Adaptive Visual Tracking for Robotic Systems
Without Visual Velocity Measurement

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Abstract

In this paper, we investigate the visual tracking for robotic systems without visual velocity measurement, simultaneously taking into account the uncertain depth information, the kinematic and the dynamic uncertainties. We propose a new image-space observer that exploits the image-space velocity information contained in the unknown kinematics, upon which, we design an adaptive controller without using the visual velocity signal where the estimated kinematic and depth parameters are driven by both the image-space tracking errors and observation errors. The major superiority of the proposed observer-based adaptive controller lies in its simplicity and the decomposition of the handling of the multiple uncertainties in visually servoing robotic systems, thus avoiding the overparametrization problem of the existing results. Using Lyapunov analysis, we demonstrate that the closed-loop system is stable and that the image-space tracking errors converge to zero asymptotically. Simulation results are provided to illustrate the performance of the proposed adaptive control scheme.

Index Terms

Visual tracking; Adaptive control; Uncertain depth information; Manipulator.

I. INTRODUCTION

It is generally believed that the incorporation of versatile sensory information (e.g., the information provided by joint position/velocity sensors, tip force/torque sensors and stereo vision systems) into the control system is an important aspect of intelligent robots. Mimicking the action of human beings, more and more manipulators are equipped with cameras to monitor their status and further to perform visual servoing control so that the system can achieve certain robustness.
against model uncertainties (see, e.g., [1], [2]). Many results in the past years have been devoted to the visual servoing problem [1]. [3]. [2]. [4]. [5]. [6]. [7]. These schemes can be classified into two categories (see, e.g., [2]). The first class, e.g., [1], [5], is known as the position-based visual servoing, which simply takes the camera as a specific task-space sensor, i.e., the end-effector position/velocity information is obtained from the camera. One possible disadvantage of this scheme, as frequently stated in the literature (e.g., [2], [7]), is the requirement of the precise/extensive calibration. The second class, e.g., [4], [6], [7], is known as the image-based visual servoing, which directly gives the information of the concerned object in the image space. The advantage of the image-based visual servoing is now well known, i.e., the possible errors in camera modeling and calibration are avoided, and the reduction of the error in the image space implies that of the error in the physical task space (or Cartesian space).

As a standard control methodology, adaptive control has been shown to be adept at treating model uncertainties and be promising to achieve aggressive performance [8]. Since 1980s, numerous adaptive controllers for robot manipulators taking into account the nonlinear robot dynamics have been proposed, see [9], [10], [11], and these controllers are all based on the linearity-in-parameters property of the manipulator model. The recent study in [12], [13], [14] shows how the linearity-in-parameters feature of the robot kinematics is exploited for performing adaptive tracking control in the presence of both the kinematic and dynamic uncertainties, and an interesting property about the overall kinematics of the manipulator using a camera to monitor the end-effector is that, if the depth of the feature point with respect to the camera frame is kept constant, the overall kinematics that describes the mapping from joint space to image space is linearly parameterized [12]. This desirable feature of the overall kinematics, unfortunately, no longer holds in the case of variable depth since the depth acts as the denominator in the overall kinematics, making the overall kinematics depend nonlinearly on the uncertain parameters [7], [15], [16], [17]. Via exploiting the respective linearity-in-parameters property of the depth and the depth-independent interaction matrix, adaptive strategies are developed in [7], [15], [16], [17], [18] to handle the uncertain camera parameters. In particular, the adaptive visual tracking problem is resolved in [15], and the adaptive solutions to the visual regulation problem are given in [17], [18], by designing appropriate adaptation laws to accommodate the uncertainties in manipulator dynamics, kinematics and the camera parameters.

