CHAPTER 15

Subtle Nonlinearity in Popular Album Charts

R. ALEXANDER BENTLEY

Department of Anthropology, 1180 Observatory Drive,
University of Wisconsin, Madison, WI 53706, USA
rabentley@students.wisc.edu

HERBERT D. G. MASCHNER

Department of Anthropology, Box 8005, Idaho State University,
Pocatello, ID 83209, USA
maschner@isu.edu

Large-scale patterns of culture change may be explained by models of self organized criticality, or alternatively, by multiplicative processes. We speculate that popular album activity may be similar to critical models of extinction in that interconnected agents compete to survive within a limited space. Here we investigate whether popular music albums as listed on popular album charts display evidence of self-organized criticality, including a self-affine time series of activity and power-law distributions of lifetimes and exit activity in the chart. We find it difficult to distinguish between multiplicative growth and critical model hypotheses for these data. However, aspects of criticality may be masked by the selective sampling that a "Top 200" listing necessarily implies.

Keywords: Culture change; self-organized criticality; multiplicative process; music albums.

1. Introduction

In this paper we test for evidence of self-organized criticality in the Billboard "Top 200" album chart (hereafter referred to as the Chart). We suspected that activity on this Chart might resemble critical models of extinction in that albums, as interconnected agents, compete to survive within a limited space on the Chart. We also evaluate the data in terms of alternative models that do not invoke self-organized criticality. Evidence for or against self-organized criticality among popular albums on album charts has the potential to lead us from market or macroevolutionary dynamics toward similar phenomena in cultural evolution reflected in the arrivals of changing styles. The analysis reveals that, while self-organized critical models may explain the large scale patterns of cultural change, all components of a system must be documented for criticality to be unambiguously apparent.

Advances in Complex Systems, Vol. 2 (2000), pp. 197–208.

Here we evaluate three possible explanations for patterns in twenty years of Chart activity, self-organized criticality, multiplicative process and Poisson process. Falling within these categories are various models which have been proposed that produce the kind of statistics discussed here. Newman and Palmer [13] review such models of extinction, some of which involve criticality and some of which do not. One important model is that of Newman [10], who has proposed an evolutionary model that reproduces the power-law behavior of the B–S model without interaction between agents. Species in this model are instead subject to external environmental stresses. We do not invoke it here, however, as Newman's model relies on a rapidly decreasing distribution of external stress events (Ref. 10, p. 251), which seems an unwarranted precondition for the Chart.

2. Self-Organized Critical Models

Self-organized critical models of extinction have been used to explain power-law distributions of species' lifespans and extinction events ("avalanches") in statistical evidence from the fossil record [4, 15, 16, 18]. Debate exists as to whether self-organized criticality is necessary to explain the power-law behavior (e.g. Refs. 9 and 11), and whether some of this fossil evidence may be a statistical artifact [6, 12]. For systems with highly resolved time series, however, such as financial markets, multi-agent models suggest that power-law behavior results from the interactions between agents [3, 7, 8]. Cultural evolution, especially the comings and goings of styles, may be governed at an abstract level by algorithms similar to those appropriate to macroevolution or market activity — styles may be seen as interconnected agents whose existences depend on each other. Products of long periods of social interaction, such as the English language, are characterized by power-law statistics that may be indicative of critical systems [2, 25]. Examples are limited, however, as the qualitative nature of cultural data makes it especially difficult to obtain solid evidence for criticality in cultural evolution.

The Bak–Sneppen (B–S) generalized model of punctuated evolution [4, 15], which generates a power-law distribution of "species" lifetimes and avalanche sizes, is a good test model for pop Chart activity because both the model and the Chart have a fixed number of positions for which interconnected agents compete. In the B–S model, species are arranged in a lattice, such that each species interacts with its nearest neighbors. Each species can be represented by a value between 0 and 1, chosen at random to start the simulation. These values can be taken to represent species' fitnesses (see Ref. 13). At each time step, the lowest fitness value in the lattice is selected, and the mutation is represented by the assignment of a new random value between 0 and 1. The nearest neighbors are also randomly assigned new fitness values in order to model abstractly their "adaptation" to the new fitness landscape created by the extinction of the species near them in the lattice. As the fitnesses in the lattice evolve to a state where they are all above a certain "gap" value, the system becomes poised for an avalanche. Avalanches occur when a value above the gap and its neighbors are replaced by random values. Since these new random replacements are likely to fall below the gap value, one of them is likely to be selected in the next time step for replacement, and subsequent time steps, until finally all values are above the gap value and the avalanche ends.

