structure and design of the sub-controllers will be shown in section 2. In section 3 the controllers will be simulated with a 14 degree of freedom vehicle model. Conclusions are given in section 4.

2. Torque Vectoring

The basic idea of torque vectoring is that given requests from the driver (steering angle, brake and acceleration pedal signals) will be processed and distributed as torque commands to the wheels of the vehicle. With
the individual torque distribution the vehicle performance, agility and safety [3] can be improved. This topic includes optimal energy management [4] of power from the combustion part and the electric drive to the wheels.

In order to distribute the torque the structure from figure 2 is used. A desired value generator gets the steering angle of the wheels δ and the desired acceleration request \( a_{req} \) from the pedals. With these inputs the desired value generator creates favored states (velocity, slip angle and yaw rate) of the vehicle. A flat feed forward controller in parallel to a combination of a PID and Linear Quadratic Gaussian (LQG) controller are calculating the desired force \( T_{\text{wheel}} \) and the desired yaw moment \( M_z \) around the vertical axis of the vehicle. The feed forward controller is used to improve the dynamics of the system. The LQG controller reduces lateral errors due to model uncertainties and parameter variations. Disturbances like side winds are canceled out with the LQG controller. The PID controller is used to cancel disturbances in longitudinal direction. In a torque distribution unit (TDU) the yaw and longitudinal commands are transferred to individual wheel torque requests \( T_{\text{wheel},i} \) for the four wheels.

2.1. Feedback control

To calculate the moment about the z-axis a linear-quadratic-Gaussian (LQG) controller [5] is used. The states \( x \) are the desired values for the slip angle \( \beta_{\text{des}} \) and the yaw rate \( \psi_{\text{des}} \). A second pair of states are the measured values for the slip angle \( \beta_{\text{meas}} \) and the yaw rate \( \psi_{\text{meas}} \). It is possible to minimize the side slip angle error \( e_\beta = \beta_{\text{des}} - \beta_{\text{meas}} \) and the yaw rate error \( e_\psi = \psi_{\text{des}} - \psi_{\text{meas}} \) while reducing the required yaw moment \( M_z \) [6]. Noise models, observer characteristics and integral behaviour can be implemented in the controller design [7]. As plant basis for the LQG controller a single track model is used. The lateral vehicle dynamics [8] are reduced to the following two equations:

\[
\dot{\beta} = \left( \frac{-a_f C_f + a_r C_r}{m v_x^2} - 1 \right) \psi - \frac{C_f + C_r}{m v_x} \beta + \frac{C_f}{m v_x} \delta
\]  \hspace{2cm} (1)

\[
\dot{\psi} = -a_f C_f \frac{1}{I_x v_x} \psi + \frac{a_r C_r - a_f C_f}{I_z} \beta
\]  \hspace{2cm} + \frac{a_f C_f}{I_z} \delta + \frac{1}{I_z} M_z
\]  \hspace{2cm} (2)

The model states are the side slip angle \( \beta \) of the vehicle and the yaw rate \( \psi \). Fixed parameters are the distance from the center of gravity (CoG) to the front axle \( a_f \) and the distance from the CoG to the rear axle \( a_r \), \( C_f \) represents the front cornering stiffness, \( C_r \) the rear cornering stiffness, \( m \) represents the mass of the vehicle and \( I_z \) the moment of inertia around the vertical axis.

For the LQG controller the model is linearised around the steering angle \( \delta \) = 0 rad. The longitudinal velocity is labeled \( v_x \) and will be set to \( v_x \) = 80 kph in order to linearise the model. \( M_z \) is the torque which is generated to turn the vehicle around the vertical axis and serves as vehicle model input.

2.2. Feed forward control

To improve the dynamics of the system a flat feed forward controller is developed. The concept of flatness [9] is useful to analyse and design linear and non-linear systems. Basically a system is called flat if all inputs \( u \) and states \( x \) can be completely described by the defined, flat outputs \( y_i \) and a number of its differentials \( y_i^{(k)} \) with respect to time. The flat outputs do not have to be the desired outputs and can be fictitious outputs without physical relevance. To determine if a system is flat it has to fulfill three conditions [10]:

1. It is possible to define the fictitious outputs \( y_i \) with \( i = 1, ..., m \) as functions of the states \( x_i, i = 1, ..., n \), inputs \( u_i \) and states \( x_i \) for a finite number of the input derivatives \( u^{(k)}_i, k = 1, ..., \alpha_i \). This can be expressed as followed:

\[
y = F(x, u_1, ..., u_{\alpha_1}, ..., u_m, ..., u_{\alpha_m})
\]  \hspace{2cm} (3)

2. The states \( x_i \) and the inputs \( u_i \) can be expressed as a function \( \Psi \) of the flat outputs \( y_i \) and a finite number of its derivatives \( y_i^{(k)}, k = 1, ..., \beta_i \) with \( \sum_{i=1}^m \beta_i \geq n \).

