Position Controller Design for Quad-rotor under Perturbed Condition


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Abstract:
There are number of failures are recorded in UAV projects because of its non-linearity nature. In Quad-rotor type Unmanned Aerial Vehicles, position controlling is one of the most critical tasks and suitable Position stabilization controller is the essential part of these types of unmanned systems. In this paper a position stabilization (i.e. x and y axes) control with noise rejection system is presented for the quad-rotor type UAV under perturbation conditions. The proposed controller is based upon Auto-tuned PID controller with extended Kalman filter for Position stabilization controlling under noisy and external disturbance conditions. Extended Kalman filter is used to filter out the sensor noises and provide more accurate feedback signal to the PID controller. The proposed control technique is simulated on Quad-rotor’s nonlinear mathematical model under perturbed and unperturbed conditions for its position controlling using MATLAB environment. Simulation results shows that proposed controller performed well and quickly maintains quad-rotor position under perturbed conditions.

Key Words: EKF, UAV, PID, stability, noise and disturbance rejection

1 Introduction:
One leading rotorcraft, a quad-rotor has been the area of interest among researchers over the past years since they are inherently suitable for many applications such as surveillance, search and rescue, inspection of the structure, defense, etc., especially where human presence is difficult or dangerous.

The quad-rotor helicopter is a (UAV) unmanned aerial vehicle with four propellers attached to four motors installed on a fixed body. A quad-rotor UAV have exceptional benefits such as small size, light weight and simple mechanical Assembly.

UAVs are being used in several applications including search and rescue missions (Ryan. A.Hedrick, 2005), wild fire surveillance (Alexis.K, 2009), monitoring over nuclear reactors (Sarris.Z, 2001), power plants inspection (Caprari. G, 2012), agricultural services (Herwitz, S.R., 2004), mapping and photographing (Naval Studies Board, 2005), marine operations (Gray.S. 2003), battle damage assessment (Girard, A., Howell, 2004), border interdiction prevention and law enforcement (Murphy.D. Cycon, 1998). Due to their small size UAVs can be useful in hazardous conditions where human life is at risk and surroundings that are inaccessible to reach.
The above mentioned applications influence the requirement of such systems which have ability of operating in harsh environments and difficult & complex mission. In unmanned aviation, helicopters and rotorcraft proved one of the best solutions because of their core capabilities such as hovering, vertical takeoff & landing and accurate positioning which is our main concern in this article while flying autonomously. So an efficient position controller is necessary requirement of these UAVs. Besides that its embedded sensor systems are noisy which make its controls more complex. To overcome these problems, EKF and PID controller are presented for removing sensor noise and the position controlling with extended disturbance rejection respectively for quad-rotor UAV systems.

1.1 Related Work:

In the recent years, position controlling of quad-rotor has remained a problem due to the constraints and unstable kinematics and dynamics. However, some of the techniques of control were developed in this field. Adaptive neural controller for handling attitude control system of UAV (D.Hazry, 2011). The X4-Flyer and STARMAC II uses LQR and PID control technique (Anil guclu, 2007). R. Xu (2006) used sliding mode and PID control for the system. Backstepping method was used for altitude, attitude and position control of a quad-rotor, the results of this technique showed a flexible control structure. Moreover, showed that quad-rotor was able to perform autonomous hovering with altitude control and autonomous take-off and landing (Samir Bouabdullah, 2007). Bouadi (2008) presented stabilizing control laws by sliding mode and backstepping approach. Syed Ali Raza(2010) applied fuzzy logic control on position, altitude and attitude. Keun Uk Lee (2011) used Dynamic Surface Control (DSC) method for altitude control and position control. This paper presents the modeling of a quad-rotor and a technique based on PID controller for position stabilization. The proposed algorithm is simulated on (MATLAB). In a real-time system, sensors and measurement devices provide noisy data. A filter is introduced into the system in order to reject noise. The proposed filter is a Kalman filter which could be a decent filter for this system. Due to the non-linear nature of the quad-rotor system a simple KF cannot be used. The extended version of the Kalman filter is EKF which is used to linearize the system States.