For pop albums, an analog to the absolute fitness values of the B–S model is not identifiable, since peak Chart position or even sales figures are dependent on time and the environment (market) and therefore are measures of relative fitness. This will force us (below) to define an avalanche differently from the B–S model, in which avalanches are defined as the total activity occurring between periods of all fitnesses above a certain threshold [14], with no other constraint on the duration of those periods. Self-organized critical extinction models are characterized by (a) a power-law distribution in the sizes of coevolutionary avalanches, (b) a power-law distribution of the lifespans of agents, and (c) self-affine time series of events characterized by a $1/f^D$ power spectrum [2]. The power-law exponent varies between models. For the B–S model, that power-law has an exponent between 1.0 and 1.5 [4]. In the model of Sole and Manrubia [17], in which interactions between species have variable strengths and signs (they can be beneficial or harmful relationships), the exponent is 2.3 ± 0.1.

Other processes may generate power-law statistics, but one important implication of self-organized criticality is that the interdependencies between agents generate the "fat tail" of the avalanche size and lifespan distributions. Our test for self-organized criticality is motivated by our belief that such interdependencies are present between albums in the Chart. For example, consider an album like Pink Floyd's *Dark Side of the Moon*, which had been on the Chart for a remarkable 566 weeks in a row by 1985. The staying power of such an album is best explained by its interaction with subsequent "spin-off" performers and albums. Acts that were inspired by *Dark Side of the Moon*, though they may themselves have been short-lived, lent sales support by association to the album that inspired them.

3. Multiplicative Processes

As an alternative to self-organized criticality, the activity of the Chart might be describable as a result of multiplicative processes. Multiplicative processes can generate power-law distributions in statistics such as the dynamics of the growth of units of complex internal structure [1]. As discussed below, a log-normal distribution of avalanche sizes or album lifespans would rule out self-organized criticality in favor of a mutiplicative process. The log-normal distribution is given by

$$f(x) = \frac{A}{x(2\pi)^{1/2}} \cdot \exp\left(-\frac{(\ln x - \bar{y})^2}{2\sigma_y^2}\right), \tag{1}$$

where A is a constant, $x \geq 0$ and $y = \ln(x)$. The values of y are normally distributed with a mean $\bar{y}$ and standard deviation σ_y. A log-normal distribution of album lifespans, for example, can be modeled by assuming that each album achieves a certain longevity on the Chart through having a certain number of attributes, each with an independent probability of being adequate to acheive that lifespan (see Ref. 23). The lifespan of the album would then be proportional to the product of the probabilities of each attribute, leading to a log-normal distribution of lifetimes. Such attributes might include, for instance, originality (or conformity), meaningful lyrics, charismatic performers, advertising, etc. Even with systems characterized by power-law statistics, self-organized criticality is not a foregone conclusion. There are many random growth processes that generate power-law distributions without invoking criticality [19]. Takayasu *et al.* [21] and Sornette [20], for example, have shown that random multiplicative growth processes have a power-law probability density function (PDF) for large x:

$$P(x) = Cx^{-1-\mu}\,, \tag{2}$$

where μ is a constant. The value of μ depends on the particular multiplicative process [20]. We discuss whether multiplicative processes may generate power-law distributions in the Chart data in Sec. 5.

4. Poisson Process

A Poisson process in Chart activity would be characterized by simple exponential distributions. With avalanche events, for example, if the probability that the activity is above the threshold during any given week is p, and the data are not correlated through time, then the expected number of avalanches of size Cx scales as

$$y = (N - x + 1) \cdot p^x\,, \tag{3}$$

where N is the total number of time steps in the time series, and the avalanche size Cx implies that x steps of above-threshold activity with average rate C occurred in a row. This is essentially a simple exponential relationship when N is large compared to x. A similar explanation can be made for an exponential distribution of album lifespans by letting p be the probability that a species will survive to the following time step.