\[
x = \Psi_x(y_1, ..., y_{\beta_1}, ..., y_m, ..., y_{\beta_m})
\]  \hspace{2cm} (4)

\[
u = \Psi_u(y_1, ..., y_{\beta_1}, ..., y_m, ..., y_{\beta_m})
\]  \hspace{2cm} (5)

3. The number of inputs has to be the same for inputs and outputs which means that \( \dim y = \dim u \).
If a system fulfills the three conditions it is called a (differential) flat system. The flat system is used to build a flat feed forward controller. Advantages of a flat feed forward controller are:

- The stability of the closed-loop system is not influenced.
- The feedback and feed forward controller can be designed independently.
- The feed forward controller is an additional tuning possibility to change the system behaviour.
- The system dynamics are improved with a flat feed forward controller.

The first step to create a flat system is to use equations 1 and 2. However $v_1$ is a known but changing parameter. By setting $v_1$ as a state it is possible to deal with the change of this variable. So the vehicle model has to be extended by the longitudinal dynamics. With the equations of the single track vehicle on the plain $(x,y)$ it is possible to fulfill the requirements for flat systems [10].

The nonlinear but flat model has the following equations:

$$v_x = \frac{F_x}{m} + \psi \beta v_x$$  \hspace{1cm} (6)

$$\dot{\beta} = \left( -\frac{a_f C_f + a_r C_r}{m v_x^2} - 1 \right) \psi$$
$$- \frac{C_f + C_r}{m v_x} \beta + \frac{C_f}{m v_x} \delta$$  \hspace{1cm} (7)

$$\Psi = -\frac{a_f^2 C_f + a_r^2 C_r}{I_z v_x} \psi + \frac{a_r C_r - a_f C_f}{I_z} \beta$$
$$+ \frac{a_f C_f}{I_z} \delta + \frac{1}{I_z} M_z$$  \hspace{1cm} (8)

Flat systems are not unique and depend on the choice of the flat outputs. One solution to get a flat system is the following declaration: The flat outputs $y_i$ of the system are the velocity in longitudinal direction ($v_x$), the side slip angle of the vehicle ($\beta$) and the yaw rate ($\psi$). By applying the three flatness criteria [10] the flat inputs $u_i$ are the sum of the forces in longitudinal direction ($F_x$), the steering angle of the front wheels ($\delta$) and the yaw torque ($M_z$). To fulfill the first condition the following equations can be derived:

$$y_1 = x_1 = v_x$$  \hspace{1cm} (9)

$$y_2 = x_2 = \beta$$  \hspace{1cm} (10)

$$y_3 = x_3 = \psi$$  \hspace{1cm} (11)

The first flatness condition $y = F(x)$ is fulfilled. To satisfy the second condition the functions $\Psi_x$ and $\Psi_u$ have to be found. $\Psi_x$ is solved with equation 9-11. $\Psi_u$ is described with the following equations:

$$u_1 = (y_1 - y_1 y_2 y_3) m$$  \hspace{1cm} (12)

$$u_2 = \frac{1}{C_f} \left[ m y_1 y_2 + \left( \frac{a_f C_f - a_r C_r}{y_1} + m y_1 \right) y_3 \right.$$  \hspace{1cm} (13)
$$+ (C_f + C_r) y_2 \bigg]$$

$$u_3 = I_z y_3 + \left( a_f^2 C_f + a_r^2 C_r \right) \frac{y_3}{y_1}$$
$$+ \left( a_f C_f - a_r C_r \right) y_2 - a_f C_f u_2$$  \hspace{1cm} (14)

The first two conditions for flatness are satisfied. The third condition is fulfilled because the number of inputs is three which is the same for the outputs. So the nonlinear model from equation 6-8 becomes flat. Using equations 12, 13 and 14 it is possible to calculate the desired inputs for the vehicle model. The first input $F_x$ from equation 12 considers the longitudinal dynamics of the vehicle. Until now this was a feed forward controller, which is not robust against disturbances and uncertainties. To improve the longitudinal dynamics a PID controller is used. The input for the PID is the difference between the desired $v_x$ and the measured $v_x$. The output of the controller is the force in longitudinal direction $F_x$.

The second input $\delta$ is the desired angular position of the front wheels. To influence the steering angle an additional actuator like electric power steering (EPS) is mandatory. Not every TR-HEV is equipped with EPS and to realize the torque vectoring controller at every TR-HEV some modifications have to be done. The flat input $\delta$ will be neglected and equation 14 has to be modified. The calculated input $u_3$ will be replaced with the real, actual angular position of the front wheels in order to calculate $u_3$.

