Design Optimization of Automotive Propeller Shafts

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Abstract

In this paper six automotive propeller shafts made of different materials are designed. Finite element modeling based on shell element is used in the eigenvalue analysis. Shaft weights, buckling torque and dynamic performance for different design alternatives, have been studied.

Simulated Annealing (SA) Algorithm is used for optimization. Results obtained clearly show the possibility of designing a single-piece propeller hybrid shaft made of fibre-reinfoced composite and aluminum tube having less weight compared to that of steel shaft and having its fundamental natural frequency above the shaft operating speed.

Keywords: Propeller shaft, Composite shaft, Hybrid shaft, Natural frequency, Optimization, Simulated annealing.

1 Introduction

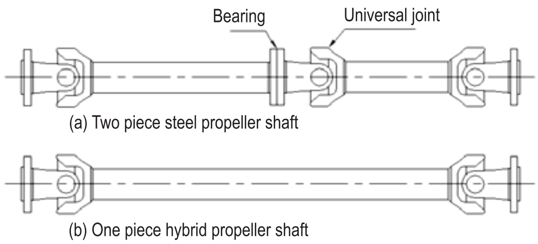

Fig. 1 Schematic diagram of the automotive propeller shaft

Although today's trend for passenger vehicles is towards front wheel drive, thus reducing the importance of the design of propeller shafts, the interest in four wheel drive for high performance cars opens a new field. Generally, most of cars, vans and small trucks have propeller shafts within about 1.25 m to 2 m of length. These shafts are running at about 6000 rpm to 7000 rpm and their torque transmission capabilities should be larger than 3000 Nm. To avoid access amplitudes of vibrations, the fundamental natural frequency of these shafts should be more than the above range of 6000 rpm to 7000 rpm. Since the fundamental natural frequency of one-piece shafts made of steel or aluminum is just at the lower limit of the above range, these shafts are made in two-pieces, as shown in Fig. 1, or some times have an additional middle bearing support. This is to increase shaft's fundamental natural frequency over the above range and thereby avoid access amplitudes of vibration. However, one-piece propeller shafts made of fibre-reinforced composites can have greater un-supported length and higher fundamental natural frequency. In addition, fibre-reinforced composite shafts have many other advantages such as reduced system weight, less noise and vibration. Many researches have studied the feasibility of using fibre-reinforced composite and hybrid shafts in automotive applications. The developments and different aspects of using composite shafts, including automotive applications, have been studied by Singh [1998], Gubran and Gupta [1996], Darlow and Creonte [1995], Gubran and Gupta

[2002]. Recent studies on composite vehiclar drive shafts are by Ajith *et al.* [2004]; Rangaswamy, *et al.*, [2002]; Badie [2006] and Montagnier and Hochard [2008].

This paper presents design and optimization based on shell finite element modeling and simulated annealing algorithm of propeller shafts made of different materials. The possibility of having one-piece propeller shaft is attempted.

2 Formulation

Finite element modeling based on nine nodded degenerated shell element is adopted for eigenvalue analysis. Torque transmission and buckling torque capabilities for different shafts have been calculated using strength of materials relations. Later optimization using simulated annealing is presented.

2.1 Natural frequency analysis

Finite element modeling, Gubran and Gupta [2005], is used for free vibration analysis of laminated/hybrid shafts. The layers of the laminated/hybrid shaft are modeled using nine-node isoparametric degenerated shell element. It is assumed that the laminate is build up by a number of laminae perfectly bonded together and there is no relative displacement between the adjacent layers. Mindlin hypothesis, i.e., a straight line element perpendicular to the middle plane before bending remains straight during bending but does not necessarily remain normal to the neutral surface of the layer. The free vibration of the un-damped shaft results into an eigenvalue problem in the form:

$$[[K] - \omega^2[M]]\{X\} = \{0\} \tag{1}$$

where $[K]$ is the global stiffness matrix; ω is the un-damped natural frequency; $[M]$ is the global mass matrix; and $\{X\}$ is the global displacement vector.

The eigenvalues are extracted using subspace iteration method. For validation of formulation and programming, several examples have been solved and results have been verified.

