Optimal PID plus fuzzy controller design for a PEM fuel cell air feed system using the self-adaptive differential evolution algorithm

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A B S T R A C T

Various control strategies for the air feed to a PEM fuel cell are investigated. Feedback and feedforward control strategies are used simultaneously in order to achieve maximum net power and prevent oxygen starvation. The control objective is to adjust the oxygen excess ratio using compressed air based on external disturbance variations. Maximum power tracking is obtained using the optimal oxygen excess ratio which is also compared to the constant oxygen excess ratio. In the feedforward strategy, a fuzzy logic controller and in the feedback a filtered PID controller is applied. The controller parameters are optimized simultaneously using a self-adaptive differential evolution algorithm. Simulation results show that the proposed technique improves the fuel cell system performance and prevents oxygen starvation by fixing the oxygen ratio at a proper setpoint.

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Introduction

The ever-increasing energy demand in industrial societies and the predictable expiration of fossil fuel reserves has led to a constant search toward new energy resources. Besides, due to environmental pollution caused by fossil fuel combustion, new energy sources such as solar, wind, geothermal or hydrogen are considered as inevitable alternatives by energy supervisors. In spite of boundless possibilities for the application of these new energy sources and environmental friendly features the main disadvantage lies in their limited availability in specific situations.

Fuel cells are proposed as one of these alternatives for energy conversion systems during recent decades. Fuel cells are electrochemical systems capable of chemical energy conversion to the electrical form without combustion. High efficiency, low process temperatures, low pollution, high safety factors, flexibility of power generation and short startup times are among the most important advantages of these systems [1]. Hydrogen and oxygen are consumed to produce electricity in a fuel cell and byproducts are discharged as water and heat. However, short lifetime and high expenses has impeded their vast applicability in common systems up to now. As a result, in order to improve the performance, enhance the lifetime and reduce the production costs, the
Several control strategies are proposed for PEM fuel cells such as PI [2], LQG [1,3,4], fuzzy sliding mode [5], fuzzy [6,7], time delay [8,9], and neural networks [10,11] which are not validated in real plants. Other studies validated in plant-wide control strategies can also be mentioned as robust [12], gain-scheduled control [13], sliding mode [14,15] and model predictive control [16–18] which are applied for the manipulation of oxygen excess ratio in PEM fuel cells with various degrees of success.

This study is focused on the design and validation of a PID (PID with low band filter) and fuzzy controller optimized by the self-adaptive differential evolution (SaDE) algorithm for controlling the oxygen excess ratio in a PEM fuel cell. The main control objectives are preventing oxygen starvation and obtaining the maximum net power output. Oxygen starvation is considered as a critical condition for fuel cell resulting in rapid degradation of the membrane and catalyst layer [19], which is controlled by keeping the oxygen excess ratio higher than a minimum value.

After the introduction of differential evolution (DE) algorithm in 1995 [20], many researchers have paid attention to the issue and as a result many improved algorithms have been under development since then. The SaDE algorithm used in this study is developed by Qin et al. [21] in 2009 and applied in antenna design [22,23], filter design [24] and transforming geocentric Cartesian coordinates [25], thereafter. This algorithm is used in control applications and fuzzy controller optimization for the first time in this study. A feed forward controller uses fuzzy logic which is a representation of human thinking process. Fuzzy logic was introduced by Zadeh [26,27], Mamdani and Assilian [28] identified frameworks for fuzzy control strategy and developed the first industrially applied fuzzy controller and Beirami and Zerafat [29] applied fuzzy control strategy for adaptation of PID controller parameters in a heat exchanger system.

This study is organized as follows: Firstly, the mathematical model of the PEM fuel cell (PEMFC) air feed system and control objective is presented (Section 2). In Section 3, the control scheme of fuzzy logic and PIDF are designed. The self-adaptive differential evolution (SaDE) algorithm and optimization of controller parameters are discussed in Section 4. Simulation results of the fuel cell model are given in Section 5. Finally, main conclusions are presented in Section 6.