However, one possible limitation of the above results which deal with the tracking problem...
is the requirement of image/visual velocity measurement in the control input. One may notice that the control inputs given in [17], [18] (which are both confined to regulation problem) do not need visual velocity measurement, yet, their extension to the more challenging tracking problem remains unclear. Also note that, in [14], the image-space velocity is avoided in the kinematic parameter adaptation, yet, the control still requires the availability of the image-space velocity. The image-space velocity is usually/commonly obtained by the standard numerical differentiation of the image-space position information. This velocity signal, however, tends to be much noisier than the joint-space velocity partly due to the relatively long processing time or delays of the image information, and thus it is undesirable to use visual velocities in the control. One possible solution is given in [19], extending the result in [12] to the case of variable uncertain depth. The limitation of [19] lies in three aspects: 1) if we hope to accommodate the uncertain dynamics based on [19], the over-parametrization problem of the robot dynamics occurs, and additionally the decomposition of the kinematic and dynamic uncertainties is impossible, 2) the determination of the controller parameters relies on some priori knowledge of the manipulator dynamics and kinematics, and 3) due to 2), it requires high control activity (due to velocity dependent feedback gain) to accommodate the variation of the depth, which means that high velocity of the manipulator will demand the undesirable high-gain feedback. So, the best result we can achieve using the scheme in [19] is still conservative. Other adaptive control schemes appear in [20], [21], [22], where the results in [20], [21] propose cascade framework, and the work in [22] proposes task-space observer-based controller to achieve image-space tracking of electrically driven robots where the desired armature current does not use the task-space velocity. The results in [20], [21], [22], in contrast to [19], take into consideration the uncertain robot kinematics and dynamics. Yet, the results of [20], [22] can only deal with the case that the depth is constant, and the controller given in [21] needs to obtain the end-effector position with respect to the manipulator base frame so as to perform the kinematic parameter estimation [refer to equation (21) in [21]] (which means that it is not a completely image-based visual servoing but a combination of image-based and position-based schemes, thus, demanding the tedious calibration and vulnerable to modeling errors). Moreover, the SDU factorization adopted in [21] results in the undesirable complexity in both the analysis and controller design.

In our opinion, decomposition for the handling of multiple uncertainties of the system is highly preferred, whose superiority may be the avoidance of overparametrization, the simplification of
the control scheme, and consequently better performance of the closed-loop system. Along this idea, in this paper, we propose an observer based adaptive control scheme for visual tracking with varying depth information (extending the constant depth case in [22], [20]) and with uncertain manipulator kinematics and dynamics. The proposed adaptive controller avoids the measurement of visual velocity and realizes decomposition of the handling of the depth, kinematic and dynamic uncertainties. Using a depth-dependent Lyapunov function, we show the stability and convergence of the closed-loop robotic system. In contrast to the velocity-dependent-gain feedback and the overparametrization problem in [19], our control scheme employs a constant-gain feedback taking into consideration the uncertain robot dynamics and kinematics in addition to the camera uncertainties and achieves decomposition of the handling of the depth, kinematic and dynamic uncertainties (avoiding the overparametrization of the robot dynamics). Moreover, the tedious calibration and vulnerability to model uncertainties of [21] (due to the kinematic parameter estimation) are conquered by the proposed completely image-based servoing controller, and additionally, the simplicity of the proposed control scheme is much preferred than the complex one in [21].

II. KINEMATICS AND DYNAMICS

In this paper, we assume that the standard fixed pinhole cameras (see, e.g., [23]) are adopted to monitor the motion of the manipulator end-effector, and the number of the feature points is $m$. In the case that cameras are employed to detect the end-effector position, the task space becomes the image space, whose units are pixels. Let $x_i \in \mathbb{R}^2$ (with units being pixels) represent the projection of the $i$-th feature point on the camera image frame, and $r_i \in \mathbb{R}^p$ denote the position of the $i$-th feature point with respect to the base frame of the manipulator, $i = 1, 2, \ldots, m$. Via the image Jacobian matrix [2] or the interaction Jacobian matrix [1], the relationship between the image velocity $\dot{x}_i$ and the feature-point velocity $\dot{r}_i$ can be written as [2], [7]

$$\dot{x}_i = \frac{1}{z_i(q)} N_i(x_i) \dot{r}_i$$

(1)

where $z_i(q) \in \mathbb{R}$ denotes the depth of the $i$-th feature point with respect to the camera frame, $N_i(x_i) \in \mathbb{R}^{2 \times p}$ is the depth-independent interaction matrix [7], $i = 1, 2, \ldots, m$, and $q \in \mathbb{R}^n$ denotes the joint position vector.
Equation (1) can be rewritten as the following compact form

\[ \dot{x} = Z^{-1}(q)N(x)\dot{r} \]  

