Transient groundwater modeling using spreadsheets

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Abstract
This study proposes a transient groundwater modeling using spreadsheet simulation (TGMSS) model for solving groundwater problems. TGMSS may be considered as a practical method and introduction to groundwater modeling that uses spreadsheets instead of conventional groundwater model codes. Irregular aquifer geometry, variable boundary conditions and sinks and/or source values, heterogeneous aquifer parameters (conductivity, storage capacities), may be evaluated in the TGMSS. Different management scenarios may be analyzed by obtaining the groundwater level of different times. Two hypothetical examples are tested with TGMSS and verified with MODFLOW. Results showed that the TGMSS and MODFLOW results were in good agreement in terms of resulting values of hydraulic heads in all cases.

Keywords: Transient; Groundwater modeling; Spreadsheet; Hydraulic head; Modflow

1. Introduction

Groundwater modeling is one of the most important topics in engineering and geosciences. Groundwater systems can be modeled using partial differential equations (PDEs). Governing equations of groundwater modeling can be solved by analytical and numerical solution methods. Analytical solutions of these equations are possible in simple and ideal cases with regularly shaped aquifers and homogeneous hydraulic properties. Aquifer systems in the groundwater modeling are a complex task since its heterogeneous structure. Many numerical solution algorithms have been developed to solve PDEs such as finite difference method (FDM) and finite element method (FEM), which have used since 1960s. Several studies have been carried out by many researchers using FDM [1–4] and FEM [5–8].

The development of computer technology may ease solving the PDE in groundwater modeling. One of the best tools for solving the PDEs is spreadsheet. There are many advantages of spreadsheets such as having numerical and visual feedback and fast calculating capabilities. One of the most important advantages of spreadsheets is its graphical interface. The solution obtained through the spreadsheet can easily be plotted at the same worksheet. Any changes in the input parameters of the solution domain will be directly reflected to the graphical representation of the solutions. Spreadsheets are user-friendly and easy to program.

Recently, popularity of spreadsheets in the solution of engineering problems has been increasing. Several studies have been carried out using spreadsheets for the last 10 years. The application of them have been carried out in different fields of engineering problems such as in solution of PDEs [9], one-dimensional transient heat-conduction problems [10], and free-surface seepage problems [11].

There are some studies in the literature [12–15] for groundwater applications, although there are many applications of spreadsheet in engineering fields. Olsthoorn [16] showed that spreadsheet is useful tool for two and three-dimensional steady-state and transient groundwater problems with homogeneous aquifer parameters and constant sinks and/or source terms.

Anderson and Bair [17] showed that spreadsheets provide an easy way for understanding groundwater problems prior to using MODFLOW. They solved one-dimensional transient and two-dimensional steady-state groundwater problems for homogeneous aquifer parameters and constant sinks and/or source terms. In addition they suggested as advanced topics that the block-centered flow (BCF) package in MODFLOW may be simulated on the spreadsheets.
In groundwater modeling on a spreadsheet, it is not necessary to write out an equation in all cells to carry out iterative calculations. Using copy and paste features of the spreadsheets, FDM equation can be copied to other cells without writing the equations to all cells individually. When the equation pasted to all cells of the solution domain, iterative calculation is started. It is carried out until the given number of iteration is finished or maximum convergence criterion has been met.

While solving the groundwater modeling system in the steady-state case is relatively easy task, but transient solutions are quite difficult in terms of time dimension in the governing equations. Inclusion of time dimension in PDEs may lead to increase CPU time if conventional methods are used. However, transient groundwater modeling problems may be solved based on iterative spreadsheet calculation since spreadsheets eliminates the matrix algebra to vector form.

The main objective of this study is to demonstrate a user-friendly and flexible groundwater modeling simulation algorithm using the FDM. Therefore, transient groundwater modeling using spreadsheet simulation (TGMSS) has been proposed. Variable grid sizes, aquifer parameters, sinks and/or source terms may be applied to groundwater modeling problems having ideal or complex geometries in the model. The results of the TGMSS are composed with MODFLOW results, which is a well-known model code for groundwater modeling.

2. Notation

\( N \) number of grid spacing in the \( x \) direction \((i=1, 2, 3, \ldots, N)\)

\( M \) number of grid spacing in the \( y \) direction \((j=1, 2, 3, \ldots, M)\)

\( \Delta x \) grid spacing in the \( x \) direction [L]

\( \Delta y \) grid spacing in the \( y \) direction [L]

\( \Delta t \) time step [T]

\( n \) number of iteration

\( m \) time indicator

\( \Delta m \) increment of time indicator

\( h \) hydraulic head, \( h \in \mathbb{H} \) [L]

\( H \) matrix of hydraulic head, \( H_{i,j} \) [L]

\( K \) matrix of hydraulic conductivity, \( K_{i,j} \) [LT\(^{-1}\)]

\( S \) matrix of specific storage, \( S_{i,j} \) [L\(^{-1}\)]

\( Q \) matrix of pumping rate, \( Q_{i,j} \) [LT\(^{-1}\)]

\( R \) matrix of recharge, \( R_{i,j} \) [LT\(^{-1}\)]