The PEMFC model

The 75 kW PEM fuel cell model used in this study is developed by Pukrushpan et al. [30,31]. The fuel cell stack consists of 381 cells, each with a 280 cm² membrane. This model has been widely accepted by researchers as a representative model to indicate real fuel cells for control objectives. The stack is combined with ancillary equipment for power generation. Fig. 1 shows the components of the fuel cell system as well as the chemical reactions involved. Increased pressures improve the reaction rate and enhance the efficiency and the fuel cell power density as well [32]. A cooling system is used to decrease the temperature of compressed air. Upon cooling, air humidity is increased to the desired value in order to prevent electrolyte dehydration. In the anode section, hydrogen is supplied while flow rate and pressure are manipulated using a control valve. Hydrogen molecules are converted to 2H⁺ and two electrons on the anode catalytic layer:

$$\text{H}_2 \rightarrow 2\text{H}^+ + 2e^- \quad (1)$$

Produced protons move toward the cathode catalytic layer through the membrane. Since the membrane is not conductive to electrons, released electrons flow in the external circuit. In the cathode, oxygen passes through the gas layer and reacts with protons and electrons in the catalyst layer resulting in water production:

$$\frac{1}{2}\text{O}_2 + 2\text{H}^+ + 2e^- \rightarrow \text{H}_2\text{O} \quad (2)$$

The general reaction taking place in the fuel cell is a combination of Eqs. (1) and (2) accompanied by heat generation:

$$\text{H}_2 + \frac{1}{2}\text{O}_2 \rightarrow \text{H}_2\text{O} + \text{heat} + \text{electrical energy} \quad (3)$$

The PEMFC air feed system model

This section deals with the investigation of dynamic behavior of variables concerning air flow control in order to prevent oxygen starvation. Oxygen, nitrogen and vapor partial pressures in the cathode are calculated using mass conservation combined with ideal gas law:

$$\frac{dP_{\text{O}_2}}{dt} = \frac{RT_e}{V_{\text{em}}M_{\text{O}_2}} (W_{\text{O}_2,\text{in}} - W_{\text{O}_2,\text{out}} - W_{\text{O}_2,\text{reactd}}) \quad (4)$$

$$\frac{dP_{\text{N}_2}}{dt} = \frac{RT_e}{V_{\text{em}}M_{\text{N}_2}} (W_{\text{N}_2,\text{in}} - W_{\text{N}_2,\text{out}}) \quad (5)$$

$$\frac{dP_{\text{V}}}{dt} = \frac{RT_e}{V_{\text{em}}M_{\text{V}}} (W_{\text{V},\text{in}} - W_{\text{V},\text{out}} + W_{\text{V},\text{gen}} + W_{\text{V},\text{membr}} - W_{\text{V},\text{out}}) \quad (6)$$

Fig. 1 – Schematic of the PEM fuel cell and ancillary components.
where, $R$ is the universal gas constant, $V_{ct}$ is the cathode volume; $T_k$ is the fuel cell temperature and $M_{O_2}$ and $M_{N_2}$ are oxygen and nitrogen molecular weights, respectively. The oxygen and nitrogen outlet mass flow rates are denoted by $W_{O_2,\text{out}}$ and $W_{N_2,\text{out}}$. The inlet mass flow rates are also shown by $W_{O_2,\text{in}}$ and $W_{N_2,\text{in}}$. The rate of oxygen reaction is also given by $W_{O_2,\text{reacted}}$. $W_{v,\text{in}}$ is the mass flow rate of vapor entering and $W_{v,\text{out}}$ is the mass flow rate of vapor leaving the cathode. $W_{v,\text{membr}}$ and $W_{v,\text{in}}$, represent the rate of vapor generated in the fuel cell, mass flow rate of water transferred across the membrane and liquid water leaving the cathode, respectively.