(2)

where \( x = [x_1^T, x_2^T, \ldots, x_p^T]^T, r = [r_1^T, r_2^T, \ldots, r_p^T]^T, Z(q) = \text{diag}[z_1(q)I_2, z_2(q)I_2, \ldots, z_p(q)I_2], \)

where \( I_2 \) is the 2 by 2 identity matrix, and

\[ N(x) = \text{diag}[N_1(x_1), N_2(x_2), \ldots, N_p(x_p)]. \]

The relationship between the feature-points’ velocity \( \dot{r} \) and the manipulator joint velocity is \[ \dot{r} = J_m(q)\dot{q} \]

(3)

where \( \dot{q} \) is the joint velocity vector, and \( J_m(q) \in R^{(pm)\times n} \) denotes the manipulator Jacobian matrix.

The combination of (2) and (3) gives rise to the overall kinematic equation \[ \dot{x} = Z^{-1}(q)J(q, x)\dot{q} \]

(4)

where \( J(q, x) = N(x)J_m(q) \) is a Jacobian matrix that is independent of the depth information.

The equations of motion of the manipulator can be written as \[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \]

(7)

where \( M(q) \in R^{n\times n} \) is the inertia matrix, \( C(q, \dot{q}) \in R^{n\times n} \) is the Coriolis and centrifugal matrix, \( g(q) \in R^n \) is the gravitational torque, and \( \tau \in R^n \) is the exerted joint torque.

Several fundamental properties of the kinematics and dynamics of the visually servoing robotic system are listed as follows, where the first three properties can be found in \[ [8], [25], \] and Property 4 is from \[ [12], [7]. \]
Property 1: The inertia matrix $M(q)$ is symmetric and uniformly positive definite.

Property 2: The Coriolis and centrifugal matrix $C(q, \dot{q})$ can be suitably selected such that $\dot{M}(q) - 2C(q, \dot{q})$ be skew-symmetric.

Property 3: The manipulator dynamics depends linearly on an unknown constant dynamic parameter vector $a$

$$M(q)\dot{\xi} + C(q, \dot{q})\xi + g(q) = Y_d(q, \dot{q}, \xi, \dot{\xi})a_d$$  \tag{8}

where $Y_d(q, \dot{q}, \xi, \dot{\xi})$ is the dynamic regressor matrix, $\xi \in \mathbb{R}^n$ is a differentiable vector and $\dot{\xi}$ is the derivative of the vector $\xi$ with respect to time.

Property 4: In the overall kinematics, the two quantities $Z(q)\chi$ and $\dot{Z}(q)\chi$ can be linearly parameterized, i.e.,

$$Z(q)\chi = Y_z(q, \chi)a_z \tag{9}$$

$$\dot{Z}(q)\chi = \dot{Y}_z(q, \dot{q}, \chi)a_z \tag{10}$$

and $J(q, x)\dot{q}$ can also be linearly parameterized, i.e.,

$$J(q, x)\dot{q} = Y_k(q, x, \dot{q})a_k \tag{11}$$

where $a_z$ and $a_k$ are the unknown constant depth and kinematic parameter vectors, respectively, $Y_z(q, \chi)$, $\dot{Y}_z(q, \dot{q}, \chi)$, and $Y_k(q, x, \dot{q})$ are the regressor matrices, and $\chi \in \mathbb{R}^{2m}$ is a vector.

III. Observer-Based Adaptive Tracking

In this section, we investigate the adaptive visual tracking for robotic systems with uncertain kinematics, dynamics and varying depth information. We will at first develop an image-space observer such that the measurement of image-space velocity can be avoided. Then, based on the designed observer, we propose an adaptive tracking controller to realize the asymptotic tracking in image space, i.e., to ensure $x - x_d \rightarrow 0$ and $\dot{x} - \dot{x}_d \rightarrow 0$ as $t \rightarrow \infty$, where $x_d$ denotes the desired trajectory in image space and we assume that $x_d, \dot{x}_d, \ddot{x}_d$ are all bounded.