### 2.3. Torque distribution

From the flat feed forward, the LQG and the PID controller a force in $x$ direction $F_x$ and a yaw torque about the vertical axis $M_z$ are calculated. These two signals have to be modified to get the torque requests of the individual wheels. For this purpose the plain dynamics of a dual track vehicle model can be used. The important relations are the following two equations:

$$F_x = F_{x,FL} + F_{x,FR} + F_{x,RL} + F_{x,RR}$$  \hspace{1cm} (15)

$$M_z = \frac{w_F}{2} (F_{x,FR} - F_{x,FL})$$
$$+ \frac{w_R}{2} (F_{x,RR} - F_{x,RL})$$  \hspace{1cm} (16)
The wheel forces $F_{x,i}$ can be rewritten as follows:

$$F_{x,FL} = F_{x,F} - \Delta F_{x,F}$$  (17)  
$$F_{x,FR} = F_{x,F} + \Delta F_{x,F}$$  (18)  
$$F_{x,RL} = F_{x,R} - \Delta F_{x,R}$$  (19)  
$$F_{x,RR} = F_{x,R} + \Delta F_{x,R}$$  (20)

Two variables $\sigma$ and $\rho$ are included to relate the front and rear forces respectively. These two variables are design parameters and can be changed during the operation of the vehicle.

$$F_{x,F} = \sigma F_{x,F}$$  (21)  
$$\Delta F_{x,F} = \rho \Delta F_{x,F}$$  (22)

Rearranging equations 15 - 22 into the following form:

$$F_{x} = 2F_{x,R} \frac{1 + \sigma}{\sigma} w_F + \rho w_R$$  (23)  
$$M_{z} = \Delta F_{x,R} \rho w_F + \rho w_R$$  (24)

So the force and moment requests are routed to force requests of the axes $F_{x,F}, F_{x,R}$ and axis-force differences $\Delta F_{x,F}, \Delta F_{x,R}$. Combining these forces results in a force request for every wheel.

$$F_{x,FL} = F_{x,F} - \Delta F_{x,F} = \frac{F_{x}}{2(1 + \sigma)} - \frac{M_{z}}{w_F + \rho w_R}$$  (25)  
$$F_{x,FR} = \frac{F_{x}}{2(1 + \sigma)} + \frac{M_{z}}{w_F + \rho w_R}$$  (26)  
$$F_{x,RL} = \frac{F_{x}}{2(1 + \sigma)} - \frac{\rho M_{z}}{w_F + \rho w_R}$$  (27)  
$$F_{x,RR} = \frac{F_{x}}{2(1 + \sigma)} + \frac{\rho M_{z}}{w_F + \rho w_R}$$  (28)

With equations 25 - 28 it is possible to determine the torque request to every single wheel. The force requests have to be multiplied with the wheel radius to get the torque request for every wheel. For the ICE driven axis it is complicated and expensive to generate a torque difference at the two wheels. This can be done with differential braking [11] which reduces the speed of the vehicle or with an active differential [12]. But both options are adding costs and need additional implementation effort. When setting $\rho \to \infty$ no torque difference at the front, ICE driven wheels is required. The yaw moment will be generated with the electric driven rear wheels. The torque request can be processed fast with the electric motors and no additional devices are necessary.

The acceleration force $F_{x}$ is shifted via $\sigma$. The vehicle energy management system has to determine the value of $\sigma$ to shift between the ICE and electric propulsion motor. With varying $\sigma$ it is possible to drive purely ICE based ($\sigma = 0$) or purely electrical ($\sigma = \infty$). A combination of both like boosting ($0 < \sigma < \infty$) or recuperating ($-1 < \sigma < 0$) is also possible. For the simulation of the torque vectoring control in section 3 $\sigma$ is set to 0 and $\rho$ is set to $10^6$. This implies that the acceleration force $F_{x}$ is purely generated by the ICE driven wheels and the yaw moment $M_{z}$ is generated by the electric driven rear wheels.

### 2.4. Desired value generation

Desired values are necessary for the LQG and PID controller. Additionally, the flat feed forward controller requires the desired flat outputs $v_{x}, \beta$ and $\psi$. The velocity in longitudinal direction depends on the drivers wish, and accordingly on the acceleration and the brake pedal state. The transformation from pedal signal to a desired velocity is very complex and involves knowledge of the drivers feelings and intentions. So the transformation part will be skipped in this paper and a requested velocity will be given.

The desired side slip angle and the yaw rate depend on the lateral vehicle dynamics. The driver influences these states with the steering wheel angle. To include the vehicle dynamics equations 1 and 2 are used (setting $M_{z} = 0$). With a given steering angle and longitudinal velocity, the side slip angle and yaw rate can be calculated. With this implementation there are two possible tuning directions:

1. Use torque vectoring to extend the region of linear vehicle behaviour [13]. So the driver has an extended, known vehicle operation region and can drive the vehicle more safely to the limits of wheel adhesion. With this strategy torque vectoring is not active in the linear driving operation - where the average driver manoeuvres the vehicle for most of the time.