Increasing the oxygen excess ratio by increasing the inlet oxygen mass inflow rate to the cathode is the most efficient way to prevent oxygen starvation. Oxygen starvation is a complicated phenomenon and an important reason of fuel cell degradation [33–35]. It must be mentioned that oxygen starvation risk is very high at high disturbance fluctuation rates [36]. As a result, the air inflow to the cathode must be increased in order to prevent sudden oxygen starvation during fast disturbance fluctuations. Oxygen excess ratio is defined as the ratio of oxygen supplied and oxygen made to react in the fuel cell stack [14,6,30,37]:

$$\lambda_{O_2} = \frac{W_{O_2,\text{in}}}{W_{O_2,\text{reacted}}}$$

(7)

The inlet mass flow rates are also defined as:

$$W_{O_2,\text{in}} = x_{O_2} \cdot W_{ca,\text{in}}$$

(8)

$$W_{N_2,\text{in}} = (1 - x_{O_2}) \cdot W_{ca,\text{in}}$$

(9)

$$W_{ca,\text{in}} = \frac{1}{1 + \nu_{\text{atm}}} \cdot k_{ca,\text{in}} (P_{atm} - P_{ca})$$

(10)

where, $k_{ca,\text{in}}$ is the inlet orifice constant and $P_{ca}$ is the cathode pressure, given by:

$$P_{ca} = P_{O_2} + P_{N_2} + P_{sat}$$

(11)

Oxygen mass fraction can also be calculated as:

$$x_{O_2} = \frac{y_{O_2} \cdot M_{O_2}}{y_{O_2} \cdot M_{O_2} + (1 - y_{O_2}) \cdot M_{N_2}}$$

(12)

where, oxygen mole fraction is denoted by $y_{O_2}$. Humidification rate is also given by:

$$\nu_{\text{atm}} = \frac{M_{v} \cdot P_{v,\text{in}}}{M_{v} \cdot P_{v,\text{in}}}$$

(13)

where,

$$P_{v,\text{in}} = \phi_{v,\text{in}} \cdot P_{sat} (T_{ca,\text{in}})$$

(14)

$$P_{v,\text{out}} = P_{sat} - P_{v,\text{in}}$$

(15)

$$M_{v} = y_{O_2} \cdot M_{O_2} + (1 - y_{O_2}) \cdot M_{N_2}$$

(16)

where, $M_{v}$ and $M_{o}$ are air and vapor molecular weights and $P_{v,\text{in}}$ and $P_{v,\text{in}}$ are also vapor and dry air pressures, respectively. Atmospheric pressures and air relative humidity at the cathode inlet are also denoted by $P_{atm}$ and $\phi_{atm}$. $T_{ca,\text{in}}$ is the inlet flow temperature to the cathode, and $P_{sat}(T_{ca,\text{in}})$ is also the vapor saturation pressure, with the following temperature dependency [31].

$$\log_{10}(P_{sat}) = (-1.6 \times 10^{-18})T^4 + (3.85 \times 10^{-7})T^3$$

$$- (3.39 \times 10^{-4})T^2 + 0.143T - 20.29$$

(17)

The outlet mass flow rates are also given by:

$$W_{O_2,\text{out}} = \frac{M_{O_2} \cdot P_{O_2}}{M_{O_2} \cdot P_{O_2} + M_{N_2} \cdot P_{N_2} + M_{sat} \cdot P_{sat}} \cdot W_{ca,\text{out}}$$

(18)

$$W_{N_2,\text{out}} = \frac{M_{N_2} \cdot P_{N_2}}{M_{O_2} \cdot P_{O_2} + M_{N_2} \cdot P_{N_2} + M_{sat} \cdot P_{sat}} \cdot W_{ca,\text{out}}$$

(19)

Since, the pressure difference between the cathode and the ambient is large in pressurized stacks. The total flow rate at the cathode exit $W_{ca,\text{out}}$ is calculated using the nozzle flow equation [38].

$$W_{ca,\text{in}} = \frac{C_{D} A_{f} \cdot P_{ca}}{\sqrt{RT_{fc}}} \left( \frac{P_{atm}}{P_{ca}} \right)^{g} \left( \frac{2}{g + 1} \right)^{\frac{g+1}{g-1}}$$

(20)

and,

$$W_{ca,\text{in}} = \frac{C_{D} A_{f} \cdot P_{ca}}{\sqrt{RT_{fc}}} \left( \frac{2}{g + 1} \right)^{\frac{g+1}{g-1}}$$

(21)

where, $g$ is the ratio of heat capacities for air, $A_{f}$ is the nozzle opening area and $C_{D}$ is the discharge coefficient of the nozzle.