5. Analysis of the Top 200 Album Chart

We analyzed data compiled by Whitburn [24] for 12,005 albums from the Billboard Album pop Chart from 1963–1985, which include the date each album entered the Chart, its peak position, and the number of weeks it stayed on the Chart. The Chart has had a variable size over the years, from 15 positions in 1955 to separate stereo and mono charts from 1959–1963, to 150 positions from 1963–1967; it has been fixed at 200 albums since 1967. Details are explained in Ref. 24. To avoid large variations in activity during the times when the Chart was small and erratic in size from week to week, we chose to consider the Chart activity beginning in 1963, by which time the Chart size had become consistent. We then normalized activity in the Chart by the number of listed albums in the Chart at a given time.

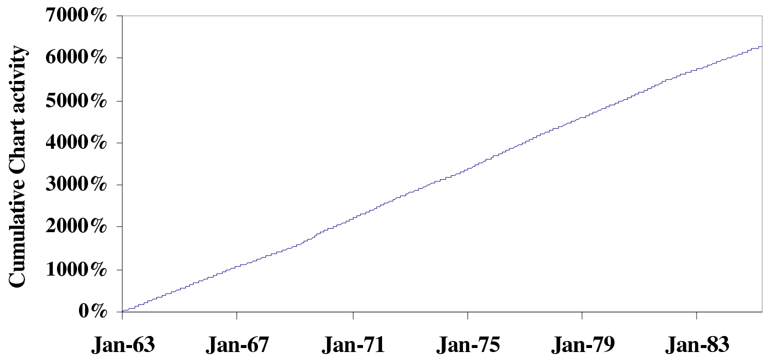


Fig. 1. The cumulative fraction of the total number of albums on the Chart that exited the Billboard pop Chart, January 5, 1963 to March 31, 1985 (1162 weeks), was determined in terms of the fraction of albums on the Chart exiting the Chart each week. The turnover rate during this period was remarkably consistent at about 5.6% per week.

The time series of cumulative exiting-album activity shows that, for over twenty years, the Chart underwent a remarkably consistent turnover rate of about 5.6% per week (Fig. 1). This is equivalent to a dominant lifespan of 10–20 weeks, which might correspond to the promotional period of new albums by the record industry. Suspecting that more interesting behavior among longer-lived albums may be hidden by the consistent turnover rate, we focus on deviations from the trend. Figure 2 shows the residual activity on the pop Chart after the linear trend of 5.6% per week has been removed.

In the next three subsections we examine the three characteristics of self organized criticality mentioned in Sec. 2, with respect to the Chart data.

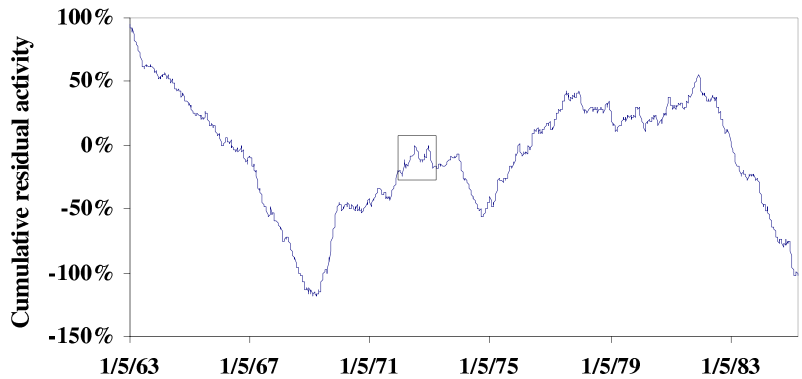
5.1. *Self-affinity*

Visual inspection of Figs. 2(a) and 2(b) suggests that the residual activity may be self-affine, as the "blow-up" of the activity in 1972 shows a pattern similar in relative scale to that of the full time series.