The displacement components at any point in the element can be represented by the element nodal displacements as:

$$(u \quad v \quad w)^T = \sum_{k=1}^{9}[H^k]\{d_e^k\} \tag{2}$$

where $\{d_e^k\} = [u^k v^k w^k \alpha_1^k \ \alpha_2^k]^T$ is the nodal displacement vector; and $[H^k]$ is the generalized shape function matrix which can be expressed as $[H^k] = N^k(r, s)([I][P])$, where $N^k(r, s)$ is the shape function; $[I]$ is the identity matrix and the matrix $[P]$ can be written as

$$[P] = t\frac{h}{2}\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$

where h is the thickness of the shell element and t is the natural coordinate with respect to the shell thickness as shown in Fig. 2.

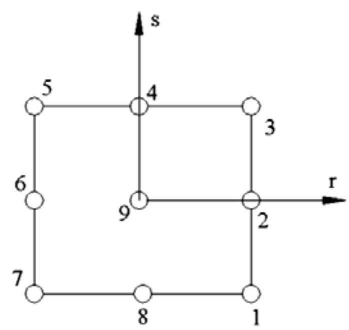

Fig. 2 The nine nodded shell finite element

The strain-displacement relationship can be expressed as:

$$\{\varepsilon\} = \sum_{k=1}^{9} [B^k]\{d_e^k\} \tag{3}$$

where $[B^k]$ is the strain–displacement matrix.

The element stiffness matrix can be obtained from the energy principle as:

$$[K_e] = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [B]^T [D][B]|J|\, dr\, ds\, dt \tag{4}$$

The matrix $[D]$ is the martial stiffness matrix defined in the local coordinates; this matrix depends on the layer material properties and fibre angle and can be written as:

$$[D] = \begin{bmatrix} D_1 & D_{12} & 0 & 0 & 0 \\ D_{12} & D_2 & 0 & 0 & 0 \\ 0 & 0 & D_3 & 0 & 0 \\ 0 & 0 & 0 & D_4 & 0 \\ 0 & 0 & 0 & 0 & D_5 \end{bmatrix}$$

and

$$D_1 = E_1/\Delta, \quad D_2 = E_2/\Delta, \quad D_{12} = E_2 v_{12}/\Delta$$

$$D_3 = G_{12}, \quad D_4 = KG_{13}, \quad D_5 = KG_{23}, \quad \Delta = 1 - v_{12}v_{21}$$

In which E_1, E_2 are the Young's longitudinal and transverse moduli and G_{12} is the shear modulus.

The consistent element mass matrix linking nodes i and j can be expressed as,

$$[M_e] = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [N_i]\rho[N_j]\, dr\ ds\ dt \tag{5}$$

where ρ is mass density of element layer.

2.2 Torque transmission

From strength of materials relation, torque transmission capacity for a shaft with shear strength τ_y, second moment of inertia J and outer radius r is given by

$$T = \frac{\tau_y}{r} J \tag{6}$$

In the case of hybrid shaft with materials 1 and 2, where the shaft with material 1 is the inner shaft and the shaft with material 2 as the outer shaft, torque transmission capacity for shaft with material 1 can be calculated as,

$$T_1 = \frac{\tau_{y1}}{r_1} J_1 = \frac{G_1 J_1}{G_1 J_1 + G_2 J_2} T \tag{7}$$

where T_1 is torque capacity of the inner shaft, τ_{y1} is the shear strength of inner shaft, r_1 is the outer radius of shaft with material 1 and J_1 and J_2 are second moment of inertia of materials 1 and 2 respectively. T is the total torque transmitted by the hybrid shaft which can be calculated as,

$$T = T_1 + T_2$$

where T_2 is the torque transmitted by shaft with material 2.

2.3 Shaft critical buckling torque

The critical buckling torque becomes important in case of thin and long shaft. For thin laminated/hybrid shaft, critical buckling torque can be calculated as,

$$T_{buc} = 22.57 \sum_{i=1}^{n\text{ layer}} (D_{22})_i^{5/8} (Q_{11})_i^{3/8} r_i^{1.25} l^{-1/2} \tag{8}$$

where $(Q_{11})_i$ and $(D_{22})_i$ are longitudinal and transverse moduli of the shaft with layer i, r_i and l are the shaft mean radius and length respectively.

2.4 Design optimization using Simulated Annealing SA

Simulated annealing procedure simulates the process of slow cooling of molten metal to achieve the minimum function value in a minimization problem. Controlling a temperature-like parameter introduced with the concept of Boltzmann probability distribution simulates the cooling phenomenon [Kirpatrick *et al.*, 1983]. According to the Boltzmann probability distribution, a system in thermal equilibrium at a temperature T has its energy distributed probabilistically according to $P(E) = \exp(-E/kT)$, where k is the Boltzmann constant. This suggests that a system at a high temperature has a uniform probability of being at any energy state, but at a low temperature it has a small probability of being at high energy state. This means by controlling the temperature T and assuming that the search process follows Boltzmann probability distribution, the convergence of the algorithm can be controlled. SA is used to find the global optimum with a high probability even for ill-conditioned functions with numerous of local minima.