Finally, the total flow rate is determined using the simplified orifice equation [31].

$$W_{ca,\text{out}} = k_{ca,\text{out}} \sqrt{P_{ca} - P_{atm}}$$

(22)

Oxygen consumption rate is proportional to the stack current and is calculated based on fundamental electrochemical principles as follows:

$$W_{ca,\text{reacted}} = M_{O_2} \frac{n_{f}}{4F} \frac{d\phi_{o_{2}}}{dt}$$

(23)

where, $n_{f}$ is the number of stack cells, $F$, Faraday constant and $I_{f}$ the stack current. Compressor rotational speed ($\omega_{cp}$) is the only dynamic state of the compressor model, which is modeled using [31]:

$$\frac{d\omega_{cp}}{dt} = \frac{1}{J_{cp}} (r_{cm} - r_{cp})$$

(24)

Where, the compressor motor inertia is denoted by $J_{cp}$ and $r_{cm}$ is the compressor motor torque and $r_{cp}$ is the external load torque. $r_{cm}$ and $r_{cp}$ are given as follows:

$$r_{cm} = \eta_{cm} \cdot \frac{k_{c}}{R_{cm}} (V_{cm} - k_{c} \omega_{cp})$$

(25)

$$r_{cp} = C_{p} \cdot T_{atm} \cdot \left( \frac{P_{sat}}{P_{atm}} \right) - 1 \cdot W_{cp}$$

(26)

where, $R_{cm}$, $k_{c}$ and $k_{c}$ are motor constants and $\eta_{cm}$ and $\eta_{cp}$ the compressor and motor mechanical efficiencies, respectively. $C_{p}$ and $V_{cm}$ are the air heat capacity at constant pressure and the inlet voltage to the compressor motor.
flow \((W_p)\) is modeled by applying Jensen and Kristensen nonlinear fitting method \([3]\) as a function of pressure ratio \((P_{sm}/P_{cm})\), upstream temperature \((T_{sm})\) and the compressor rotational speed \((\omega_p)\). Dynamical behavior of the supply manifold air pressure is expressed as follows:

\[
\frac{dP_{sm}}{dt} = \frac{RT_{sm}}{M_p}V_{sm}(W_p - k_{ca,sm}(P_{sm} - P_{ca}))
\]  

(27)

where,

\[
T_{cp} = T_{sm} + \frac{T_{sm}}{\eta_{cp}} \left( \frac{P_{sm}}{P_{cm}} \right)^{\frac{1}{\gamma_1}} - 1
\]  

(28)

where, \(V_{sm}\) and \(T_{cp}\) are the supply manifold volume and the outlet gas temperature from the compressor, respectively. Details of the compressor model used in this study can be found elsewhere \([31]\).

**Control objective**

Oxygen excess ratio \(\lambda_{O_2}\) can be considered as an index of system efficiency \([30]\). Oxygen starvation results in quick degradation of the stack as a result of hot spot generation, membrane surface burning and the reduction of power generation. The measure generally considered to prevent oxygen starvation is simultaneous termination of reactant inflow and outflow from the stack \([31]\). For \(\lambda_{O_2} < 1\), oxygen starvation occurs. However, \(\lambda_{O_2} \gg 1\) means high oxygen availability at the cathode which improves power generation in the stack. At the same time, if oxygen availability is very high, net power \((P_{net})\) will be reduced due to the elevation of compressor power consumption known as oxygen saturation. Since the air compressor consumes the highest power among fuel cell auxiliary components, others may be neglected in the calculation of net power:

\[
P_{net} = P_{at} - P_{cm}
\]  

(29)

where,

\[
P_{at} = V_{at}I_{at}
\]  

(30)

and

\[
P_{cm} = \tau_{cm}\omega_p
\]  

(31)

where, \(P_{at}\) and \(P_{cm}\) are the stack power generation and compressor power consumption, respectively and \(V_{at}\) is also the stack voltage.

The PEMFC system explained in Section 2 has two inputs; One is the compressor motor voltage \((V_{cm})\) as the inlet to the control system and the other is the stack current \((I_{st})\) as the disturbance. Based on Eqs. (7)-(28), \(\lambda_{O_2}\) is dependent on the compressor motor voltage and the stack current and since \(I_{st}\) is the disturbance, \(\lambda_{O_2}\) must be manipulated by \(V_{cm}\).

