Parameter identification of a discretized biased noisy sinusoidal signal

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\textbf{Abstract}

A discrete time, on-line approach is proposed for the simultaneous identification of the frequency, the amplitude and the phase parameters in continuous noisy sinusoidal signals. The proposed technique takes the exact discretization of a continuous sinusoidal signal and generates exact computation formulas for the frequency, bias, amplitude and phase parameters, calculated in terms of quotients involving functions of the output and of some of its delayed values. The formula yields exact parameter calculations in the noiseless case, and it can be synthesized by means of time-varying filters whose outputs are obtained online. When the treated signal is affected by additive noises, a least squares-based “invariant filtering” is proposed, resulting in an enhancement of the signal to noise ratio of the factors conforming the basic algorithm. The scheme is proven and analysed by means of a laboratory experiment, a noise analysis and a comparison against other existing methods, where the proposed method shows accurate results with fast convergence rates.

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\textbf{1. Introduction}

The problem of parameter estimation in sinusoidal waves has attracted much attention due to its importance in theoretical studies and real-world applications, from areas as civil engineering \cite{1,2}, robotics and automatic control \cite{3,4}, bio-medics \cite{5,6}, signal processing and power systems \cite{7,8} among others. It should be emphasized that numerous approaches have been developed for frequency estimation in sinusoidal signals ranging from Time domain-based schemes, Kalman filtering to nonlinear adaptive estimation (see \cite{9–11}). Some works include innovative theoretical comparisons with traditional techniques (see \cite{5}). A rather complete survey examining different approaches and performance of the algebraic methods with many other previously proposed approaches can be found in the articles by Trapero et al. \cite{12,13}.

Fast (non-asymptotic) continuous time algorithm for algebraic identification of the frequency parameter in continuous sinusoidal signals have been successfully tested, and compared, among other competitive approaches in \cite{13,14}. The algebraic approach considers the problem as an exact computation formula for the unknown parameters when interpreted in a noiseless continuous time-domain environment. When suitably combined with classical low-pass filters in an “invariant manner” an enhancement is obtained of the quality of the algorithm response against additive measurement noises. This approach achieves good results when the algorithm is provided by a sufficient quantity of signal samples. However, the case of parameter...
identification with a poor quantity of supplied samples implies a different treatment, considering a discrete time case. The advantage of sinusoidal waves is the fact that the function describing their behaviour in discrete time (without bias) is given in terms of an exact second order difference equation, in which the discrete-time equivalent identification approach can be implemented, and the external facts as the constant bias can be solved by means of algebraic manipulations.

In this article, a discrete-time fast identification approach for continuous signals is developed and tested in a laboratory experiment dealing with a test sinusoidal signal. The methodology can be applied in hardware restriction processes where a fast frequency computation is required. Concretely, this paper deals with the problem of estimating the angular frequency \( \omega \), the bias parameter \( b \), the amplitude \( A \), and the phase \( \phi \) of a sinusoidal signal of the form \( x(t) = A \sin(\omega t + \phi) + b + \xi(t) \), \( \xi(t) \) an additive noisy signal, by means of exact discretization. This scheme allows us to transform a continuous sinusoidal signal into a third order difference equation. The algebraic estimation approach is applied on the difference equation and by means of algebraic manipulations the parameters of the signal are obtained as a function in terms of the output and of its immediate delayed terms. This methodology is based on the fast identification method of discrete-time linear systems used in [15] for the area of servo-vision control.

Some numerical simulations were done to test the methodology behaviour under different noisy signals, and a comparison test was carried out to compare this technique with another fast estimation method. A laboratory experiment is carried out for the frequency estimation, under noisy measurements, of a sampled sinusoidal signal. The proposed approach is suitably combined with invariant filtering aimed at reducing the signal to noise ratio. The bias, amplitude and the phase parameters are simultaneously calculated once the angular frequency is accurately determined.

The paper is organized as follows: In Section 2, the difference equation of a noiseless biased sinusoidal signal is obtained, and the identification approach is proposed for the noiseless case. Sections 3 and 4 consider the case of a biased and noisy signal case, proposing the invariant filtering approaches for the parameter identification process, as well as the procedure for the amplitude and phase parameters identification. In Section 5, the methodology is illustrated with some numerical simulations; to set the efficiency of the approach in different circumstances, this section tests the identification method with different signal to noise ratios and sampling frequencies, using a percentage error index. Section 6 deals with some comparisons with other fast and effective classic discrete time frequency identification procedures. Section 7 shows some experimental results and, finally, some conclusions and suggestions for further work are collected in Section 8.

