STABILIZED INVERSE DIFFUSION EQUATIONS AND SEGMENTATION OF VECTOR-VALUED IMAGES.

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Abstract

We further investigate and generalize the stabilized inverse diffusion equations ("SIDEs"), which were introduced and analyzed in [6, 7]. We demonstrate their robustness and applicability to the segmentation of color images.

1. INTRODUCTION.

The upsurge in using evolutions specified by partial differential equations (PDE's) as image processing procedures for tasks such as edge enhancement and segmentation, among others (see [3, 4, 8] and references therein) is largely due to their versatility in efficiently achieving selective nonlinear filtering. While the analysis of these techniques is most often performed in the continuous setting, where an image is identified with a function of two continuous spatial variables, the implementation of such equations generally involves their discrete approximation. Following Weickert [9], Lindeberg [3], and other researchers, we concentrated in [6] on semi-discrete scale spaces (i.e., continuous in scale (or time) and discrete in space). More specifically, we introduced a new family of semi-discrete evolution equations which stably sharpen edges and suppress noise, which we termed "stabilized inverse diffusion equations" (or "SIDEs"). The scale space of such an equation is a family of segmentations of the original image, with larger values of the scale parameter t corresponding to segmentations at coarser resolutions. Initially, the finest possible segmentation is assumed: each pixel is a separate region. In the course of evolution, two neighboring regions are merged whenever the difference between their intensity values becomes equal to zero. The intensity value u_i of the i-th region evolves according to

$$\dot{u}_i = \frac{1}{m_i} \sum_{j \in A_i} F(u_j - u_i) p_{ij}, \qquad (1)$$

where

- m_i is the area of the i-th region (i.e., the number of pixels in it);
- A_i is the set of the indices of all the neighbors of region i;
- p_{ij} is the length of the boundary between regions i and j;
- F is a monotonically decreasing odd function, discontinuous at zero, and non-negative for positive values of the argument.

The connections of this equation to other non-linear image restoration and segmentation techniques, such as region-merging methods [2, 4] and Perona-Malik equation [5], are discussed in [6]. It is also shown in [6] that SIDEs have many important properties, such as stability, computational efficiency, and robustness. These properties are well illustrated by applying SIDEs to the segmentation of synthetic aperture radar (SAR) imagery in which speckle noise is a well-known problem that has defeated many algorithms. SAR image formation also has a natural blur associated with it, due to the finite aperture used in forming the image. A prototypical SAR log-magnitude image of two textural regions–forest and grass–is shown in Fig. 1, (a). The two-region segmentation produced by SIDE (Fig. 1, (b)) is very accurate, thus demonstrating robustness to both blurring and large-amplitude noise.

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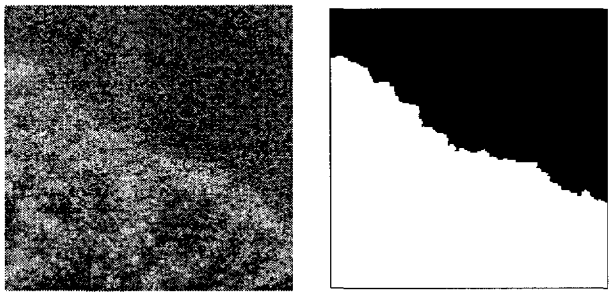


(a) (b)

Figure 1: (a) SAR image and (b) its segmentation.

2. SEGMENTATION OF COLOR IMAGES.

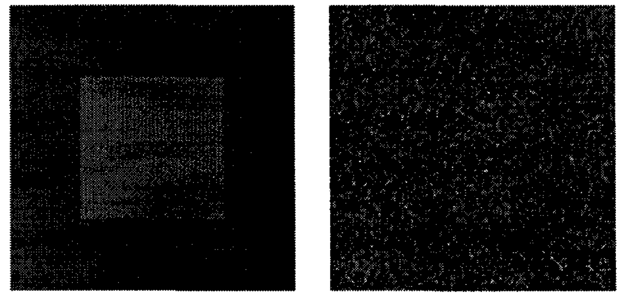
We now use one of the theoretical properties mentioned in [6] in order to generalize our evolution equations to vector-valued images. It is easily checked by differentiation that Eq. (1) is a (weighted) gradient descent equation for the global energy

$$\mathcal{E} = \sum_{j \in A_i} E(||u_j - u_i||)p_{ij}, \qquad (2)$$

where E is an antiderivative of the force function F and is therefore concave everywhere except at zero and non-differentiable at zero. The norm $||\cdot||$ here stands simply for the absolute value of its scalar argument. Now notice that we still can use Eq. (2) if the image under consideration is vector-valued, i.e., if $\vec{u}_j, \vec{u}_i \in \mathbb{R}^n$. In this case, the descent equation is:

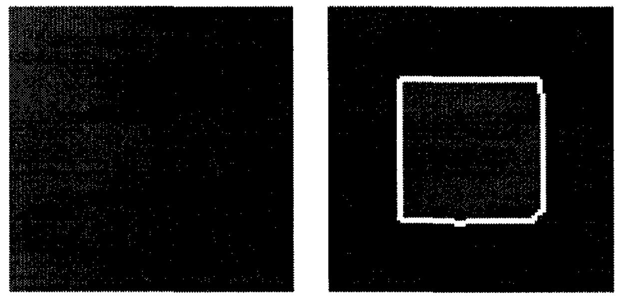
$$\dot{\vec{u}}_i = \frac{1}{m_i} \sum_{j \in A_i} \frac{\vec{u}_j - \vec{u}_i}{||\vec{u}_j - \vec{u}_i||} F(||\vec{u}_j - \vec{u}_i||)p_{ij}. \qquad (3)$$

It can be verified that this equation inherits many useful properties of the scalar equation (1), such as the conservation of mean, the local maximum principle for each channel, reaching the steady state in finite time, and–for functions F which are infinite at zero–well-posedness almost everywhere. The proofs from [6] carry over with minor modifications. Vector-valued SIDEs are also robust to severe noise, as we show in Figures 2 and 3. The color image in Figure 2, (a) consists of two regions: two of its three color channels undergo an abrupt change at the boundary between the regions. More precisely, the {red,green,blue} channel values are $\{0.1, 0.6, 0.9\}$ for the background and $\{0.6, 0.6, 0.5\}$ for the square. Each channel is corrupted with white Gaussian noise whose standard deviation is 0.4 (Figure 2, (b)). The image in Figure 3, (a) is the result of evolving a vector-valued SIDE on the noisy image, until exactly two regions remain. The final boundary, superimposed onto the initial image, is depicted in Figure 3,



(a) (b)

Figure 2: (a) A test image and (b) its noisy version.



(a) (b)

Figure 3: (a) The two-region segmentation of the noisy image in Figure 2; (b) the boundary, superimposed onto the noise-free image.

(b). Just as in the scalar case, the algorithm is very accurate in locating the boundary. Moreover, since the state dimension is being reduced during the evolution, vector-valued SIDEs are fast (just like scalar ones), being much faster (per iteration) than other non-linear diffusions.

3. FUTURE WORK.

Another paper presented at this conference [1] explores other ways of using SIDEs for segmentation of color images and compares SIDEs to different methods, in the context of an important medical imaging application. We are now working on the application of the present method to segmentation of dermatoscopic images.

Color images are only one instance of vector-valued images. We are also currently investigating the application of our technique to texture segmentation.

4. REFERENCES

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