The control objective is the maximum output power generation for various disturbance conditions. The relationship between the net power output and \(\lambda_{O_2}\) for different disturbance conditions is called the performance curve. From Fig. 2, the maximum power output for each current ends in \(\lambda_{O_2}\) (optimal \(\lambda_{O_2}\), shown by ‘****’ in Fig. 2). Those \(\lambda_{O_2}\) values generating the maximum power are in the 1.98–2.53 range for 100–300 A stack currents. \(\lambda_{O_2} = 2\) is proposed in some studies \([31]\) for simplicity. In other studies \([2,37,39]\), the reference \(\lambda_{O_2}\) is assumed as a variable with the aim of maximum power generation for each load. In the first scenario, the control objective is to keep \(\lambda_{O_2}\) at \(\lambda_{O_2} = 2\) which has been the subject of most studies up to now. In the second scenario, a lookup table is generated using \(\lambda_{O_2}\) which results in the maximization of \(P_{net}\) for the inlet \(I_{st}\) (Fig. 3). Obtained values are \(\lambda_{O_2}\) setpoints for tracking. The controller task is also tracking the reference \(\lambda_{O_2}\) generated by the lookup table.

**The control scheme**

The fuel cell system has a complicated and nonlinear behavior due to multiple inputs. So, in order to manipulate the oxygen excess ratio as the control objective, a proper control scheme is required to manipulate the compressor motor voltage \((V_{cm})\). As a result, PIDF plus fuzzy controller is implemented in this study with control parameters optimized by SaDE algorithm. The proposed control scheme is given in Fig. 4.

**Feedforward fuzzy logic control**

Fuzzy controllers are nonlinear having a special structure with successful applications in engineering problems. These controllers perform similar to expert humans while controlling a system.

In feedforward control, the measurement of stack current as the inlet disturbance to the fuel cell and its implementation as additional information results in the enhancement of PID (feedback loop) control performance \([40]\). This measurement can be considered as an early warning of \(\lambda_{O_2}\) deviation from the setpoint. Based on this warning, the feedforward controller will have sufficient time to vary the manipulated variable \(V_{cm}\) before deviation.
Fuzzy logic control design
The fuzzy logic controller used in this study is a Mamdani fuzzy inference system with a single input and a single output. The stack current ($I_{st}$) is selected as the input and the compressor motor voltage ($V_{cm}$) as the output ($U_{ff}$). The input range is 100–300 A and the output range varies in the 100–235 V range. For input and output variables, triangular membership functions and for the sides, S-shape and Z-shape functionalities are assumed. Twelve membership function parameters are optimized using the SaDE algorithm. The input and output fuzzy linguistic variables are denoted by Very High (VH), High (H), Medium (M), Low (L) and Very Low (VL). Five IF-THEN rules are used in the fuzzy logic system given in Table 1. Defuzzification is also implemented by the center of gravity (COG) technique [41].

Feedback PIDControl
A feedforward controller must be accompanied by a feedback controller so that the steady-state offset value is ensured to be zero [40]. Feedback control has proved its performance in many applications with the exception of cases with very slow feedback dynamics. It is worth mentioning that, imprecise adjustment of controller parameters can result in instability [40].

The main problem of the differentiating part in a PID controller is the amplification of high frequency noise due to measurement inefficacy resulting in high output values. To overcome this problem, a low-band filter is used to limit the high frequency gain and noise [42]. The transfer function of the PID controller with a low-band differentiating filter is:

$$u = K_p + K_i \frac{1}{s} + K_d \frac{N}{1 + N\frac{1}{s}}$$

(32)

$K_p$, $K_i$ and $K_d$ are the proportional, integral and differential gains, respectively and $N$ is the filter coefficient.

Optimization tools
DE algorithm
DE is a population-based, stochastic algorithm using global optimization which is implemented for solving several

Table 1 – The fuzzy rules.