For achieving the above objective, we design the following image-space observer

$$\dot{x}_o = \dot{Z}^{-1}(q)\dot{J}(q, x)\dot{q} - \frac{1}{2} \dot{Z}^{-1}(q)\dot{Z}(q) \times [(x_o - x) - 3(x - x_d)] - \alpha (x_o - x) \tag{12}$$
where \( x_o \) denotes the observed signal of the image-space position, \( \alpha > 0 \) is a positive design constant, and \( \hat{Z}(q), \dot{\hat{Z}}(q) \) are the estimates of \( Z(q), \dot{Z}(q) \), respectively, which are obtained by replacing \( a_z \) in \( Z(q), \dot{Z}(q) \) with its estimate \( \hat{a}_z \). The employment of the second term on the right side of (12) is to account for the varying nature of the depth \( Z(q) \). The estimated Jacobian matrix \( \hat{J}(q, x) \) is obtained by replacing \( a_k \) in \( J(q, x) \) with its estimate \( \hat{a}_k \).

The closed-loop observer dynamics can be written as

\[
\Delta \dot{x}_o = \hat{Z}^{-1}(q) \hat{J}(q, x) \dot{q} - Z^{-1}(q) J(q) \dot{q} - \frac{1}{2} \hat{Z}^{-1}(q) \dot{\hat{Z}}(q) \times (\Delta x_o - 3 \Delta x) - \alpha \Delta x_o
\]

where \( \Delta x_o = x_o - x \) and \( \Delta x = x - x_d \). Equation (13) can be further formulated as,

\[
Z(q) \Delta \dot{x}_o = \left[ Z(q) - \dot{\hat{Z}}(q) \right] \dot{\hat{Z}}^{-1}(q) \hat{J}(q, x) \dot{q} + \hat{J}(q, x) \dot{q} - J(q, x) \dot{q} - \frac{1}{2} Z(q) \dot{\hat{Z}}^{-1}(q) \dot{\hat{Z}}(q) (\Delta x_o - 3 \Delta x) - \alpha Z(q) \Delta x_o
\]

Let us rewrite (14) as (by Property 4)

\[
Z(q) \Delta \dot{x}_o + \frac{1}{2} \dot{\hat{Z}}(q) (\Delta x_o - 3 \Delta x) = -Y_z (q, \dot{\hat{Z}}^{-1}(q) \hat{J}(q) \dot{\hat{Z}}(q) \dot{q}) \Delta a_z + Y_k (q, x, \dot{q}) \Delta a_k + \frac{1}{2} \dot{\hat{Z}}(q) (\Delta x_o - 3 \Delta x) - \frac{1}{2} Z(q) \dot{\hat{Z}}^{-1}(q) \dot{\hat{Z}}(q) (\Delta x_o - 3 \Delta x)
\]

\[
- \alpha Z(q) \Delta x_o
\]

where \( \Delta a_z = \hat{a}_z - a_z \) and \( \Delta a_k = \hat{a}_k - a_k \) are the depth and kinematic parameter estimation errors, respectively, and the vector \( \Pi \) can be interestingly written as (again by Property 4)

\[
\Pi = \frac{1}{2} \left[ \dot{\hat{Z}}(q) - \dot{\hat{Z}}(q) \right] (\Delta x_o - 3 \Delta x) + \frac{1}{2} \left[ \dot{\hat{Z}}(q) - Z(q) \right] \dot{\hat{Z}}^{-1}(q) \dot{\hat{Z}}(q) (\Delta x_o - 3 \Delta x)
\]

\[
= -\frac{1}{2} Y_z (q, \dot{q}, \Delta x_o - 3 \Delta x) \Delta a_z + \frac{1}{2} Y_z (q, \dot{\hat{Z}}^{-1}(q) \dot{\hat{Z}}(q))(\Delta x_o - 3 \Delta x) \Delta a_z
\]
In this way, the closed-loop observer dynamics can be written as

\[
Z(q)\Delta \dot{x}_o + \frac{1}{2}\dot{Z}(q)(\Delta x_o - 3\Delta x) = -\alpha Z(q)\Delta x_o - Y^*_z \Delta a_z + Y_k(q, x, \dot{q})\Delta a_k
\]  