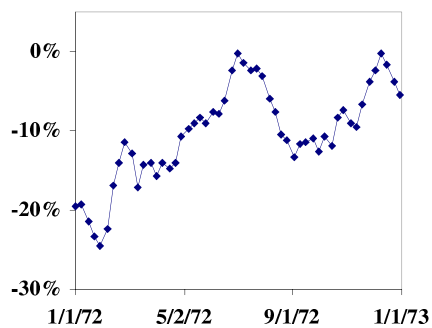
The self-affine nature of the residual series, $\xi(t)$, was tested by examining its Fourier spectrum. A fast Fourier-transform (FFT) was performed on $\xi(t)$ over $N =$ 1162 weeks. In the FFT, the magnitude $P(f)$ is

$$P(f) = \frac{1}{\sqrt{N}} \sum_{\tau=1}^{N} C(\tau) \exp\left(\frac{-i2\pi f \tau}{N}\right), \tag{4}$$

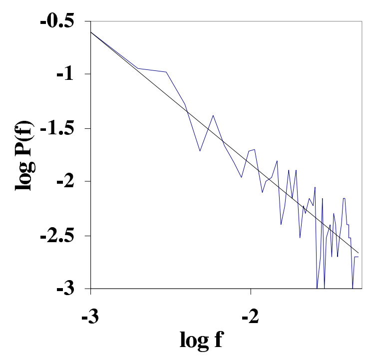
where $C(\tau) = \langle \xi(t)\xi(t+\tau) \rangle$ is the autocorrelation function, and the integral of $P(f)$ over all frequencies is one. The frequency f is in terms of the temporal unit t of the time series, so in this case f is in cycles/week. We find a $1/f^D$ power spectrum with $D = 1.23 \pm 0.09$. This indicates that the time series of cumulative residual activity is self-affine (Ref. 22, p. 148). This self-affinity could mean that the pop Chart activity may consist of two components, a fairly steady turnover



(a)



(b)



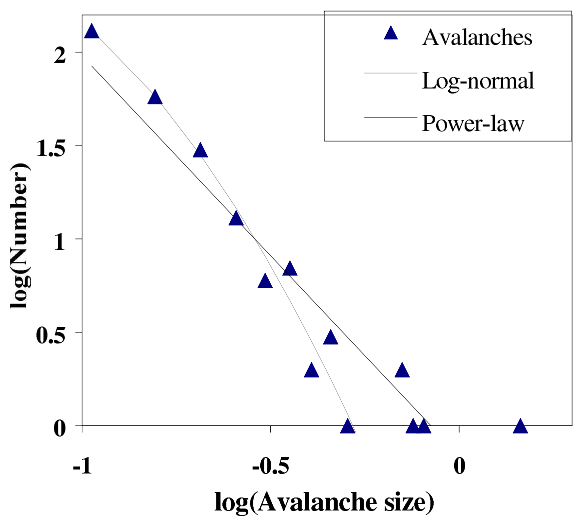
(c)

Fig. 2. Cumulative residual activity with linear trend removed. (a) Cumulative residual activity series after a linear trend of 5.6% per week was removed. (b) A "blow-up" of the time series in (a), showing the weekly data for 1972. (c) Power spectrum of the series in (a), with a least-squared-fitted slope of -1.23 ± 0.09. The three figures show that self-similarity and a $1/f^D$ power spectrum become apparent in the cumulative activity after a linear trend is removed.

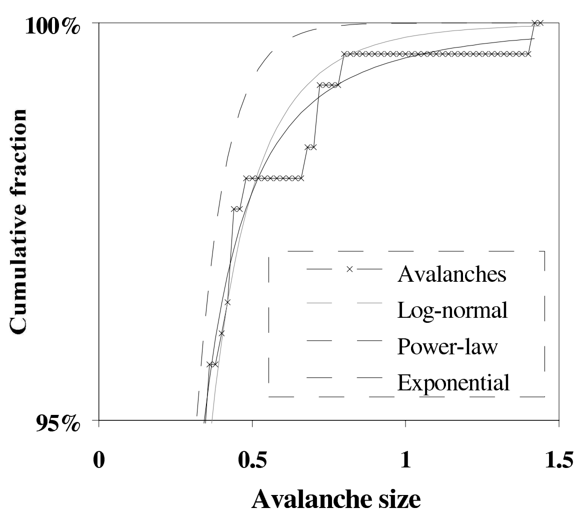
component averaging 5.6% per week and a residual component with characteristics of self-organized criticality. Alternatively, the power spectrum could be generated by multiplicative processes rather than critical phenomena (Ref. 23, pp. 156–165). We discuss this consideration below.