<table>
<thead>
<tr>
<th>If the stack current ($I_{st}$) is:</th>
<th>Then the compressor motor voltage ($V_{cm}$) is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low (VL)</td>
<td>Very Low (VL)</td>
</tr>
<tr>
<td>Low (L)</td>
<td>Low (L)</td>
</tr>
<tr>
<td>Medium (M)</td>
<td>Medium (M)</td>
</tr>
<tr>
<td>High (H)</td>
<td>High (H)</td>
</tr>
<tr>
<td>Very High (VH)</td>
<td>Very High (VH)</td>
</tr>
</tbody>
</table>

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engineering problems due to simplicity and high efficiency. More information on DE applications can be found elsewhere [43,44]. In DE algorithm, the population consists of NP vectors $X_{i,G}$ ($i = 1, \ldots, NP$). Each $X_{i,G}$ is a D-dimensional vector representing a candidate solution. The $X_{i,G}$ vectors are initially randomized within the search space with a uniform distribution. The population of the forthcoming generations is produced by mutation, crossover and selection operators. Based on these operators, various DE algorithms are generated. Different strategies are in mutation with the most common as:

$$V_i = X_i^c + F \left( X_{r_1}^c - X_{r_2}^c \right), \quad r_1 \neq r_2 \neq r_3 \quad (33)$$

Three vectors ($r_1$, $r_2$ and $r_3$) are thus selected randomly from the population and the $r_3 - r_1$ vector is multiplied by an $F$ factor and added to $r_1$ to generate the trial vector. $F$ is the mutation control parameter which is a positive number close to unity. The selection of the appropriate mutation strategy is dependent on the problem. The mutation operator in DE, has a more pronounced role compared with other operators contrary to other algorithms and is identified by trial and error. In order to remove this shortcoming, a strategy candidate pool consisting of a number of various strategies is applied instead of the time consuming trial and error strategy. Upon mutation, the crossover operator is implemented to generate offsprings from the previous trial population. In this algorithm, two crossover operators are defined; One is exponential and the other binomial crossover which is used here:

$$u_{j,G} = \begin{cases} u_{j,G}, & \text{if } (\text{rand}(0,1) \leq CR) \text{ or } (j = j_{rand}), \quad j = 1, 2, \ldots, D \quad (34) \\ X_{i,G}, & \text{otherwise} \end{cases}$$

The crossover rate (CR) is randomly chosen from the $[0, 1)$ range showing the parameters’ share copied from the mutant vector. $j_{rand}$ is a random integer from the $[1, D]$ range. The selection process in DE is different from other algorithms. In other evolutionary algorithms, the remaining members for the next generation are selected based on probability, while selection in DE is performed deterministically between the offspring and parent vectors based on fitness. This approach is usually called a selection tournament given as follows:

$$X_{i,G+1} = \begin{cases} U_{j,G}, & \text{if } f(U_{j,G}) \leq f(X_{i,G}) \quad (35) \\ X_{i,G}, & \text{otherwise} \end{cases}$$

where, $f(X_{i,G})$ and $f(U_{j,G})$ are fitness functions of the offspring and parent vectors. The stopping criterion for DE is usually the number of generations or the number of objective function evaluations.

**SaDE algorithm**

Estimation of DE parameters entails a trial and error approach which imposes an extra computational load on the solution procedure. The SaDE algorithm can adjust $F$ and CR during the
Fig. 6 — SaDE algorithm.
algorithm performance and can select the mutation strategy. In SaDE, both the generation of trial vectors and control parameters become gradually self-adapted by learning from previous generations. In this paper, a strategy candidate pool is used that consists of four generation strategy trial vectors as follows:

\[ u_{ij} = \begin{cases} x_{ij} + F(x_{bestj} - x_{ij}) + F(x_{1j} - x_{ij}) + F(x_{2j} - x_{ij}) + F(x_{3j} - x_{ij}) & \text{if } \text{rand}(0, 1) < \text{CR} \\ x_{ij} & \text{otherwise} \end{cases} \]  

\[ (36) \]

“DE/rand/bin”:

\[ u_{ij} = \begin{cases} x_{ij} + F(x_{bestj} - x_{ij}) & \text{if } \text{rand}(0, 1) < \text{CR} \text{ or } j = j_{\text{rand}} \\ x_{ij} & \text{otherwise} \end{cases} \]  

“DE/rand-to-best/2/bin”:

\[ (37) \]
and $S_{k,G}$ is the success rate of generated trial vectors with the $k$th strategy in previous LP generations from generation $G$ successfully entered the next generation.