(17)

where the combined depth regressor \(Y^*_z\) is defined by

\[
Y^*_z = Y_z(q, \hat{Z}^{-1}(q)\dot{J}(q)q) + \frac{1}{2}Y_z(q, \dot{q}, \Delta x_o - 3\Delta x) - \frac{1}{2}Y_z(q, \hat{Z}^{-1}(q)\dot{\hat{Z}}(q)(\Delta x_o - 3\Delta x))
\]  

(18)

Next, we develop an adaptive controller based on the observed signal generated by the observer (12). At this place, the depth kinematics (6) and the decomposition property (5) will be exploited for the adaptive controller design.

Let us define the joint-space reference velocity \(\dot{q}_r\) as

\[
\dot{q}_r = [\dot{J}(q, x_o) + \frac{1}{2}\text{diag} [x_o - x_d] [\dot{J}_z(q) \otimes [1, 1]^T]]^+ [\dot{X}(q)\dot{x}_r]
\]  

(19)

where \(\dot{J}^+ = \dot{J}^T [\dot{J}^* \dot{J}^*]^T\) is the standard generalized inverse of \(\dot{J}^*\), the image-space reference velocity \(\dot{x}_r = \dot{x}_d - \alpha(x_o - x_d)\), and the use of \(\dot{J}(q, x_o)\) (which is obtained by replacing \(x\) in \(\dot{J}(q, x)\) with the observed image-space position \(x_o\)) instead of \(\dot{J}(q, x)\) is to avoid the image-space velocity measurement in the derivative of \(\dot{q}_r\). Differentiating (19) with respect to time gives the reference acceleration \(\ddot{q}_r\)

\[
\ddot{q}_r = \dot{\dot{J}}^+ [\dot{Z}(q)\dot{x}_r + \dot{X}(q)\dot{x}_r - \dot{J}^*\dot{q}_r]
\]  

(20)

Note that neither \(\dot{q}_r\) nor \(\ddot{q}_r\) involves the measurement of visual velocity \(\dot{x}\).

**Remark 1**: The modified estimated Jacobian matrix \(\dot{J}^*\) exploits the information contained in the depth kinematics (6). The use of this modified estimated Jacobian matrix instead of the usual estimated Jacobian matrix \(\dot{J}(q, x)\) is to accommodate the effect of the varying depth information.

Then, define the following sliding vector

\[
s = \dot{q} - \dot{q}_r
\]  

(21)
Using $\hat{J}^*$ to multiply both sides of (21) and exploiting Property 4 gives

$$\hat{J}^* s = \hat{J}(q, x) \dot{q} + \frac{1}{2} \hat{Z}(q)(x_o - x_d + 2\Delta x_o) - \hat{Z}(q) \dot{x}_r$$

$$= Z(q) [\ddot{x} - \dot{x}_d + \alpha(x_o - x_d)]$$

$$+ \frac{1}{2} \hat{Z}(q)(x_o - x_d + 2\Delta x_o) + Y_k(q, x, \dot{q}) \Delta a_k$$

$$+ \left[ \frac{1}{2} Y_z(q, \dot{q}, x_o - x_d + 2\Delta x_o) - Y_z(q, \dot{x}_r) \right] \Delta a_z$$

(22)

Now we propose the control law as

$$\tau = Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \hat{a}_d - \hat{J}^* T K \hat{J}^* s$$

(23)

where $K \in R^{n \times n}$ is a symmetric positive definite matrix, $\hat{a}_d$ is the estimate of $a_d$, which is updated by the adaptation law

$$\dot{\hat{a}}_d = -\Gamma_d Y_d^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) s$$

(24)

where $\Gamma_d$ is a symmetric positive definite matrix, and the adaptation laws for the estimated parameters $\hat{a}_k, \hat{a}_z$ are given as

$$\dot{\hat{a}}_k = \Gamma_k Y_k^T(q, x, \dot{q}) (\Delta x - \Delta x_o)$$

(25)

$$\dot{\hat{a}}_z = \Gamma_z Y_z^* \Delta x + Y_z^* \Delta x_o$$

(26)

where $\Gamma_k, \Gamma_z$ are both symmetric and positive definite matrices.

