# Jensen-Rényi Divergence Measure: Theoretical and Computational Perspectives

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*Abstract* — **We analyze the theoretical properties of the Jensen-Rényi divergence, and we discuss its application to parametric classification. Numerical results on medical images show the the potential of the proposed divergence measure in image registration.**

## I. Theoretical and Computational analysis

**Definition 1** *Let $\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_n$ be n probability distributions of X and $\boldsymbol{\omega} = (\omega_1, \omega_2, \ldots, \omega_n)$ be a weight vector such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \geq 0$. The Jensen-Rényi divergence is defined as*

$$JR_\alpha^{\boldsymbol{\omega}}(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_n) = R_\alpha\Big(\sum_{i=1}^{n} \omega_i \boldsymbol{p}_i\Big) - \sum_{i=1}^{n} \omega_i R_\alpha(\boldsymbol{p}_i),$$

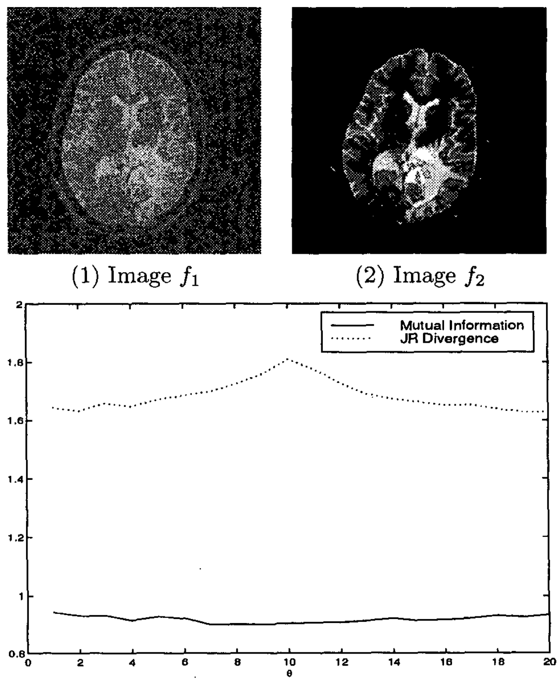
*where $R_\alpha(\boldsymbol{p})$ is the Rényi entropy.*

Inspired by the good performance of the mutual information measure [1], and looking to address its limitations in often difficult imagery, we proposed recently an information theoretic approach to ISAR image registration [2]. Compared to the mutual information based registration techniques [1], the Jensen-Rényi divergence adjusts its weight and exponential order to control the measurement sensitivity of the joint histogram. This flexibility ultimately results in a better registration accuracy. Maximization of the Jensen-Rényi divergence is a very general criterion, and no limiting constraints are imposed on image contents. The most fundamental properties of this divergence measure are convexity and symmetry.

The goal of image registration is to find a spatial transformation such that a similarity metric between two or more images taken at different times, from different sensors, or from different viewpoints achieves its maximum. Given two images $f_1, f_2 : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$, where $\Omega$ is a bounded set (usually a rectangle), the goal of image registration in the context of the Jensen-Rényi divergence is to determine the spatial transformation parameters $(\ell^*, \theta^*, \gamma^*)$ such that

$$\begin{aligned} (\ell^*, \theta^*, \gamma^*) &= \arg \max_{(\ell,\theta,\gamma)} JR_\alpha^{\boldsymbol{\omega}}(f_1, \mathcal{T}_{(\ell,\theta,\gamma)} f_2) \qquad (1) \\ &= \arg \max_{(\ell,\theta,\gamma)} JR_\alpha^{\boldsymbol{\omega}}(\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_n), \end{aligned}$$

where $\boldsymbol{p}_i = \boldsymbol{p}_i(f_1, \mathcal{T}_{(\ell,\theta,\gamma)} f_2)$, and $\mathcal{T}$ is a Euclidean transformation with translational parameter $\ell = (\ell_x, \ell_y)$, a rotational parameter $\theta$ and a scaling parameter $\gamma$.




(1) Image $f_1$ (2) Image $f_2$

(3) Dissimilarity measure $D(f_1, f_2)$

Figure 1: Registration result in the presence of the noise

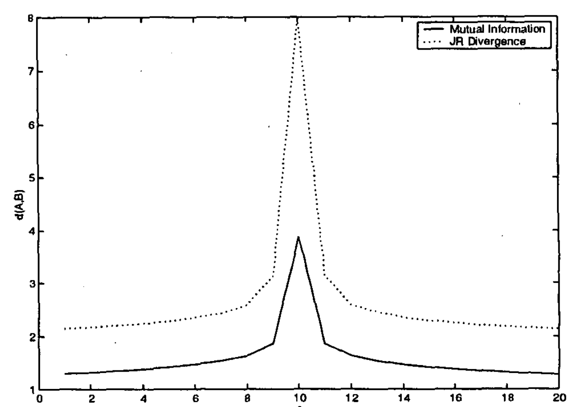



Figure 2: Mutual information vs. Jensen-Rényi

## References

[1] P. Viola and W. M. Wells, "Alignment by maximization of mutual information," *Int. Jour. of Comp. Vision*, vol. 24, 1997.

[2] Y. He, A. Ben Hamza, H. Krim, "A generalized divergence measure for robust image registration," *IEEE Transactions on Signal Processing*, vol. 51, no. 5, May 2003.



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