Algorithm:

Step 1: Select an initial temperature T, and initial solution, X_o. Let $f_o = f(X_o)$ denote the corresponding objective value. Set $i = 0$and go to step 2.
Step 2: Set $i = i + 1$ and generate new solution, $(X_i' = X_i + r^* V_i)$ where, r is random number and V_i, is step length. Evaluate $f_i = f(X_i')$.
Step 3: If $f_i < f_{i-1}$, go to step 5, else accept f_i as the new solution with the probability $e^{(-\Delta E/T)}$ (where $\Delta E = f_i - f_{i-1}$) and go to step 4.
Step 4: If f_i was rejected in step 3, set $f_i - f_{i-1}$ and go to step 5.
Step 5: If satisfied with the current objective value f_i, stop.

Otherwise, adjust the temperature $(T' = T.r_T)$ where r_T is temperature reduction rate and go to step 2.

The important parameters which govern the successful working of the simulated annealing procedure are the initial temperature T, temperature reduction rate r_T and the number of iterations performed at a particular temperature. If a large initial temperature T is chosen, it requires a large number of iterations for convergence. On the other hand, if a small initial temperature T is chosen the search is not adequate to thoroughly investigate the search space before converging to the true optimum. A large value of n is recommended in order to achieve quasi-equilibrium state at each temperature, but the computation time will be more. Optimization of composite drive shafts using Simulated Annealing SA was carried out as follows:

1. **Objective Function**: minimization of shaft weight.
2. **Design constraints**:
   (i) shaft bending natural frequency,
   (ii) torsional buckling capacity and
   (iii) torque transmission capacity.
3. **Design variables**:
   (i) ply angles and their stacking sequence,
   (ii) ply thickness and
   (iii) shaft outer diameter which is limited to 90 mm.

3 Results and Discussion

3.1 Analysis of different design alternatives

As the fundamental natural frequency of one-piece shaft made of steel or even aluminum can not be higher than 6000 rpm, the speed at which generally these shafts are running, these shafts are made of two-pieces with central support and universal joint in-between. This gives an additional weight to the system and hence more fuel consumptions and less pay loads. In recent year's improvement in reinforcing fibres and resin properties, fibre-reinforced composites are replacing an increasing number of metal components. The advantage that composite can have over metallic components include, lower weight, better fatigue properties, elimination of corrosion, reduction in assembly piece parts, less vibration and noise and low coefficient of thermal expansion. Initially in the design of fibre-reinforced composite shafts, boron/epoxy and glass/epoxy were used. However with the developments in fibre processing technology, graphite fibres replaced boron fibres which were costly and difficult to process. Since in automotive applications the cost of the component is an important consideration, hybridization with glass/epoxy have been tried which proved cost effective. Hybridization with steel and aluminum tubes provides better joints and coupling with metallic parts and better damage requirements.

This study gives a comparison between six shafts made of different materials. Shaft mean diameter is kept as 0.075 m and assuming a 4000 Nm transmitted torque. In the first stage of design, number of plies and wall thickness for composite tube and/or aluminum or steel tube are calculated based on shear strength of shaft material (given in Table 1). In the second stage of design and based on calculated wall thickness and shaft length, which is kept the same for all shafts, natural frequencies and buckling torque have been calculated.

Later, plies stacking sequences and thickness have been optimized for proper placement of shaft natural frequency, torque transmission capacity and minimum shaft weight. The effect of shaft length on the performance of shafts, made of different materials, has been studied.