A small constant value, $\epsilon = 0.01$ is added to SaDE in order to prevent null probability. It is obvious that a larger $S_{k,G}$ indicates a higher probability value for the selection of the $k$th strategy in the current generation. At the next step, control parameters will become self-adapted. The selection of $NP$ is totally dependent on the nature of the problem being performed through simple estimations. Among other parameters, CR is very sensitive to the problem type and can have a significant role in obtaining the correct solution. The $F$ value has a great impact on the rate of convergence, which is selected randomly with a probability proportional to a normal distribution with an average of 0.5 and a variance equal to 0.3, denoted by $N(0.5, 0.3)$. The control parameter $K$ in “DE/current-to-rand/1” strategy is also selected randomly from the [0, 1] range.

The crossover rate control parameter is selected with a normal distribution probability and a standard variance equal to 0.1. $CR_{m}$ is also given an initial value of 0.5 equal for all strategies. The $CR$ value with a successful trial vector in the previous generation is registered in the crossover rate memory ($CR_{m}$) for every strategy and the previous generation LP. The $CR_{m}$ value for every generation and the $K$th strategy is calculated by averaging the registered values in the $CR_{m}$ memory and $CR$ is also obtained using a normal distribution probability with a $CR_{m}$ average and a variance equal to 0.1. The flowchart of the SaDE algorithm is given in Fig. 6. More information on SaDE can be found elsewhere [21].

### Encoding

A number of parameters are required for encoding the candidate answers for fuzzy and PIDF controllers. Simultaneous optimization of fuzzy and PIDF controllers is performed to obtain improved results. In this study, optimization of proportional ($K_{p}$), Integral ($K_{i}$) and differential ($K_{d}$) coefficients of the PIDF controller and membership functions of the fuzzy controller are performed using SaDE algorithm.

Encoding the position of membership functions has been performed by different techniques in the literature [45,46]. Fig. 4 shows the encoding approach used in this study. The position of membership functions are represented by $a_{j}^{i}$, where $j$ shows the number of $\epsilon_{j}$ and $i$ a special point from $\epsilon_{j}$. The initial and end points of triangular membership functions are shown by $i = 1$ and $i = 3$, respectively and the point $\epsilon_{j} = 1$ is represented by $i = 2$. For Z-Shape membership functions, the point separated from unity (Fig. 7) is $i = 1$ and the point touching zero is indicated by $i = 2$. For S-shape membership functions, the point separated from zero is $i = 1$ and the point touching unity is $i = 2$. In many studies, membership function parameters for each $\epsilon_{j}$ are defined independently in the

![Fig. 8 - The optimized membership functions of the fuzzy feedforward controller (a) input membership functions, (b) output membership functions.](image)
candidate solution for the optimization problem leading to an extra step for changing or removing impossible solutions bringing about large computational loads. Special rules are devised in order not to confront impossible solutions. We assume that the sequence of membership function linguistic variables does not change and each value from the output range has at least one nonzero member in one of its membership functions in order to simplify the solutions encoding and algorithm acceleration by eliminating impossible solutions. To this end, the sequence of points indicates the position of membership functions (Fig. 7).

The optimization problem parameters for the fuzzy controller are $x_1$ through $x_{12}$ where $x_i$ shows the relative distance between the membership function points in the $[0, 1]$ range.

Fig. 9 – The comparison of three control strategies with a constant oxygen excess ratio: (a) disturbance variation, (b) stack voltage variation, (c) oxygen excess ratio variation using PIDF plus Fuzzy FF, (d) oxygen excess ratio variation using PI + FF, (e) oxygen excess ratio variation using FF.
range. The membership function point values are calculated as follows: \( a_1 \) is the initial point in the range and the other points are previous values plus 
\[ L_i = \frac{1}{\sum x_i} \times L, \]
where \( L \) is the length of the controller output range. Finally, \( x_1, x_14 \) and \( x_15 \) are defined for proportional, Integral and differential parameters of the PIDF controller, respectively.