### 2. Difference equation of a single sinusoidal wave

Let \( x(t) \) be a continuous sinusoidal signal of the form
\[
x(t) = A \sin(\omega t + \phi)
\]
where \( A \in \mathbb{R} \), is the amplitude, \( \phi \in \mathbb{R} \), is a constant phase component and \( \omega \in \mathbb{R} \) denotes the angular frequency of \( x(t) \). This continuous signal is sampled with a period \( T \) yielding the following expression:
\[
x(kT) = x(t)|_{t=kT} = A \sin(\omega kT + \phi)
\]

By applying well known trigonometrical identities we obtain:
\[
x(kT - 2T) = A \sin(\omega kT - \omega T - \omega T + \phi)
\]
\[
= \cos(\omega T)x(kT - T) - A \sin(\omega T) \cos(\omega kT - T + \phi)
\]

Using the fact that \( \cos(\theta_1) \sin(\theta_2) = 1/2(\sin(\theta_2 + \theta_1) + \sin(\theta_2 - \theta_1)) \) and substituting in (2), we obtain after simple manipulations:
\[
x(kT) = 2 \cos(\omega T)x(kT - T) - x(kT - 2T)
\]

where (3) is a difference equation which expresses (1) in an exact discretization form. For the sake of simplicity, define \( \mu = \cos(\omega T) \). Thus, the estimation of \( \mu \) is expressed as
\[
\hat{\mu}(kT) = \frac{x(kT - 2T) + x(kT)}{2x(kT - T)} = \frac{n_{00}(kT)}{d_{00}(kT)}
\]
and
\[
\hat{\omega}(kT) = \frac{1}{T} \arccos \left( \frac{n_{00}(kT)}{d_{00}(kT)} \right)
\]

That is, the value of \( \omega \) is directly obtained by some measurements of the output signal and its delayed terms.

### 3. A biased and sinusoidal signal

Consider now a biased sinusoidal signal with an exact discretization process:
\[
x_0(kT) = A \sin(\omega kT + \phi) + b
\]
with \( b \in \mathbb{R} \) being an additive offset component. \( x(kT) \) can be written as follows:
\[
x(kT) = x_0(kT) - b
\]

Using the relation between \( x \) and \( x_0 \) in (3) leads to the following expression:
\[
2\mu_0(x_0(kT - T)) = x_0(kT - 2T) + x_0(kT) - 2b(1 - \mu)
\]

Applying a time-shift operation in (5) and subtracting to eliminate the constant term we have:
\[
2\mu_0(x_0(kT - T) - x_0(kT - 2T)) = x_0(kT) - x_0(kT - T) + x_0(kT - 2T) - x_0(kT - 3T)
\]

The frequency can be then estimated as follows:
\[
\hat{\mu} = \frac{1}{2} \frac{x_0(kT) - x_0(kT - T) + x_0(kT - 2T) - x_0(kT - 3T)}{x_0(kT - T) - x_0(kT - 2T)}
\]
\[
= \frac{n_{00}(kT)}{d_{00}(kT)}
\]
i.e.,
\[
\hat{\omega}(kT) = \frac{1}{T} \arccos \left( \frac{n_{00}(kT)}{d_{00}(kT)} \right)
\]
Therefore, an exact frequency estimation of the biased sinusoidal signal can be obtained after three sampling times.

3.1. Bias, amplitude and phase identification

Assuming that the frequency component is known, then the bias parameter can be calculated from (5):

$$\dot{b}(kT) = \frac{x_0(kT) - 2Ax_0(kT - T) + x_0(kT - 2T)}{2(1 - \mu)}$$ (7)