The organization of the paper is structured as the section 1 that refers to the mathematical modeling, which follows the discussion on Kinematics and Dynamics of quadrotor. Experimental results and simulations have been provided in section 2. Section 3 is regarding the proposed Control and filter technique for disturbance and noise rejection.

Mathematical Modeling of Quad-rotor:

2 Kinematics and Dynamics of Quad-rotor:

2.1 Quad-rotor Kinematics

The quad-rotor UAV has four fixed pitch rotors which are mounted on the four ends of a simple cross frame. A pair of motors (Q1 and Q3) rotates clockwise while other pair of motors (Q2 and Q4) rotates counter-clockwise direction as shown in figure 1. This combination of motor rotation counters the opposite torques produced by motors. The rotation of these propellers generate a vertical lifting force upwards which raises quad-rotor body in the air and it can moves in pitch, roll, yaw, hover, takeoff and landing positions. For the vertical takeoff and landing, the rotation speed of all four rotors gradually increased and decreased respectively. Pitch and roll movements can be achieved by the
altering the speed of any one pair of motors while other motor pair speed remain constant. Yaw movement can be achieved by altering the speed of both motors pairs in quadrotor.

2.2 Quad-rotor Dynamics

As we have discussed quad-rotor consists of four motors that are used to control the 6 degrees of freedom (DOF), so we need to understand the dynamics of the motor. Propeller motor dynamics is identified and validated in (N. Guenard et al, 2006). A first-order transfer function is sufficient to present the dynamics of rotor used in quad-rotor type unmanned system.

\[
G(S) = \frac{K}{s^2 + 1} \quad (2.0)
\]

In modeling quadrotor dynamics, there are two frames that have to be defined as a reference which is Earth Inertial frame (E frame) and quadrotor fixed-body frame (F frame). The frames are shown in Figure 2. The dynamics of quadrotor can be describe in many different ways such as quaternion, Euler angle and direction matrix. The Quadrotor orientation can be defined by three Euler angles which are roll angle (\(\phi\)), pitch angle (\(\theta\)) and yaw angle (\(\psi\)). These three Euler angles form the vector \(\Omega^T = (\phi, \theta, \psi)\). Similarly the position of the vehicle in the inertial frame is define by the vector \(\mathbf{q}^I = (x, y, z)\) and the transformation of vectors of the fixed frame to the inertial frame is given by the resultant transformation matrix of \(z, y\) and \(x\) axis, \(R\) as below:

\[
R = \begin{bmatrix}
c\psi c\theta & c\phi & -s\psi c\theta - s\psi c\phi & c\psi s\theta c\phi + s\psi s\phi \\
c\psi s\theta & s\phi & c\psi s\theta s\phi + c\psi c\phi & c\psi s\theta c\phi - c\psi s\phi \\
-s\theta & s\phi & c\theta c\phi & c\theta s\phi \\
c\theta c\phi & -s\phi & c\theta s\phi + c\theta c\phi & c\theta c\phi
\end{bmatrix} \quad (2.1)
\]

Where, \(c\) and \(s\) denotes the \(\cos\) and \(\sin\) function respectively.

The thrust force, \(F\) generated by each motor \(j\) is defined as following:

\[
F_j = b_j \omega_j^2 \quad (2.2)
\]

With \(j=1, 2, 3 \& 4\)

Where \(b\) is the thrust factor and \(\omega_j\) is the rotational speed of motor \(j\). The total thrust force applied to the airframe from the four motors is given by:

\[
T_F = \sum_{j=1}^{4} |F_j| = b \sum_{j=1}^{4} \omega_j^2 \quad (2.3)
\]

The differential equation for acceleration of the quadrotor can be described in equation 2.4 as:

\[
\ddot{q} = \begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
\ddot{z}
\end{bmatrix} = g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - R \frac{T}{m} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2.4)
\]