5.2. *Avalanches of Chart activity*

For our analysis, we define an avalanche as an event during which the rate of activity was sustained above a chosen threshold over a succession of consecutive time intervals (weeks), and the size of the avalanche as the total number of albums that exited on the Chart during that period. Avalanches defined in this way vary in temporal duration, which is similar to B–S model avalanches. With the pop Chart activity, the threshold was chosen to be the average turnover rate of 5.6%



(a)



(b)

Fig. 3. Avalanche events in the Billboard pop Chart, January 5, 1963 to March 31, 1985, defined as periods of activity sustained above the average exit rate of 5.6% of the Chart per week. Avalanche size is expressed as proportion of the total number of albums on the Chart at one time. (a) Avalanche sizes shown in a regular histogram. The bin size for the avalanche size is a proportion of 0.05. For the power-law fit, the slope is -2.14 ± 0.25, $r^2 = 0.873$. The power-law fit is significantly better than for the exponential relationship (not shown), for which $r^2 = 0.569$. (b) Cumulative plot of avalanches, showing the largest 5% of avalanche sizes, to distinguish between log-normal and power-law distributions. All of the shown cumulative functions fit the avalanche distribution up to about 95%, and differences between possible distribution functions are only apparent for large avalanches. The power law may be the closest to the distribution of the few largest avalanches.

Table 1. Effect of threshold on avalanche distribution. The power law slope in the avalanche distribution depends on the threshold (as a portion of the Chart exiting per week) used to define avalanche events. The threshold also affects the correlation coefficients for power law and exponential fits to the distribution. As the threshold moves outside the range of 4%–7%, the waning number of avalanches makes it difficult to produce a good histogram of avalanche sizes.

Threshold	Avalanches	Power law slope	R^2 for log-log plot	R^2 for semilog plot
4%	179	-1.34 ± 0.13	0.820	0.648
4.5%	210	-1.64 ± 0.16	0.832	0.581
5%	248	-2.03 ± 0.18	0.883	0.664
5.5%	256	-2.20 ± 0.26	0.860	0.547
5.6%	256	-2.14 ± 0.25	0.873	0.569
6%	232	-2.48 ± 0.19	0.947	0.772
6.5%	199	-2.70 ± 0.36	0.877	0.660
7%	155	-2.65 ± 0.76	0.844	0.656

per week. The result is a distribution of avalanches with an extended tail, possibly a power law (Fig. 3). Power-law and log-normal functions both fit considerably better than an exponential function to avalanche distribution. This is significant because avalanches defined this way for random noise should be exponentially distributed. As a test, we looked at the distribution of avalanches (using 5.6% threshold) for the values of the weekly time series in randomly shuffled order. The result was an exponential distribution resembling equation (3). For the real data, the avalanche distribution is either a power-law or log-normal with significantly more large events, either of which is definitely distinguishable from an exponential distribution. As Fig. 3 shows, it is hard to tell whether the distribution is log-normal or a power law.

We noticed that the extended tail in the distribution of avalanche sizes occurred for a range of different thresholds, with slopes becoming steeper as the threshold is increased from 4% to 7% per week (Table 1). The relationship in all cases within this range is better fitted with a power-law (Table 1, log-log plot) or log-normal than with an exponential function (Table 1, semilog plot), which is the expected result for a random time series. Outside this range, there are not enough avalanches for a good histogram because either there are only a few mostly small avalanches or a few very large avalanches. We want to make the point that the slope of the power-law distribution of avalanche size is highly dependent on the choice of threshold value, as the average avalanche size increases as the threshold is lowered (Table 1). There is a qualitative similarity in this effect to the B–S model, in which the average avalanche size increases as the fitness threshold is brought closer to its critical value [14]. Nonetheless, avalanche sizes on the Chart are clearly either power-law or log-normally distributed for a range of thresholds.