Now, supposing that the parameters $\mu$, $b$ are identified, let us construct the following equation system using (4) and a time shift operation in (4). That is

$$x_0(kT) - b = A\cos(\phi)\sin(\omega kT) + A\sin(\phi)\cos(\omega kT)$$
$$x_0(kT - T) - b = A\cos(\phi)\sin(\omega kT - \omega T) + A\sin(\phi)\cos(\omega kT - \omega T)$$

solving for $A\cos(\phi)$, $A\sin(\phi)$, we have:

$$A\cos(\phi) = \frac{|x_0(kT) - b|\cos(\omega kT) - |x_0(kT - T) - b|\cos(\omega kT)}{\sin(\omega kT)\cos(\omega kT) - \sin(\omega(kT - T))\cos(\omega kT)}$$
$$A\sin(\phi) = \frac{|x_0(kT - T) - b|\sin(\omega kT) - |x_0(kT) - b|\sin(\omega(kT - T))}{\sin(\omega(kT - T))\cos(\omega kT) - \sin(\omega(kT - T))\cos(\omega(kT - T))}$$

$$A = \sqrt{(A\sin(\phi))^2 + (A\cos(\phi))^2}$$

$$\phi = \arctan 2(A\cos(\phi), A\sin(\phi))$$

where $\arctan 2(A\cos(\phi), A\sin(\phi))$ represents the two quadrant arc tangent function.

4. Parameter identification under additive noise

Under noiseless conditions, the proposed formulas have an ideal behaviour which is not presented in a practical framework where additive noises may arise. Notice that the lumped parameters $\mu$, $b$, $A\cos(\phi)$, $A\sin(\phi)$ are in terms of a quotient of time functions; in other words, any of the last parameters can be expressed as:

$$\alpha = \frac{n_x(kT)}{d_x(kT)}$$ (12)

where $\alpha$ is the parameter to be estimated. Under noisy measurements, we assume that $n_x(kT)$, $d_x(kT)$ are affected by an additive zero mean random signal with unknown statistical parameters. Since the proposed identification formulas require the accurate correspondence of the derived model with the measured data, the process of parameter estimation under noisy conditions can produce poor results. Then, to improve the identification scheme it is necessary to enhance the Signal to Noise Ratio (SNR) of the estimator (12) by means of filtering strategies. A very effective methodology is the “invariant filtering approach”, introduced in [16]. In this scheme, two low pass filters are used (one for the numerator signal while the other is applied to the denominator signal). In this form, the invariant effect does not affect the estimated value while the noise effects are reduced. The nature of the invariant filtering, as proposed, is purely linear. To enhance the filtering schemes, in this case, it is proposed a nonlinear invariant filtering method, this method will be analysed in the remainder of this section.

Consider the expression (12), it can be rewritten as follows:

$$d_x(kT)\alpha = n_x(kT)$$ (13)

Due to the noisy measurements, there exists an equation error, denoted by $\varepsilon(kT)$, such that (13) takes the form:

$$d_x(kT)\tilde{\alpha} = n_x(kT) + \varepsilon(kT)$$ (14)

where $\tilde{\alpha}$ is the estimate value of $\alpha$.

For the minimization of the equation error, we propose the following cost function:

$$J(\tilde{\alpha}, kT) = \frac{1}{2}\sum_{i=0}^{k} e^2(iT)$$ (15)

Taking the partial derivative with respect to $\tilde{\alpha}$ in (15) to minimize $J$, yields

$$\frac{\partial J}{\partial \tilde{\alpha}} = \frac{1}{2}\sum_{i=0}^{k} e(iT) = \frac{1}{2}\sum_{i=0}^{k} \frac{\partial^2 e^2(iT)}{\partial \tilde{\alpha}^2} = \frac{1}{2}\sum_{i=0}^{k} (d_x(iT)\tilde{\alpha} - n_x(iT))d_x(iT)$$ (16)

Given that $J(\tilde{\alpha}, kT)$ is a convex function [17], then it has a global minimum that satisfies: $\frac{\partial J(\tilde{\alpha}, kT)}{\partial \tilde{\alpha}} = 0$ for all $kT$.

Thus $\tilde{\alpha}$ is expressed as:

$$\tilde{\alpha}(kT) = \frac{\sum_{i=0}^{k} n_x(iT)d_x(iT)}{\sum_{i=0}^{k} d_x^2(iT)}$$ (17)

**Remark.** Since there is a lack of complete measurements for the first three sampling times, the above formula is valid from the fourth sample (as said in the identification formula). Then, it is necessary to wait this amount of time to start the filtering process.

In presence of additive measurement noises, the invariant filtering enhances the SNR in both the numerator and the denominator signals, increasing the numerical conditioning of the parameter estimation formula. Moreover, this methodology eliminates singularities in the denominator signal due to the zero-crossing phenomenon.

**Remark.** The uniform filtering of numerator and denominator was formerly introduced as “invariant filtering” by Reger et al. [16].