2. Tune the vehicle parameters ($m, l_{z}, a_{f}, a_{r}, C_{f}$ and $C_{r}$) in the desired value generation model. It is possible to set an understeering, neutral or over-steering behaviour. Or with proper parameter selection the vehicle can behave like a different vehicle. For example it is possible to have a sedan style vehicle behaving like a small, compact vehicle with reducing the mass, the moment of inertia and the distances to the front and rear axle of the desired value generator.

Besides the linear calculation of the desired side slip angle and the desired yaw rate, borders [13] for these values are included to operate the vehicle more safely.
If the vehicle leaves the regions for $\beta$ and $\psi$ the tire forces will start to saturate which can cause a loss of vehicle control. This dangerous situation will be suppressed with limiting $\beta_{des}$ and $\psi_{des}$.

3. Simulation results

For the simulation a 14 degree of freedom (DOF) vehicle model is used. The tire characteristics are calculated with a modified Dugoff model [14]. Four different vehicle configurations are compared during the simulation. The first one is a vehicle with no control of the motors at the rear axle. The second configuration controls the electric motors with the proposed controller design from section 2. The third configuration controls the yaw rate with a PID controller like in [3]. In the fourth design a PI controller is used to control the side slip angle rate as in [15].

For the desired value generation the parameter variation is empirically developed. The distance from the center of gravity to the rear axle is multiplied with a factor of 1.2 and the moment of inertia about the vertical axis $I_z$ will be multiplied with a factor of 0.6. With these changes the vehicle turns faster but has a more understeering characteristic. The desired values for the side slip angle $\beta_{des}$ and the yaw rate $\psi_{des}$ will have the same amplitude with less time delay.

As driving scenario a sinusoidal movement with delay [16] is used. The advantage of this test is the given steering trajectory and a better reproducibility compared to a double lane change test. For the first test the initial driving velocity of the vehicle is set to 80 kph and the vehicle will operate at a dry road. A sinusoidal movement will be applied to the steering wheel with amplitude 60 degree and frequency 0.7 Hz. After reaching the second extremum the steering angle will be held for 0.5 seconds before finishing the sinusoidal period. As outputs to the steering angle trajectory, the yaw rate and the side slip angle are displayed in figure 3 a-c. This maneuver leads the vehicle to a moderate nonlinear region of operation.

As a second test the steering wheel input has the same shape but an amplitude of 120 degrees and an initial velocity of 120 kph. This is a exceptional maneuver which will normally not be performed but it is used to check the vehicle behaviour in a extreme situation.

The vehicle without a torque vectoring controller follows poorly the desired values. The PID based yaw rate controller from [3] does not limit the side slip angle and the side slip angle rate based PI controller [15] has problems to follow the desired yaw rate. The flat feed forward and LQG controller is a compromise to track the yaw rate and side slip angle and has the smallest track-

![Figure 3. Simulation results for 80 and 120 kph](image_url)
Table 1. Parameters of the simulation model

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<th>Parameter</th>
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<td>$P_{eMot}$</td>
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</tr>
<tr>
<td>$\mu$</td>
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</table>

The simulation model parameters are defined as follows:
- $a_f$: distance front axle to CoG in m
- $a_r$: distance rear axle to CoG in m
- $C_f$: Cornering stiffness of the front axle in N
- $C_r$: Cornering stiffness of the rear axle in N
- $w_F$: width of the front axle in m
- $w_R$: width of the rear axle in m
- $m$: mass of the vehicle in kg
- $I_z$: moment of inertia around vertical axis in kg
- $T_{eMot}$: maximal torque of one electric motor in Nm
- $P_{eMot}$: maximal power of one electric motor in kW
- $\mu$: adhesion coefficient between road and wheel

In the extreme maneuver the proposed controller design from section 2 can not really follow the desired values because of the saturation of the wheel forces. Never the less this design has still the safest vehicle behaviour and reduces the overshoot in the yaw rate and slip angle faster than the uncontrolled, the yaw rate PID controlled and the side slip angle rate PI controller. For comparison the most important parameters of the simulation are written in table 1.

4. Conclusion

The control structure for torque vectoring in a through the road hybrid electric vehicle has been developed and was compared with other solutions [15] and [3]. The concept of a flat feed forward controller is explained and improves the dynamic of the vehicle. Disturbances and modeling errors are rejected with a PID controller for the longitudinal dynamics and with a LQG controller for the lateral vehicle dynamics. Concept for desired values and torque distribution to the wheels has been shown. Simulation results showed improvements of the vehicle behaviour if the car is equipped with torque vectoring. The next step is to include the nonlinear tire characteristics into the controller design and test the control structure for torque vector within a real vehicle.

References