Table 1 Shafts material properties

Shaft Material	Longitudinal Modulus, E_1, Gpa	Transverse Modulus, E_2, Gpa	Shear Modulus, G_{12}, Gpa	Poisson Ratio, υ_{12}	Density, kg/m^3	Shear Strength, Mpa
Steel	210	210	84	0.30	7830	420
Aluminum	70	70	28	0.28	2600	210
Graphite/epoxy	130	10	7	0.25	1500	75

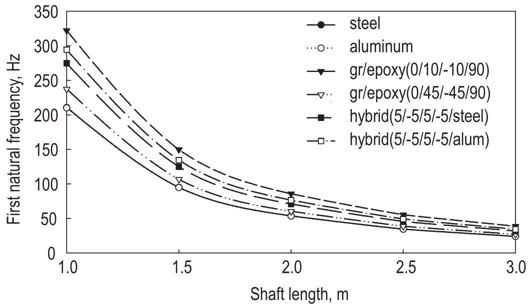

Fig. 3 Variation of fundamental natural frequency with shaft length

Results presented in Fig. 3, show the variations of the natural frequencies, for different design alternatives, with shaft length. These clearly show that the natural frequencies for steel and aluminum shafts are almost the same. This is obvious due to equal E/ρ ratios for both cases. However, for same torque transmission capacity, the weight of aluminum shaft is lesser by about 30% than that of steel shaft. The above observations are different for the case of shafts made of composite materials. In case of composite materials, properties are much dependent on orientations of fibres and individual thickness of different plies in the configuration. For example, shear strength is maximum for shafts with ply angles of 45° and decreases as the ply angle shifts towards 0°. However, longitudinal modulus, E_1, which has direct contribution to the shaft natural frequency, decreases as the ply angle shifts towards 45°. In the present study two cases of shaft configurations have been presented. The first one is of $(0°/10°/-10°/90°)$ while the second is of $(0°/45°/-45°/90°)$. Both shafts are designed to have the same torque transmission capacity. Due to the presence of 45° ply angles in the second shaft (i.e., $(0°/45°/-45°/90°)$), it is having more shear strength compared to the first one (i.e., $(0°/10°/-10°/90°)$). Therefore, it is obvious to have lesser shaft wall thickness and hence less weight, by about 60%, compared to the first. However, the first shaft is found to have fundamental natural frequency higher than the second by about 26%. This means that, placing ply angles of 45° in the shaft configuration lowers its fundamental natural frequency and decreases its weight. However, the fundamental natural frequencies of both of composite shafts are higher than that of steel and aluminum shafts.

The possibility of increasing shaft fundamental natural frequency and decreasing shaft weight is examined by hybridization of composite tube with steel tube (i.e., $5/-5/5/-5/$steel) and aluminum tube (i.e., $5/-5/5/-5/$Al). A reduction in shaft weight by 26% and an increase in the fundamental natural frequency by 7% are observed for hybridization of composite with aluminum compared to that of hybridization of composite with steel. However, comparison of hybridization of composite with aluminum to that of steel shaft, an increase in the shaft natural frequency by about 40% and a reduction in shaft weight by 12% is observed.

Taking a case of 1.5 m long propeller shaft, with mean diameter of 0.075 m, running in speed range of 6000 rpm to 7000 rpm and transmitting a 4000 Nm torque, it is found that, steel and aluminum shafts are having their fundamental modes at about 94.860 Hz and 94.586 (i.e., 5690 rpm). This is less than the lower limit of the operating speed. Practically, for these shafts to be used they have to cross the first