5.3. *Album lifespans*

We now turn to distribution of album lifespans on the Chart (Fig. 4). As Fig. 4 shows, however, both the log-normal and power law functions fit the data reasonably

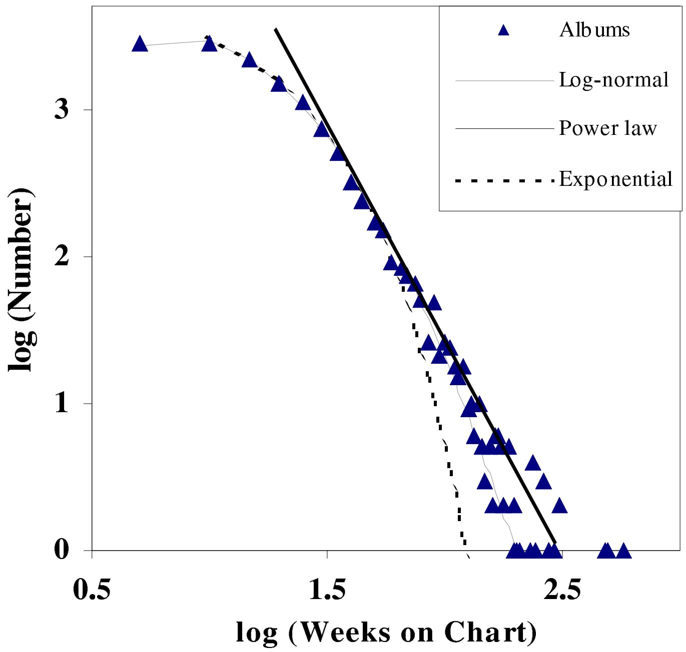


Fig. 4. Lifespans of albums on the Billboard Chart, 1963–1985. The bin size is five albums. Beyond 20 weeks, the slope converges asymptotically to -2.70 ± 0.12, $r^2 = 0.913$. The power-law fit for albums living 20 weeks or more is better than for the exponential relationship, but the log-normal distribution fits the entire distribution.

well. The difference between these two is rather important, because while neither distribution requires interconnections between albums [13], a log-normal distribution suggests this is not a critical system at all. While only log-normal fits the left side of the distribution, it is the proper form of the tail that we are most concerned with. An indication that albums on the pop Chart interact in a critical system would be a power-law distribution of lifespans analogous to the species lifespans in self-organized critical models of evolution. If the distribution of album lifespans in Fig. 4 indeed has a power-law tail, its exponent is 2.70 ± 0.12, which is higher than that of the B–S one-dimensional and two-dimensional models, with exponents of 1.58 and 1.28, respectively (Ref. 14, Fig. 30). The plateau in the lifespan distribution up to 20 weeks is due to the dominance of the 5.6% per week rate of turnover in the Chart, the reciprocal of which is an expected lifespan of about 18 weeks. However, it is the longer lifespans that interest us here, since in a non-critical system, we would expect the distribution to fall off as an exponential function rather than the existing power-law. A power-law tail in the lifespan distribution would be additional evidence for two components of Chart activity, with the effect of a steady turnover giving way to a critical-type power-law in the longer lifespans.

We find it unlikely that multiplicative processes would produce a power-law distribution of album lifespans. Assuming that an album's position on the Chart is a function of weekly sales rate, it might be that the sales rate for each album, as a stochastic multiplicative process, could generate a power-law distribution of album lifespans on the Chart. For an album to be on the Chart on a given week, its sales must be above the sales rate, x_{201}, of the 201^st album tracked by Billboard. Supposing that sales are distributed in the power-law form (2), the probability that an album's sales are above this threshold in a given week is

$$P(x > x_{201}) = \int_{x_{201}}^{\infty} \frac{C}{x^{1+\mu}} dx = C x_{201} \,. \tag{5}$$

The probability that it remains on the Chart for n weeks in a row is then

$$P(n) = C x_{201}^{-n\mu} \,, \tag{6}$$

which is an exponential with respect to n. The value of x_{201} varies week to week, but unless x_{201} were to decrease with time, Eq. (4) does not resemble a power-law with respect to n.

