Alfred O. Hero, *University of Michigan*, and
Hamid Krim, *North Carolina State University*

Mathematical Methods in Imaging

Mathematical methods have had a long history in imaging. For example, as recently pointed out by Glasbey and Mardia [3], in the later part of the 19th century Sir Francis Galton applied warping to align and average portrait photographs of known criminals with the hope of discovering the archetypical "criminal face" [2]. (Galton was a polymath geneticist, anthropologist, philosopher, and Africa explorer who established the field of biometry. Among his other exploits were founding the fields of eugenics, differential psychology, and fingerprint analysis; pioneering the field of meteorology; developing rank-order statistics; and inventing the statistical methods of correlation and regression [1].) More recent examples of mathematics in imaging include: analytical methods for tomographic image reconstruction, differential geometric methods for shape and surface representation, and projective geometries for three-dimensional and group theoretic representations of image invariants. The articles in this special issue highlight the imaging applications of some emerging mathematical approaches including fractals and self-similarity, nonlinear partial differential equations (PDEs), anisotropic diffusion, variational methods, pattern theory, stochastic point processes, and random graph theory.

The special issue opens up with three articles on applying PDEs to images. In the first article Tschumperlé and Deriche present vector-valued diffusion PDEs for restoration and noise removal in color images. These multivalued PDEs are ingeniously constructed to produce a solution (image restoration) having desired local geometrical properties. In the second article Pollak presents a nonlinear diffusion approach to multiscale segmentation and noise suppression. Among other interesting results, the author shows that nonlinear scale-space curve evolution is fast and optimal in the sense of maximizing a norm-penalized maximum likelihood (ML) criterion. Ben Hamza, Unal, and Krim round out this group of articles by providing a unifying framework for PDE diffusion approaches covering both stochastic (Markovian) and deterministic (energy constrained) image models. Using insights gained from this unifying perspective they present an interesting hybrid gradient descent flow algorithm based on a combination of negentropy and total variation norms of the gradient field.


©2002 CORBIS

The special issue continues with two articles on stochastic image modeling. The article by Pesquet-Popescu and Lévy Véhel presents stochastic fractal and multifractal models for texture in images. These models are related to fractional Brownian motion (fBM) random field representations, and the authors discuss extensions of these representations to useful nonisotropic models which can be applied to noncontinuous digitized imagery. Srivastava presents an intriguing new class of mixture models for capturing observed heavytailed marginal densities of image gradients. These models are Bessel K forms which exploit the fact that natural images contain many separate objects that can be modeled as components following a Poisson point process.

The issue concludes with two articles on image analysis. Descombes and Zerubia present a marked point process framework as an alternative

1053-5888/02/$17.00©2002IEEE

to Markov random fields to better capture the geometrical constraints and non-Gaussian noise attributes of natural images. The authors describe a novel birth and death process for expanding and pruning random graphs defining a path through the configuration space, e.g., a road network on a roadmap or the edges of buildings on a digital elevation map (DEM). We wrap up this special issue with a article by Hero, Ma, Michel, and Gorman, who review the intriguing asymptotic theory of random graphs and use this theory to motivate minimal graph algorithms for pattern matching, image registration, and image retrieval. The key property underlying these algorithms is that good minimal graphs are "entropic": their total weight can be used as a pattern discriminant which converges to the Rényi entropy of the points connected by the graph.

References

[1] "Obituary of Sir Francis Galton D.C.L. D.Sc F.R.S.," *J. Royal Statistical Society*, pp. 314-320, Feb. 1911.

[2] F. Galton, "Composite portraits," *J. Anthropol. Inst. Great Britain and Ireland*, vol. 8, pp. 132-142, 1878.

[3] C.A. Glasbey and K. Mardia, "A penalised likelihood approach to image warping (with discussion)," *J. Royal Statistical Society, Ser. B*, vol. 63, pp. 465-514, 2001.

obituary

continued from page 12

three honorary degrees from European universities.

To the Polytechnic community, Papoulis was a memorable figure on campus, known for his dramatic lectures and creativity in incorporating mathematical ideas into electrical engineering.

"Prof. Papoulis was a human dynamo," recalls former student Charles M. Rader, a senior staff member of MIT's Lincoln Lab. "He had this wonderful way of bringing mathematics to life, whether by relating a geometrical diagram to a real-life situation or by outrageously claiming to know Greek mathematicians from the first millenium."

Longtime colleague Dr. Mischa Schwartz, the Charles Batchelor Professor Emeritus of Electrical Engineering at Columbia University, remembers Papoulis as "a gifted applied mathematician. His research and books demonstrated his unique ability to carry out complex engineering analysis in a relatively simple, insightful way."

Another former colleague, 1992 Nobel Laureate Rudolph A. Marcus, the Arthur Amos Noyes Professor of Chemistry at California Institute of Technology, calls Papoulis a "lifelong friend. I enjoyed his careful and well-constructed logic in our discussions over the years, a logic in which he came to the essence of any problem."

Although known for his scientific intellect, Papoulis also loved the arts, especially music and the theater. "My father often said that if he had another career it would be as an actor," says his daughter Irene. "He was proud when people said that his lectures were performances." Papoulis's twin loves of education and the arts influenced his five children: Irene, a professor at Trinity College; Helen, a kindergarten teacher in California; James, a composer in New York; Ann, a dancer in Slovenia; and Mary, a violinist in Montana. He is also survived by his sister, Eleftheria; brother, Leftheri; and six grandchildren.