**Remark.** The proposed nonlinear filtering can be modified, without affecting its invariance property, by means of some nested additional summations, simultaneously applied in both numerator and denominator signals. This can be viewed as an extra low pass filtering process in order to increase the filtering effect. This extra filtering may be helpful in very noisy environments achieving better estimations. However, this procedure may produce a short delay on the time estimation. For instance, the nonlinear filtering with an additional extra summation becomes:
\[ \hat{x}(kT) = \frac{\sum_{i=0}^{k} \sum_{j=0}^{i} n(jT) d(jT)}{\sum_{i=0}^{k} \sum_{j=0}^{i} d(jT)} \] (18)

The nonlinear invariant filtering approach can be performed in a similar way for the estimation of \( b, A, \phi \). In the case of \( A, \phi \), the filtering process has to be applied in (8) and (9) respectively.

5. Simulation results

A sinusoidal wave with an amplitude of 179.6 [V] and a frequency of 377 [s\(^{-1}\)], which is equivalent to a frequency of 60 [Hz] was taken to apply the proposed identification procedure. The sampling period was set to be \( T = 1.4 \text{ ms} \) (12 samples per period) and the SNR ratio was 30 [dB]. The nonlinear invariant filtering was defined by Eq. (18). Fig. 1 shows the signal used and its frequency estimation. Amplitude and phase identified values are shown in Fig. 2. The parameter identification procedure is also verified for the same signal with a bias component of 20 [V], as shown in Figs. 3 and 4 for the frequency, amplitude, and phase, respectively. The computation of all parameters is simultaneous regarding to the frequency case. Notice that the desired identification value is reached as soon as all the data values are available with a good accuracy.

5.1. Robustness against additive noisy signals

In this subsection, some numerical simulations of the proposed identification scheme were carried out. The simulations consisted in testing the identifier with the presence of different signal to noise ratio levels to show the robustness achieved by the nonlinear filtering. The SNR is given as follows:

\[ \text{SNR(dB)} = 20 \log_{10} \frac{A}{r_{\text{max}}} \]

where \( r_{\text{max}} \) is the peak value of the noisy signal. In this case we just tackle the frequency estimation process, since the rest of the parameters after the frequency estimation is achieved. The robustness analysis consists of running some simulations varying two parameters, the number of samples per period and the signal to noise ratio, in order to measure the estimation error, which will be given by

![Fig. 1. Frequency estimation for an unbiased sinusoidal signal.](image1)

![Fig. 2. Estimation of the amplitude and phase parameters.](image2)
The Mean Absolute Percentage Error (MAPE) is calculated as follows:

$$MAPE = \frac{1}{N} \sum_{k=1}^{N} \frac{|\omega - \hat{\omega}(k)|}{|\omega|}$$

where $N$ is the number of samples taken for the simulation, $\hat{\omega}(k)$ is the estimated value at the sample $k$, and $\omega$ is the actual value of the frequency. Fig. 5 shows the MAPE index evolution for these variations for $N = 40$ samples. Notice that the methodology reduces its accuracy when
increasing the sampling rate according to the Cramér-Rao bound [12]. Also, the identifier’s robustness against different additive noise levels is shortened when sampling rate is increased. It is advisable to take into consideration the noisy environment of the signal to decide a possible reduction in the sampling rate. The performed analysis shows that this approach is suitable for measurement conditions in which a short quantity of samples per period is available and a fast parameter estimation (a quarter of cycle or less) is demanded.

With the aim of analyse the bias and variance of the estimations, we ran Monte Carlo simulations with different values of SNR. The frequency estimation was carried out over four cycles of the signal and using 9 samples per period. The procedure was repeated 10,000 times for each value of SNR. Fig. 6 represents the histograms obtained

![Fig. 6. Histogram analysis for different signal to noise ratio values. In the left part, the analysis is performed with a bias and, on the right part, the same analysis is given without bias.](image-url)
for SNR of 25, 40 and 60 [dB] and a bias of the 10% of the peak value of the signal. From the figures we can conclude that the proposed technique is suitable for a short quantity of samples per period and it is obtained acceptable results up to 25 dB. The bias effect for these conditions does not affect in a significant manner the estimations.

On the other hand it is getting better estimates for the version without bias as long as it is below 5% of the amplitude of the signal. In order to obtain a good performance of the method, it is not advisable to use more than 16 samples per period and SNR larger than 20 [dB].