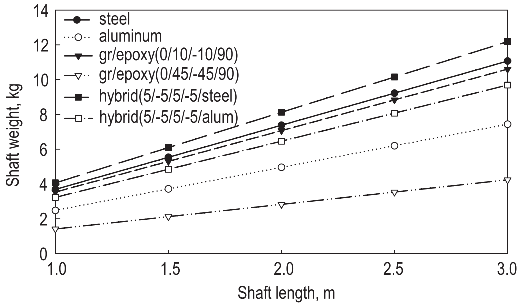

Fig. 4 Variation of weight with shaft length

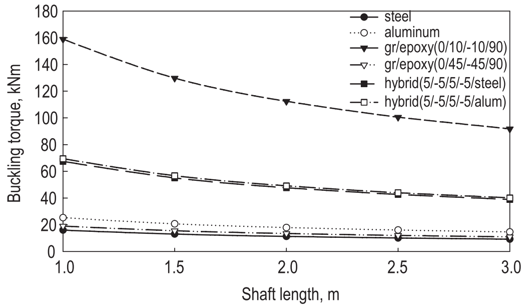

Fig. 5 Variation of buckling torque with shaft length

critical speed, this will give rise to excessive vibrations and stresses. To increase the first fundamental mode of these shafts, generally they are made in two pieces with universal joint and middle support. However, this gives rise to additional system weight, maintenance problems and produces noise and vibrations that are transmitted to the vehicle through the center bearing. Fibre – reinforced composite shaft with $(0°/10°/-10°/90°)$ configuration is having first bending natural frequency of 150 Hz (i.e., 9000 rpm) which is beyond the upper limit of the required operating speed range. Therefore this shaft can work safely under its first fundamental natural frequency. Moreover, fibre-reinforce shaft weight of this configuration is 5.30 kg which is about 4.3% less than the weight of steel shaft. The other fibre-reinforced composite shaft i.e., $(0°/45°/-45°/90°)$ is having more torsional strength due to the presence of $\pm 45°$ ply angles, this is also clear by less weight (as shown in Fig. 4) compared to $(0°/10°/-10°/90°)$ shaft. However, the first bending natural frequency is 106 Hz (i.e., about 6360 rpm) which is on the lower limit of the shaft operating speed range. The fundamental natural frequency of hybridization of composite and steel shaft is found to be 124 Hz, (i.e., about 7440 rpm) which is on the upper limit of the operating speed range. For hybridization of composite and aluminum the first bending natural frequency is 135 Hz, (i.e., about 8100 rpm) which is beyond the range of the operating speed and the shaft weight is 4.85 kg (as shown in Fig. 4). As shown in Fig. 5, the calculated critical buckling torque for different shafts design is grater than the required transmitted torque taking into consideration a factor of safety of 2.25 (Kraus [1988]).

3.2 Design optimization

An ideal composite drive shaft should be light in weight, of low cost to manufacture, and reliable for service. Propeller shafts in automobile are used for transmitting power from engine to the differential of the rear wheel drive vehicle. An optimal design of composite drive shaft could be achieved by selecting the proper variables, which can be identified for safe structure against failure and to meet the design requirements. As the length and outer radius of drive shafts in automotive applications are limited due to spacing, the design variables include the inside radius, layers thickness, number of layers, fiber orientation angle and layers stacking sequence. In optimal design of the drive shaft these variables are constrained by the lateral natural frequency, torsional strength and torsional buckling. The tailorability of elastic constants in composites provides numerous alternatives for the variables to meet the required stability and strength of a structure.

In recent year's improvement in reinforcing fibres and resin properties, fibre-reinforced composites are replacing an increasing number of metal components. The advantage that composite can have over

Table 2 Material properties for optimized shafts, [Rangaswamy, 2002]

Material Property	Steel	HM – Carbon Epoxy	E-Glass Epoxy
Long. Modulus, E_1, Gpa	210	190	50
Trans. Modulus, E_2, Gpa	210	7.7	12
Shear Modulus, G_{12}, Gpa	84	4.2	5.6
Poisson Ratio, υ_{12}	0.3	0.3	0.3
Density, kg/m^3	7830	1600	2000

metallic include, lower weight, better fatigue properties, elimination of corrosion, reduction in assembly piece parts, less vibration and noise and low coefficient of thermal expansion. Initially in the design of fibre-reinforced composite shafts, boron/epoxy and glass/epoxy were used. However with the developments in fibre processing technology graphite fibres replaced boron fibres which were costly and difficult to process. Since in automotive applications the cost of the component is an important consideration, hybridization with glass/epoxy have been tried which proved cost effective. Hybridization with steel and aluminum tubes provides better joints and coupling with metallic parts and better damage requirements.

Since the fiber orientation angle that offers the maximum bending stiffness which leads to the maximum bending natural frequency is to place the fibers longitudinally at zero angle from the shaft axis, on the other hand, the angle of $\pm 45^\circ$ orientation realizes the maximum shear strength and 90° is the best for buckling strength. The main design goal is to achieve the minimum weight while adjusting the variables to meet a sufficient margin of safety, which is translated in a bending natural frequency higher than the operating speed, both of the critical torsional buckling torque and torque transmission capacity must be higher than the applied torque.

To present optimization of composite drive shafts, the case study presented by [Rangaswamy, 2002] based on Genetic Algorithm (GA) is taken. However, in this work, Formulation based on shell finite element modeling for the eigenvalue analysis and optimization based on Simulated Annealing (SA) algorithm is used. Drive shafts made of three different materials, namely, Steel, HM-Carbon Epoxy and E-Glass Epoxy with material properties are designed and optimized. Material properties are kept as those taken by [Rangaswamy, 2002] and presented in Table 2. Drive shafts, for passenger cars, small trucks, and vans, are about 1.25 m in length and generally made from steel. Torque transmission capability should be larger than 3500 Nm, and the fundamental natural bending frequency of the shaft should be higher than 6500 rpm to avoid whirling vibration. The outer diameter restricted to 100 mm due to space limitations and it is taken as 90 mm.