In other words, even if the power-law PDF (5) were the probability of sales for a given week, it would not explain a power-law distribution of album lifetimes, which are the number of weeks *in a row* above a threshold sales rate. Thus any random process that might produce a power-law PDF for album sales still does not explain the power-law tail of album lifespans, unless x_{201} becomes smaller with time over the lifetime of an album, which of course does not make sense.

6. Discussion

Despite the good fit of the log-normal function to the lifespan distribution, we do not abandon our suspicion that popular album choice may be a self-organized critical phenomenon, due to the ubiquitous interdependencies in popular music culture, especially in the form of imitation. It may be that evidence for criticality is obscured by the nature of the data. This is because the Top 200 by definition is a limited representative of the entire system. This can be illustrated by considering the Top 200 as a real-life B–S model. In the B–S model, the critical state is reached when the species in the system all take on fitness values above a critical threshold, which happens to be 0.67 for the one-dimensional model [14]. Avalanches are triggered by species with random fitnesses, likely to be lower than the threshold, replacing high-fitness species and their neighbors, and thus being likely to be reselected for replacement in the subsequent time step. The Chart, comprising the top 200 best-selling albums, is filled with fit species like the B–S model in the critical state. However, the fitness of new albums introduced to the Chart is anything but random. Albums are momentarily very fit when they enter the Chart. Best-selling albums are replaced by other best-selling, highly fit albums. In other words, observation limited to the 200 best albums at any time eliminates the fundamental generator of avalanches and power-law lifespans. Unlike species in the B-S model, the newest albums on the Chart are *not* necessarily the most likely to be replaced the following week.

This leads to an important point. Criticality may only be visible with data that include all the members of the system, all the way down to the most unfit competitors. Within a ranking of, say, the top 10,000 albums, there would be masses of unsuccessful albums making the bottom levels of such a chart for momentary periods. There would also be more long-lived albums with sales below Top 200 levels but in the Top 10,000 for a long time. This is speculation, of course. The main point is that a continuously-updated ranking of the topmost echelon of a competitive hierarchy is unlikely to exhibit a power-law distribution of lifespans, even for a self-organized critical system. It is likely that the variance of album sales is highly dependent on sales themselves, perhaps analogous to the way the standard deviation of the growth rates of firms has an inverse-power law dependence on firm size [1]. In other words, to see the full range of variance, the smallest competitors must be included in the data.

Finally, despite our adherence to the hypothesis of self-organized criticality, the evaluation of log-normal fits in addition to possible power-laws has been instructive. We note that data similar to ours have been described as power-law in form without reference to the possibility of log-normality. For example, Barabasi and Albert (Ref. 5, Fig. 1(a)) describe the collaboration graph of movie actors as having a power-law tail, yet the distribution looks log-normal.

7. Conclusion

By analogy with the B–S model, we speculated that the pop Chart has a critical nature because albums contemporary with one another may be interconnected in such a way that the demise or success of one can affect others in the Chart. However, since the Chart draws the best-selling albums in all genres, interconnections are not prevalent and evidence of self-organized criticality is subtle at best. An important lesson from this study may be that the data must include the least successful albums if criticality is to become apparent. Potential interconnections might be highlighted by isolating a single music genre or record label, and isolating the effect of such relationships is the goal of further research.

The implications of finding evidence for or against criticality on the Billboard Album pop Chart are significant for future behavioral and cultural studies. Social scientists are beginning to recognize that there are organizational principles in human socioeconomic systems that underlie both interactions between individuals and complex, macro-regional networks. How popular music choices evolve has bearing on how we look at oral traditions, fashion, bestseller book lists, ancient pottery styles, religion, and many other competitive categories of modern and past social systems. For archaeology, for example, our results present a challenge due to the difficulty in seeing all the artifact styles that were not successful and are sparsely represented in excavations.