Substituting the control law (23) into the manipulator dynamics yields

$$M(q) \ddot{s} + C(q, \dot{q}) s = -\hat{J}^* T K \hat{J}^* s + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \Delta a_d$$

(27)

where $\Delta a_d = \hat{a}_d - a_d$ is the dynamic parameter estimation error.
The full closed-loop dynamics can be written as the following cascade form

\[
\begin{align*}
Z(q)\Delta \dot{x}_o + \frac{1}{2} \dot{Z}(q)(\Delta x_o - 3\Delta x) \\
= -\alpha Z(q)\Delta x_o - Y_z^* \Delta a_z + Y_k(q, x, \dot{q}) \Delta a_k \\
Z(q)\Delta \dot{x} + \frac{1}{2} \dot{Z}(q)(x_o - x_d + 2\Delta x_o) \\
= -\alpha Z(q)(x_o - x_d) - Y_k(q, x, \dot{q}) \Delta a_k - Y_z^* \Delta a_z + \dot{j}^* s \\
M(q)\dot{s} + C(q, \dot{q}) s \\
= -\dot{j}^* T \dot{j}^* s + Y_d(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \Delta a_d
\end{align*}
\] (28)

We are presently ready to formulate the following theorem.

**Theorem 1:** The observer (12), the control (23) and the adaptation laws (24), (25), (26) for the visually servoing robotic system (4), (7) guarantee the stability and convergence of the image-space tracking errors, i.e., \( x - x_d \to 0 \) and \( \dot{x} - \dot{x}_d \to 0 \) as \( t \to \infty \).

**Proof:** Following [10], for the third subsystem in (28), we consider the Lyapunov-like function candidate

\[
V_1 = \frac{1}{2} s^T M(q) s + \frac{1}{2} \Delta a_d^T \Gamma_d^{-1} \Delta a_d
\] (29)

whose time derivative along (27) can be written as (exploiting the skew symmetry of \( \dot{M}(q) - 2C(q, \dot{q}) \), i.e., Property 2)

\[
\dot{V}_1 = -s^T \dot{j}^* T \dot{j}^* s \leq 0
\] (30)

which implies that \( s \in L_{\infty} \), \( \dot{j}^* s \in L_2 \), and \( \dot{a}_d \in L_{\infty} \). The fact that \( \dot{j}^* s \in L_2 \) and \( Z(q) \) is uniformly positive definite yields the result that \( \int_0^t s^T \dot{j}^* T Z^{-1}(q) \dot{j}^* s dr < \bar{I}_M \), \( \forall t \geq 0 \) for some positive constant \( \bar{I}_M \).

For the upper two subsystems in (28), let us consider the following depth-dependent Lyapunov-
like function candidate

\[
V_2 = \frac{1}{2} \Delta x^T Z(q) \Delta x + \frac{1}{2} \Delta x_o^T Z(q) \Delta x_o \\
+ \frac{1}{2} \Delta a_k \Gamma_k^{-1} \Delta a_k + \frac{1}{2} \Delta a_z \Gamma_z^{-1} \Delta a_z \\
+ \frac{1}{\alpha} \left[ \bar{I}_M - \int_0^t s^T \hat{J}^* T Z^{-1}(q) \hat{J}^* s dr \right] \tag{31}
\]

where the term \( \Pi^* \geq 0 \) on the right side of the above equation is inspired by [27] [i.e., the positive definite function used in the proof of Lemma 4.1 in [27] (p. 118)].