6. A comparative estimation study

In order to illustrate the proposed method, we now compare its performance with two recent and reliable strategies. The two stage autocorrelation approach [18] is a novel technique in which the measured signal is transformed to another noisy sinusoidal signal; in this second stage, autocorrelation functions are applied to obtain the frequency estimate. This technique exhibits a high performance in presence of gaussian noise having a better performance than conventional autocorrelation methods with a low computational complexity, allowing a fast implementation. Also, the estimator has shown to be superior to other approaches, such as the Discrete Fourier Transform approach [19], the MUSIC estimation method [20] and the Principal Component estimation method [21]. On the other hand, we compared the method with the contraction mapping method [22], which exhibits fast frequency estimation results and a good level of noise immunity. This methodology is less sensitive to initial conditions and computing restrictions than most of the least square regression methods (see [23,24] for further information). This method uses an autoregressive filter and a forgetting factor denoted as $\eta$.

6.1. Two stage autocorrelation approach

In this subsection, a brief description of the Two Step Autocorrelation Method is given. Consider the discretized form of the unbiased sinusoidal (3). A first stage autocorrelation function is given by

$$\lambda_{k,m} = \sum_{n=m+1-k}^{N-m-k} x(nT - kT)[x(nT) + x(nT + 2kT)]$$

$k = 1, 2, \ldots, m$. $m = 5, 6, \ldots, M$

$M = [(N - 1)/2] \geq 5$

The second stage autocorrelation functions with lags 1 and 2 are

$$\Lambda_{1,m} = \sum_{k=4}^{m} \lambda_{k-1,m} (\lambda_{k,m} + \lambda_{k-2,m})$$

$$\Lambda_{2,m} = \sum_{k=5}^{m} \lambda_{k-2,m} (\lambda_{k,m} + \lambda_{k-4,m})$$

Since the mathematical expectations $E\{\Lambda_{1,m}\}$, $E\{\Lambda_{2,m}\}$ are proportional to $\cos(\omega T)$ and $\cos(2\omega T)$, respectively, the following additive procedure is applied to improve the estimation process.

$$\Lambda_1 = \sum_{k=5}^{m} \Lambda_{1,m}; \quad \Lambda_2 = \sum_{k=5}^{m} \Lambda_{2,m}$$

and the frequency estimation is given by:

$$\hat{\omega} = \frac{1}{T} \arccos \left( \frac{\Lambda_2 + \sqrt{\Lambda_2^2 + 8\Lambda_1^2}}{4\Lambda_1} \right)$$

To test the Two Stage Autocorrelation Algorithm, a numerical simulation with the parameters $A = 179$, $\omega = 376.99$ [s$^{-1}$], $\phi = 1.5$, 14 samples per cycle and a zero mean additive noise of the 3 per cent of the peak sinusoidal amplitude was carried out. This algorithm tends to work better in larger sampling periods, thus, the methodology is not affected in this sampling rate. Fig. 7 depicts the comparison for the frequency estimation process between the two stage correlation approach and the proposed approach. As depicted, the estimations obtained by the two stage correlation method are quite good, even in presence of noise; however, the nature of the filtering scheme requires a minimum of 12 samples to start the identification process, and this restriction makes a forced time delay which is important in fast estimation under a short quantity of samples per cycle available.

![Fig. 7. Comparison results with respect to the two stage autocorrelation approach. $x$ represents the actual frequency, $x^\prime$ is the proposed method and $x_{CM}$ is the estimation using the two stage autocorrelation approach.](image)

![Fig. 8. Comparison results between the contraction mapping method ($f_{CM}$) and the proposed methodology.](image)
6.2. Contraction mapping frequency estimator

This methodology considers the frequency estimation based on the parameter $a \triangleq \cos(\omega T)$. The algorithm depends on a main tuning parameter $\eta \in (0, 1)$, which is used to give the convergence rate of the identifier. Denote $\hat{a}$ as the estimation value of $a$. The algorithm is described follows:

$$\theta(\hat{a}(k)) = -\frac{1 + \eta^2}{2\eta} \hat{a}.\quad x_s(k) = x(kT) - 2\theta(\hat{a}(k))\eta x_s(k - 1) - \eta^2 x_s(k - 2)$$