References

[1] Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Salinger, M. A. and Stanley, H. E., Power law scaling for a system of interacting units with complex internal structure, *Phyl. Rev. Lett.* **80**, 1385–1388 (1998).

[2] Bak, P., *How Nature Works: The Science of Self-Organized Criticality* (Springer, New York, 1996).

[3] Bak, P., Paczuski, M. and Shubik, M., Price variations in a stock market with many agents, *Physica* **A246**, 430–453 (1997).

[4] Bak, P. and Sneppen, K., Punctuated equillibrium and criticality in a simple model of evolution, *Phyl. Rev. Lett.* **71**, 4083–4086 (1993).

[5] Barabasi, A.-L. and Albert, R., Emergence of scaling in random networks, *Science* **286**, 509–512 (1999).

[6] Kirchner, J. W. and Weil, A., No fractals in fossil extinction statistics, *Nature* **395**, 337–338 (1998).

[7] Lux, T. and Marchesi, M., Scaling and criticality in a stochastic multi-agent model of a financial market, *Nature* **397**, 498–500 (1999).

[8] Mantegna, R. N. and Stanley, H. E., Scaling behaviour in the dynamics of an economic index, *Nature* **376**, 46–49 (1995).

[9] Newman, M. E. J., Self-organized criticality, evolution, and the fossil extinction record, *Proc. Roy. Soc. London* **B263**, 1605–1610 (1996).

[10] Newman, M. E. J., A model of mass extinction, *Journal of Theoretical Biology* **189**, 235–252 (1997a).

[11] Newman, M. E. J., Evidence for self-organized criticality in evolution, *Physica* **D107**, 293–296 (1997b).

[12] Newman, M. E. J. and Eble, G. J., Power spectra of extinction in the fossil record, *Proc. Roy. Soc. London* **B266**, 1267–1270 (1999).

[13] Newman, M. E. J. and Palmer, R. G., Models of extinction: a review, *Philosophical Transactions of the Royal Society B* (1999) submitted. Electronic pre-print, LANL archive, adap-org/9908002.

[14] Paczuski, M., Maslov, S. and Bak, P., Avalanche dynamics in evolution, growth, and depinning models, *Physical Review* **E53**, 414–443 (1996).

[15] Sneppen, K., Bak, P., Flyvbjerg, H. and Jensen, M. H., Evolution as a self-organized critical phenomenon, *Proc. Natl. Acad. Sci. USA* **92**, 5209–5213 (1995).

[16] Sole, R. V. and Bascompte, J., Are critical phenomena relevant to large-scale evolution? *Proc. Roy. Soc. London* **B263**, 161–168 (1996).

[17] Sole, R. V. and Manubria, S. C., Extinction and self-organized criticality in a model of large-scale evolution, *Physical Review* **E54**, R42–R45 (1996).

[18] Sole, R. V., Manrubia, S. C., Benton, M. and Bak, P., Self-similarity of extinction statistics in the fossil record, *Nature* **388**, 764–767 (1997).

[19] Sornette, D., Linear stochastic dynamics with nonlinear fractal properties, *Physica* **A250**, 295–314 (1998a).

[20] Sornette, D., Multiplicative processes and power laws, *Physical Review* **E57**, 4811–4813 (1998b).

[21] Takayasu, H., Sato, A. H. and Takayasu, M., Stable infinite variance fluctuations in randomly amplified Langevin systems, *Physical Review Letters* **79**, 966–969 (1997).

[22] Turcotte, D. L., *Fractals and Chaos in Geology and Geophysics* (Cambridge University Press, 1997).

[23] West, B. J. and Deering, B., *The Lure of Modern Science: Fractal Thinking* (World Scientific, Singapore, 1995).

[24] Whitburn, J., *Joel Whitburn's Top Pop Albums, 1955–1985* (Record Research, Menomonee Falls, WI, 1985).

[25] Zipf, G. K., *Human Behavior and the Principle of Least Effort* (Addison-Wesley, Cambridge MA, 1949).