The derivative of \( V_2 \) along the upper two subsystems in (28) is

\[
\dot{V}_2 = -\alpha \Delta x_o^T Z(q) \Delta x_o - \alpha \Delta x^T Z(q)(x_o - x_d) \\
- (\Delta x - \Delta x_o)^T Y_k(q, x, \dot{q}) \Delta a_k \\
- (\Delta x^T Y_z^* + \Delta x_o^T Y_z^*) \Delta a_z \\
+ \Delta a_k \Gamma_k^{-1} \dot{a}_k + \Delta a_z \Gamma_z^{-1} \dot{a}_z \\
+ \Delta x^T \hat{J}^* s - \frac{1}{\alpha} s^T \hat{J}^* T Z^{-1}(q) \hat{J}^* s \tag{32}
\]

Substituting the adaptation laws (25) and (26) into (32) gives

\[
\dot{V}_2 = -\alpha \Delta x_o^T Z(q) \Delta x_o - \alpha \Delta x^T Z(q) \Delta x \\
- \alpha \Delta x^T Z(q) \Delta x_o + \Delta x^T \hat{J}^* s \\
- \frac{1}{\alpha} s^T \hat{J}^* T Z^{-1}(q) \hat{J}^* s \tag{33}
\]

Using the following two standard inequalities

\[
-\Delta x^T Z(q) \Delta x_o \leq \frac{1}{2} \Delta x^T Z(q) \Delta x + \frac{1}{2} \Delta x_o^T Z(q) \Delta x_o \\
\Delta x^T \hat{J}^* s \leq \frac{1}{4} \alpha \Delta x^T Z(q) \Delta x + \frac{1}{\alpha} s^T \hat{J}^* T Z^{-1}(q) \hat{J}^* s
\]

we can rewrite (33) as

\[
\dot{V}_2 \leq -\frac{\alpha}{4} \Delta x^T Z(q) \Delta x - \frac{\alpha}{2} \Delta x_o^T Z(q) \Delta x_o \leq 0 \tag{34}
\]

The inequality (34) gives the result that \( \Delta x \in L_2 \cap L_\infty \), \( \Delta x_o \in L_2 \cap L_\infty \), \( \dot{a}_k \in L_\infty \) and \( \dot{a}_z \in L_\infty \). If \( \hat{J}^* \) is non-singular, we obtain \( \dot{q}_r \in L_\infty \) from equation (19) since \( \hat{Z}(q) \) and \( \dot{x}_r \) are
both bounded. From the boundedness of \( s \), we have \( \dot{q} \in L_\infty \). From (12), we get the boundedness of \( \dot{x}_o \), giving rise to the boundedness of \( \dot{x}_r \). From the adaptation laws (25) and (26), we derive the boundedness of \( \dot{a}_k \) and \( \dot{a}_z \), which means that \( \dot{Z}(q) \) and \( \dot{J}^* \) are bounded. Therefore, we obtain \( \dot{q}_r \in L_\infty \) from (20). From the closed-loop dynamics (27), we obtain the boundedness of \( \dot{s} \) since \( M(q) \) is uniformly positive definite, which, plus the boundedness of \( \dot{q}_r \), yields the boundedness of \( \ddot{q} \). Then, from the overall kinematics (4) and its differentiat ion

\[
\ddot{x} = Z^{-1}(q)J(q)\ddot{q} - Z^{-1}(q)\dot{Z}(q)Z^{-1}(q)J(q)\dot{q} + Z^{-1}(q)\dot{J}(q)\dot{q},
\]

we obtain \( \dot{x} \in L_\infty \) and \( \ddot{x} \in L_\infty \). We also obtain the boundedness of \( \dddot{x}_o \) from the differentiation of equation (12). Hence, \( \Delta x_o, \Delta x, \Delta \dddot{x}_o, \) and \( \Delta \dddot{x} \) are all uniformly continuous. From the result in [27] (p. 117), we obtain \( \Delta x_o \to 0 \) and \( \Delta x \to 0 \) as \( t \to \infty \). Then, from Barbalat’s Lemma [8], we have \( \Delta \dddot{x}_o \to 0 \) and \( \Delta \dddot{x} \to 0 \) as \( t \to \infty \).

**Remark 2:** The avoidance of visual velocity measurement is achieved at the kinematic level, thus, resulting in the decomposition of the handling of the kinematic and dynamic uncertainties. In addition, the closed-loop system is fully cascaded, i.e., the kinematic loop and dynamic loop are designed separately.