$$\hat{a}(k + 1) = \frac{\sum_{j=1}^{k} y_s(j - 1) + \eta^2 y_s(j - 2)}{(1 + \eta^2)\sum_{j=1}^{k} y_s^2(j - 1)}$$

$$\dot{\omega}(k) = \frac{\arccos(\hat{a})}{T}$$

$$x_s(-1) = 0$$

$$x_s(-2) = 0$$

$$\hat{a}(1) = a_0 \text{ Arbitrary}$$

$$\hat{a}(2) = a_1 \text{ Arbitrary}$$

To test both algorithms, we used the simulation signal given in the last section for both algorithms. The design parameters values of the contraction mapping method (CM) were set to be $\hat{a}(1) = .9$. $\hat{a}(2) = .85$. $\eta = .95$. The numerical results of the frequency estimation algorithms are depicted in Fig. 8. Both algorithms converge in a fast time, however, the convergence rate in the Contraction Map Method depends on the design parameters, where two of them are arbitrary initial conditions for the identified value, while our method do not need any tuning gain parameter. Furthermore, the nonlinear invariant method is faster and can estimate all the parameters with a good precision. The MAPE index obtained was 1.1988 for the contraction mapping method and 0.021 for the proposed method.

7. Experimental results

Finally, in order to assess the behaviour of the identification approach in an experimental framework. A sinusoidal signal generated by a laboratory function generator BK Precision Model 4085, was measured by means of a digital oscilloscope Tektronix Model TDS1012. The sinusoidal wave was described by the following function:
\[ x(kT) = 4 \sin(2\pi(10kT + 0.9)) + 2 + \zeta(kT) \] [V]

with a sampling value \( T = 6.25 \times 10^{-3} \) [s] with an additive noise of diverse nature (Signal Generator, Data acquisition process, etc.) denoted as \( \zeta(kT) \). The methodology was the same applied in the simulation instance. Fig. 9 shows the signal and the identified frequency. Despite the fact that there is no statistical data available for the noise, the invariant filtering has a good behaviour; the identifier reaches a vicinity of the real value in a fourth part of the sinusoidal signal period. On the other hand, the amplitude, bias and phase parameters are depicted in Fig. 10.

8. Concluding remarks

In this paper, a methodology for the fast computation of the frequency, bias, amplitude and phase parameters of noisy sinusoidal signals was presented. The algorithm is based on an exact discretization of the nominal signal, which, under noiseless conditions, not many sampling times are needed for convergence. The algorithm relies on the estimated frequency value to immediately calculate the unknown bias, phase and amplitude parameters. This algorithm is easy to implement and has a high efficiency due to its low computational cost.

For the frequency identification, the solution implies devising a difference equation of the sampled sinusoidal signal, which permits us to obtain exact responses in the noiseles case. This can be achieved by simple algebraic manipulations.

In the main case, additive zero mean measurement noise, with no additional statistical information, was assumed and by employing a suitable low pass invariant filtering on the algebraic algorithm constitutive parts, the signal to noise ratio was substantially improved for the correct application of the identification algorithm. This filtering improved the response of the identifier and made possible to obtain correct values even in experimental circumstances. The proposed nonlinear invariant filtering scheme has the benefits of eliminating the zero crossing problems which may arise in linear invariant filtering approaches. Suitable methodologies as finding an average value from the addition of the absolute value in a number of past values of numerator and denominator signals can also be used as invariant filtering methods.

The bias, amplitude and phase estimation process requires the knowledge of the frequency which, as in practical fast algebraic identification frameworks is obtained after an “epsilon” setting time given by the order of the difference equation (which is known in contrast with the continuous time algebraic identification approach [12]).

When finding auxiliary equations by means of delayed expressions, these can be chosen for different time delays. For instance, in all the cases here presented, we applied a single backward shift operator to obtain as many equations as unknowns. However, in more complex cases, the required additional equations could be obtained from using arbitrary time delays of the original model equation. The results are similar.

This method is suitable for applications where there is a short amount of available samples per period and a fast and accurate identification scheme is needed, for high sampling ratios, it is better to use the continuous time scheme [12].

The developments here presented can be extended to deal with finite sums of sinusoidal signals with different frequencies, or even for the challenging case of frequency identification within a modal identification framework. Also, since the parameter estimation methodology is not adequate for high sampling rates, the problem can be related in the context of numerical denoising schemes related to algebraic filtering schemes [25], which have shown good numerical results dealing with noisy signals. These topics can be proposed for future research.

References