Where \(g\) is gravity (9.81m/s\(^2\)) and \(m\) is the weight of the quadrotor.
Vector \( T \) describes the torque applied to the quadrotor’s body. Torque can be calculated by using equation 2.5. So, the vector \( T \) can be defined as:

\[
T = FL
\]  

\[
T = \begin{bmatrix}
   Lb(\omega_3^2 - \omega_4^2) \\
   Lb(\omega_1^2 - \omega_2^2) \\
   d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2)
\end{bmatrix}
\]  

Where \( F \) and \( \omega_j^2 \) are the force produced from the propeller while \( L \) is the length of the quadrotor’s arm. \( b \) and \( d \) are thrust factor and drag factor respectively.

Vector \( T_G \) described as the gyroscopic torque. The gyroscopic torque is produced by the effect of rotation of the motors. The vector \( T_G \) defined as:

\[
T_G = I_M \cdot \left( \hat{\omega} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \cdot (\omega_2 + \omega_4 - \omega_1 - \omega_3)
\]  

Where, \( I_M \) is the motor inertia.

Using equation 1.6 and 1.7 with the inertia matrix \( I \) (a diagonal matrix with the inertias \( I_{xx}, I_{yy} \) and \( I_{zz} \) on the main diagonal), a second set of differential equations is obtained:

\[
I \cdot \ddot{\omega} = -(\hat{\omega} \times I \cdot \dot{\omega}) - T_G + T
\]  

As discussed, the movements of quadrotor are achieved by varying the speeds of the motors. The rotational speed of each motor \( \omega_i \) denotes as the input variable for the transformation of the quadrotor movements using the obtained mathematical model. Therefore, the input variables can be defined as following:

\[
\begin{align*}
   u_1 &= b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\
   u_2 &= b(\omega_4^2 - \omega_2^2) \\
   u_3 &= b(\omega_3^2 - \omega_1^2) \\
   u_4 &= d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2)
\end{align*}
\]  

Where \( u_1 \) is equal to \( T_r \) as in equation 1.3 which denotes to the thrust force applied to the quadrotor body; \( u_2 \) denotes the force which leads to the roll torque; \( u_3 \) denotes the force which leads to the pitch torque and \( u_4 \) denotes the force which leads to the yaw torque.

However, remember that the gyroscopic torque also produced from the rotational velocities of the motors. Defined that vector \( u = (u_1, u_2, u_3, u_4) \) is the input variables. The total gyroscopic torques \( g(u) \) effected on quadrotor is:

\[
g(u) = \omega_2 + \omega_4 - \omega_1 - \omega_3
\]
Where, $\omega$ is the motor’s speed.

By combining equation 2.4 and equation 2.8, overall dynamic model yield in the form of equation 2.11.

$$
\begin{align*}
\ddot{x} &= -(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \frac{u_1}{m} \\
\dot{y} &= -(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \frac{u_1}{m} \\
\ddot{z} &= g - (\cos \phi \cos \theta) \frac{u_1}{m} \\
\dot{\phi} &= \frac{l_{yy} - l_{zz}}{l_{xx}} \phi g(u) + \frac{L}{l_{xx}} u_2 \\
\dot{\theta} &= \frac{l_{xx} - l_{zz}}{l_{yy}} \theta g(u) + \frac{L}{l_{yy}} u_3 \\
\dot{\psi} &= \frac{l_{xx} - l_{yy}}{l_{zz}} \psi g(u) - \frac{1}{l_{zz}} u_4
\end{align*}
$$

For Position controlling only X & Y axis equations will be taken in account as shown in Equation (2.12),

$$
\begin{align*}
\ddot{X} &= (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{u_1}{m} \\
\dot{Y} &= (\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \frac{u_1}{m}
\end{align*}
$$

3. Proposed Controller Design for Perturbed and Unperturbed system

Technique of Auto-tune PID controller is proposed for good position controllability of quad-rotor under the perturbed and unperturbed conditions. In addition Externded Kalman Filter (EKF) is also proposed to filter the noise caused by the sensors and system.