**Remark 3:** Compared with the existing results [12], [15], [19], [21], the main new point of our result lies in the proposed observer (12), the definition of the reference velocity (19), the image-space-velocity-free adaptation laws (25) and (26), and the proposed depth-dependent Lyapunov function (31) plus the associated stability analysis. The adaptation law (25) for updating \( \hat{a}_k \) coincides with the one in [22], [20] except for the fact that in the case of varying depth, the regressor matrix \( Y_k(q, x, \dot{q}) \) involves the image-space position signal, yet, the result in [22], [20] only consider the simpler case of constant depth. The control law (23) and the dynamic adaptation law (24) is basically the same as the one in [12] (i.e., an extension of [10] to account for the uncertain kinematics), just with new reference velocity and acceleration.

**Remark 4:** The projection versions of the kinematic parameter adaptation law (25) and the depth parameter adaptation law (26) can be adopted such that the estimated kinematic parameter

\footnote{The task-space observer and the desired armature current given in [22] (which deals with the adaptive control of electrically driven robots) make us believe that one can obtain the solution for rigid robots (a reduced case of electrically driven robots) from [22] and would find that the adaptation law (25) is the same as this solution.}
is in an appropriate domain and the estimated depth $\hat{Z}(q)$ is uniformly positive definite [15, 17].

IV. SIMULATION RESULTS

In this section, we present the simulation results to show the performance of the proposed observer-based adaptive controller, and we consider a typical 3-DOF anthropomorphic robot with a fixed camera to monitor the robot end-effector position (see Fig. 1). The focal length of the camera is set as $f = 0.15$ m and the scaling factor of the camera $\beta = 900$. The three axes of the camera frame (denoted by $X_C, Y_C$ and $Z_C$, respectively) are assumed to be aligned with the axes $Y_0, Z_0$ and $X_0$ of the manipulator base frame, respectively, yet, there is an offset $d_C = 5$ m along the axis $Z_C$ between the origins of the two frames. The lengths of the three links of the manipulator are $l_1 = 2.0$ m, $l_2 = 2.0$ m, and $l_3 = 2.0$ m, respectively. The mass and inertia properties of the manipulator are not listed due to the space limitation. The sampling period is chosen to be 2 ms.

The controller parameters are determined as $K = 0.01I$, $\alpha = 20.0$, $\Gamma_d = 10.0I$, $\Gamma_k = 0.5I$ and $\Gamma_z = 0.5I$. The initial values of the parameter estimates are chosen as $\hat{l}_2 = \hat{l}_3 = 3.0$ m, $\hat{d}_C = 3.0$ m, $\hat{f} = 0.1$ m, and $\hat{\beta} = 700.0$. The initial dynamic parameter estimate is chosen as $\hat{a}_d = [0^T, 25, 0]^T$. The desired trajectory in the image space is given as

$$x_d = \begin{bmatrix} 52 + 10 \cos(\pi t/3) \\ 65 + 10 \sin(\pi t/3) \end{bmatrix} \quad (35)$$

The simulation results are shown in Fig. 2, Fig. 3, and Fig. 4. From Fig. 2 and Fig. 3, we see that the image-space tracking errors indeed converge to zero asymptotically. Fig. 4 gives the actual depth information and the estimated depth information, respectively, during the motion of the manipulator. It seems that the estimated depth tries to track the actual depth. Although convergence of the depth estimation error does not occur, the image-space trajectory tracking convergence is still guaranteed.

V. CONCLUSION

In this paper, we examine the visual servoing problem for robotic systems with uncertain depth information, uncertain kinematics and dynamics, and the visual velocity is assumed to be unavailable. To achieve visual tracking without visual velocity measurement, we propose a novel
Fig. 1. A 3-DOF anthropomorphic robotic system with a fixed camera

Fig. 2. Image-space tracking errors
Fig. 3. The desired and actual trajectories in the image space

Fig. 4. The actual and estimated depths
image-space observer and an adaptive controller based on the observed signal, which results in a cascade closed-loop robotic system. Using a depth-dependent Lyapunov function, plus the standard Lyapunov-like function for analyzing the Slotine and Li adaptive controller, we demonstrate that the closed-loop system is stable and the image-space tracking errors asymptotically converge to zero. We also show the asymptotic convergence of the image-space observation errors. Simulation results are given to show the performance of the proposed observer-based adaptive controller.

REFERENCES