In simulated annealing, an initial temperature of 100 and a temperature reduction factor of 0.8 are selected [Gubran and Gupta, 2002]. As shown in Table 3, the optimal design of a steel drive shaft with outer diameter of 90 mm and 400 Nm torque transmission capacity is found to have a fundamental bending natural frequency of 9420 rpm. This is higher than the recommended value of 6500 rpm. Torsional buckling torque and shaft weight are found to be 47452 Nm and 8.60 kg respectively. It is to be noted here optimization aspects are limited for the design of steel drive shaft. However, in designing HM-Carbon Epoxy and E-Glass Epoxy drive shafts, the objective is to obtain a drive shaft with minimum weight, the constraints are imposed on bending natural frequency and torsional buckling and transmitted torque. Results presented in Table 3, show a saving in shaft weight for that made of HM-Carbon Epoxy by about 88% compared to that of steel shaft. However, drive shaft made of E-Glass Epoxy has a weight reduction by about 73% compared to that of steel shaft.

Table 3 Optimized results

	Shaft Material	d_0, mm	Optimum Configuration, Plies (n_k)	Optimum Thickness t_k, mm	Total Thickness, mm	N_{cr} rpm	$T_{\max}$ Nm	T_{buck} Nm	Weight, kg	Saving %
Present work	Steel	90	–	–	3.32	9420	4000	47452	8.600	–
Rangaswamy, 2002	Steel	90	–	–	3.32	9323	3501	43857	8.600	–
Present work	Carbon/ Epoxy	74.3	[16.32/−59.26/ 58.35/−23.93]s	[0.29242/0.8188/ 0.25382/0.29803]s	2.25	9600	4000	4000	1.01201	88.20
Rangaswamy, 2002	Carbon/ Epoxy	90	[65/25/68/63/36/ 40/39/74/39]s	0.12 × 17 plies	2.04	9270	3656	3765	1.12	86.90
Present work	Glass/ Epoxy	90	[10.63/−15.00/ 17.57/−19.42]s	[0.86538/0.3356/ 0.26341/0.23345]s	3.39	8400	4000	4000	2.30971	73.14
Rangaswamy, 2002	Glass/ Epoxy	90	[46/64/15/13/39/ 84/28/20/27]s	0.4 × 17 plies	6.80	6514	3525	29856	4.4	48.36

Improved results compared to that of [Rangaswamy, 2002 and Ajith, 2004], are obtained by allowing thicknesses of different plies to vary as well as shaft outer radius. This is clearly shown in Table 3, for the case of E-Glass Epoxy shaft, where a reduction in the shaft weight by about 73% is obtained compared to that of 45% obtained by [Rangaswamy, 2002 and Ajith, 2004].

For drive shafts made of thin tubes of large radius, the shear strength which specify the transmitted torque capacity is relatively easy to satisfy. Therefore the main design factors are the bending natural frequency and the torsional buckling, which are functions of the longitudinal and hoop bending stiffness, respectively.

4 Conclusions

In this study, design and optimization of six automotive propeller shafts made of different materials have been studied and compared. Modeling based on degenerated shell finite element for the eigenvalue analysis and optimization based on Simulated Annealing algorithm is considered. Drive shafts made of steel and fibre reinforced composites, HM-Carbon Epoxy and E-Glass Epoxy are designed and optimized using SA algorithm. Optimized results obtained for carbon-epoxy and glass-epoxy are compared with that of steel drive shaft. From the results, the following main conclusions are made:

1. The fundamental natural frequency of a hybrid shaft, made of fibre-reinforced composites and aluminum, is 40% higher than that of steel and aluminum shafts. This makes it possible to design one-piece automotive propeller shaft which leads to reduced system weight, less components and hence less maintenance problems.
2. A weight saving of about 88% and 73% is obtained by optimizing the design of carbon-epoxy and glass-epoxy drive shafts compared to that of steel drive shaft.
3. Making ply thickness and shaft outer diameter as design variables give more improved results.
4. Results obtained show that SA can be used effectively and efficiently in optimization of fibre-reinforced composite structures.
5. Designing propeller driveshafts with fibre-reinforced composite materials gives the designer more design alternatives for proper critical speed placements and required torque transmission.

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