Figure 3 shows the overall block diagram of Proposed Control Algorithm for Stabilizing quadrotor position control.

The main aim of this research is to design controller which maintains the stability of a quad-rotor under the circumstances of external disturbance. To handle this problem a suitable auto-tune PID controller is proposed which stabilize the quadrotor under different disturbance condition during flying.

The other common problem is quad-rotor is its sensor noises which cause to gave wrong feedback information to the controller and make system unstable. To overcome this problem EKF is used in feedback loop to filter out the noises and provide noise free signal to the controller.

3.1 Extended Kalman for noise Rejection

The kalman filter states the general problem of finding the estimates of the discrete time process of a linear system while EKF is used if the dynamics of the system or output of the system is nonlinear. In
our case quad-rotor is nonlinear system so we must adapt EKF to filter out the noisy sensor data. EKF lies on the principles of linearization of the current estimation error mean and covariance (Bishop, 2001).

Considering a standard state space model of a nonlinear system as shown in figure 4.

\[
x_{k+1} = f(x_k, u_k) + w_k \\
\hat{y}_k = h(x_k) + v_k
\]  

(3.1)  
(3.2)

Where, ‘\(x_k\)’ is a state vector, ‘\(\hat{y}_k\)’ is a measured process, ‘\(w_k\)’ and ‘\(v_k\)’ are the process and measurement noises respectively. \(f(\cdot)\) and \(h(\cdot)\) are generic nonlinear functions.

The extended Kalman filter is used to estimate unmeasured states and the actual process outputs. The figure (5) shows the estimated state and estimated measured output.

Likewise the standard kalman filter, the EKF also uses two step prediction and correction algorithm. The time update equations of EKF are

\[
\hat{x}_{k+1} = f(\hat{x}_k, u_k) \\
P_{k+1} = A_kP_kA_k^T + Q
\]  

(3.3)  
(3.4)

Where ‘\(\hat{x}_{k+1}\)’ is prior state estimate.

The time update equations uses the state and covariance estimate ‘\(P_{k+1}\)’ from previous time step ‘\(k\)’ to the current time step ‘\(k+1\)’. The measurement update equations of EKF are

\[
K_k = P_kC_k^T(C_kP_kC_k^T + R)^{-1} \\
\hat{x}_{k+1} = f(\hat{x}_k, u_k) + K_k[\hat{y}_k - h(\hat{x}_k)] \\
P_{k+1} = A_k(1 - K_kC_k)P_k
\]  

(3.5)  
(3.6)  
(3.7)

Where ‘\(K\)’ is the Kalman gain correction vector and ‘\(A_k\)’ and ‘\(C_k\)’ cannot be used directly. With this type of limitation either Taylor series is applied or Jacobian is used. The Jacobians are defined as

\[
A_k = f'(\hat{x}_k, u_k) \\
C_k = h'(\hat{x}_k)
\]  

(3.8)  
(3.9)

Where \(f(\cdot)\) can be evaluated as in the eq. (3.10).

\[
f'(x) = \begin{bmatrix}
df_1/dx_1 & \ldots & \ldots & \ldots & \ldots & df_n/dx_1 \\
\vdots & \ddots & \ldots & \ldots & \ldots & \ddots \\
\vdots & \ldots & \ddots & \ldots & \ldots & \ddots \\
\vdots & \ldots & \ldots & \ddots & \ldots & \ddots \\
\vdots & \ldots & \ldots & \ldots & \ddots & \ddots \\
df_n/dx_1 & \ldots & \ldots & \ldots & \ldots & df_n/dx_n
\end{bmatrix}
\]  

(3.10)
3.2 PID controller For Perturbed Conditions:

The output of EKF produces a noiseless signal in feedback loop which combine with the input reference signal to produce the error signal for the controller as shown in Figure (3). Here PID controller which is regulated by auto-tune method within range between 0 to 1 for its all parameters which are Kp, Ki, Kd. For the realization of the desired task only the principal requirement is to work on the position of quad-rotor. Only stipulation is the dynamics of axes X and Y, which represents the positional checking of the quad-rotor.

\[
\begin{align*}
\dot{X} &= (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi) \frac{u_x}{m} + \dot{\xi}_{ux}(t) \\
\dot{Y} &= (-\cos \psi \sin \phi + \sin \psi \sin \theta \cos \phi) \frac{u_y}{m} + \dot{\xi}_{uy}(t)
\end{align*}
\]

(3.11)

Where, \(\dot{\xi}_u(t)\) is the disturbance factor. Taking ideal condition supposing \(\dot{\xi}_u(t) = 0\).

For Position controlling \(\psi = 0\) and by using small angle approximation to the eq. 3.11 yields:

\[
\begin{align*}
\dot{X} &= -\phi \frac{u_x}{m} \\
\dot{Y} &= \theta \frac{u_y}{m}
\end{align*}
\]

(3.12)

In above equation \(\phi, \theta\) and \(\frac{u_i}{m}\) are constant values, where with respect of time \(\phi\) and \(\theta\) change their values and give a constant value at a particular time so above equation can be rewritten as:

\[
\begin{align*}
X(S) &= \frac{K_2}{s^2} \\
Y(S) &= \frac{K_2}{s^2}
\end{align*}
\]

(3.13)

For controller design, the tracking error signal for X and Y will be:

\[
\begin{align*}
E_x &= X_{ref} - X \\
E_y &= Y_{ref} - Y
\end{align*}
\]

(3.14)

PID controller is proposed to control the position of quad-rotor. So the control law can be chosen as:

\[
U(s) = K_p E(s) + \frac{K_i}{s} E(s) + K_d s E(s)
\]

(3.15)

Therefore close loop transfer function for the complete system will become:

\[
\begin{align*}
Y(S) &= \frac{K_4 K_d s^2 + SK_p K_j + K_i K_j}{S^3 + K_d K_j s^2 + (S K_p + K_i) K_j}
\end{align*}
\]

(3.16)

Where \(K_j\) can be \(K_1\) or \(K_2\) for X and Y axis position respectively. Equation (3.16) is the Position controller equation for X and Y axis, which represents actual, linearized and controlled output.
4. Results & Simulations

4.1 Disturbance Rejection:

Figure 6 shows the disturbance applied on a system and the PID controlled System output. Figure 7 represents the overall Position Control system for disturbance rejection and satisfies the response of control system tracking the actual path under perturbs and the system could track the actual path with almost minimal tracking error. However, Controller responding time was found to be very quick in stabilizing the system.

4.2 Noise Rejection:

This part shows the estimate of the trajectory of EKF, figure 8 shows the Gaussian noise added in the way of real trajectory and to evaluate the performance of EKF figure 9 shows the real and measured trajectory of the particular system with the rejection of the noise.

5. Conclusion:

This article presents the complete mathematical modeling and position control of quad-rotor. Design of controller is based on PID with auto-tune method. Finally EKF is introduced by which the estimates of parameters could converge successfully with their right value at the time of the desired exit is deductible under the terms of the difficulties due to the noise. According to simulations, there is evidence that the effectiveness of the control method is verified that the controller has offered to return the entire system to stabilize the situation where there is any kind of disturbance and noise imposed quad-rotor.

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Figures:

Figure 1: Quad-rotor UAV with its dimensions
Figure 2: Quad-rotor Body and Earth Inertial Frame

Figure 3: Proposed Control System
Figure 4: Nonlinear system, with input and a measurement noise

\[
x_k = f(x_{k-1}, u_k) + w_k
\]

**Extended Kalman Filter**

Figure 5: EKF system

Figure 6: Disturbance and its effect

Figure 7: Overall Position system
Figure 8: Noise Traced

Figure 9: Noise Rejection by Using Extended Kalman Filter