**Simulation results**

The PIDF plus Fuzzy control scheme is applied to the 75 kW PEMFC system. The numerical parameters used in the simulation are given in Table 2. PIDF controller parameters and the parameters regarding the membership functions of the fuzzy controller are optimized using SaDE algorithm. The optimized membership functions for the fuzzy controller are given in Fig. 8. The main control objectives are: (a) Preventing oxygen starvation by keeping \( \lambda_{O_2} \) constant at \( \lambda_{O_2} = 2 \), and (b) acquiring the maximum power tracking by the optimal \( \lambda_{O_2} \) to achieve the best system performance.

**Controller performance with constant \( \lambda_{O_2} \)**

The first simulation is performed by keeping \( \lambda_{O_2} \) constant and stepwise changing of the disturbance covering the whole fuel cell performance range (Fig. 9). Despite load fluctuations, the control system adjusts \( \lambda_{O_2} \) at the setpoint. Fig. 9 (c)–(d) shows the simulation results for controlling \( \lambda_{O_2} \) using feedforward [30,31], PI plus feedforward [30,31] and PIDF plus fuzzy feedforward control strategies. Also, Fig. 10 indicates a magnified version of the oxygen excess ratio graph using various control techniques. Based on the results, the proposed techniques have improved values of

<table>
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<th>ITAE</th>
<th>ISE</th>
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![Fig. 10](image)

**Fig. 10** — The magnified plot of oxygen excess ratio variations at: (a) 10 s, (b) 20 s.

![Fig. 11](image)

**Fig. 11** — PIDF plus Fuzzy FF control scheme for the case of maximum power tracking: (a) optimal oxygen excess ratio variation, (b) stack voltage variation.
rising time, undershoot and overshoot compared with previous studies [30,31]. The comparison of the total performance criteria is given in Table 3. As it is obvious, the PIDF plus fuzzy feedforward has a better response compared to other techniques.

Besides, the proposed technique has a better performance toward the instantaneous disturbance rise. The settling time for PI plus feedforward and pure feedforward are 230 and 430 ms, respectively reduced to 100 ms in the PIDF plus fuzzy feedforward control scheme (Fig. 10(a)). A sudden decrease of the disturbance results in 110 and 60 ms settling times for PI plus feedforward and feedforward control schemes, respectively reduced to 20 ms in the PIDF plus fuzzy feedforward type (Fig. 10(b)). The results show that the PIDF plus fuzzy feedforward controller is a promising alternative for oxygen supply to the fuel cell especially in case of continual disturbance variations.

Variable \( \lambda_{O_2} \) tracking for maximum power efficiency

Based on Section 2.2, the variable reference \( \lambda_{O_2} \) based on disturbance variations is derived in order to achieve the maximum efficiency and reduction of fuel cell expenses. \( \lambda_{O_2} \) must be manipulated according to disturbance variations in order to obtain the maximum net power. However, the risk of oxygen starvation in case of sudden high power demands is increased as a result of oxygen shortage in the stack. As a result, the controller capacity for tracking the variable \( \lambda_{O_2} \) setpoint is very important. Using the same disturbance variation characteristics in the first simulation, the proposed controller is tested for the case of variable \( \lambda_{O_2} \) (Fig. 11). The results show that the proposed control scheme has performed successfully for setpoint tracking and noise rejection. Finally, the net power efficiency is calculated for all techniques (Fig. 12). The results show that the second scenario (maximum power tracking) results in a higher net power by 1% compared with the first scenario (constant oxygen excess ratio). Based on the results, the proposed control strategy has a higher efficiency compared with other techniques in both scenarios.

Conclusions

In this study, the hybrid PIDF plus fuzzy feedforward control technique is developed for oxygen excess ratio to manipulate the air inflow to the PEMFC. Controller parameters are optimized simultaneously using SaDE algorithm. The control objective is preventing oxygen starvation and obtaining the maximum fuel cell efficiency. The results show that, the proposed control scheme performs better compared with other methods for fuel cell control under various disturbance fluctuations. Also, this technique reduces the settling time, overshoot and undershoot significantly in comparison with other techniques. The calculation of the optimal oxygen excess ratio as the system setpoint results in the enhancement of net power by 1% compared with other control strategies or constant oxygen excess ratio resulting in an improved fuel cell efficiency.

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References


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