\( W \) matrix of sinks and/or sources, \( W_{i,j} \) [T\(^{-1}\)]

\( Q_k \) vector of boundary conditions which is a function of \( \Omega(x, y) \) \((k = 1, 2, 3, \ldots)\) [L]

3. Mathematical model

The governing equation of a vertically integrated Darcy’s flow in a two-dimensional confined, compressible, isotropic, heterogeneous aquifer is:

\[
\frac{\partial}{\partial x} \left( K \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial h}{\partial y} \right) \pm W = S \frac{\partial h}{\partial t}
\]  

(1)

One of the well-known solution methods to solve Eq. (1) is the FDM. FDM solution technique can be classified into two groups: implicit and explicit methods. Implicit methods are interpolative and require a solution of simultaneous equations for each time interval using matrix algebra and require finding the inverse of the matrix for each time interval. Thus, computations are complicated and time consuming. Explicit methods are extrapolative and require a simple solution of an algebraic equation for one unknown for each point and for each time interval. The CPU time is usually much smaller than the implicit technique.

FDM solution technique can be further classified into two groups: alternating direction explicit method (ADEM) [18] and alternating direction implicit method (ADIM) [19].

ADEM equations can be written as:

\[
S_{ij} \frac{H_{i,j}^{m+1} - H_{i,j}^m}{\Delta t} = K_{i,j}^+ \left( H_{i+1,j}^m - H_{i,j}^m \right)
\]

+ \( K_{i,j}^- \left( H_{i-1,j}^m - H_{i,j}^m \right) \) + \( W_{i,j} \)

(2)

and

\[
S_{ij} \frac{H_{i,j}^{m+1} - H_{i,j}^m}{\Delta t} = K_{i,j}^- \left( H_{i-1,j}^m - H_{i,j}^m \right)
\]

+ \( K_{i,j}^+ \left( H_{i+1,j}^m - H_{i,j}^m \right) \) + \( W_{i,j} \)

(3)

where

\[
W_{i,j} = \pm \frac{Q_{ij}^{m+\frac{1}{2}}}{\Delta x \Delta y H_{i,j}}
\]  

(4)

In order to obtain a stable solution for explicit groundwater modeling, the stability criterion of \( \frac{KH}{b} \left( \frac{\Delta \nu}{\Delta t} \right) + \left( \frac{\Delta \nu}{\Delta t} \right)^2 \leq \frac{1}{2} \) must be satisfied [20].

The ADIM is a cycle of calculation and consists of two stages in Eqs. (2) and (3). In each stage, weight is placed on
two flux terms at \( t = (m+1)\Delta t \) and two flux terms at \( t = m\Delta t \).

In the first stage, the flux terms in the \( x \) and \( y \) directions are taken \( t = (m+1)\Delta t \) and \( t = m\Delta t \), respectively. In the second stage, the flux terms in the \( x \) and \( y \) directions are taken \( t = m\Delta t \) and \( t = (m+1)\Delta t \), respectively. There is no instability problem in this method.

By arranging Eqs. (2) and (3), Eq. (5) which consists of one-stage solution algorithm can be obtained as:

\[
H_{ij}^m = \frac{\left[ CC(H_{i,j}) + CE(H_{i+1,j}) + CW(H_{i-1,j}) + CS(H_{i,j+1}) + CN(H_{i,j-1}) + W_{ij} \right]}{(CE + CW + CS + CN + CC)}
\]

where

\[
CC = \frac{S_{ij}}{\Delta t}
\]

\[
CE = \left( \frac{1}{2\Delta x^2} \right) [K_{ij} + K_{i+1,j}]
\]

\[
CW = \left( \frac{1}{2\Delta x^2} \right) [K_{ij} + K_{i-1,j}]
\]

\[
CS = \left( \frac{1}{2\Delta y^2} \right) [K_{ij} + K_{i,j+1}]
\]

\[
CN = \left( \frac{1}{2\Delta y^2} \right) [K_{ij} + K_{i,j-1}]
\]

\[
W_{ij} = \pm \frac{Q_{ij}}{\Delta x \Delta y H_{ij}}
\]

TGMSS uses the iterative alternating direction implicit method (IADIM). So, it does not need any stability criteria since Eq. (5) is iteratively solved with neighboring cells simultaneously.

### 4. Model development

The general structure of problem domain for the TGMSS, which uses FDM, can be seen in Fig. 1. The FDM equations in the cell is easily generated as much as it is required depending on the \( \Delta x \) and \( \Delta y \), which are the size of grids in the solution domain.

The TGMSS model is divided into rectangular grid intervals both in \( x \) and \( y \) directions in order to carry out the iterative spreadsheet calculation. The TGMSS model takes \( \Delta x \), \( \Delta y \), \( \Delta t \), aquifer parameters (e.g. hydraulic conductivity, storage capacity, initial hydraulic heads), iteration number and/or maximum change (\( \varepsilon \)), time \( (m\Delta t) \) as input parameters. The flowchart of TGMSS can be seen in Fig. 2.

The TGMSS consist of two loops as in Fig. 2; inner loop and outer loop. The inner loop computes the hydraulic heads for given time interval until the maximum convergence criterion \( (\varepsilon) \) is met, then, the outer loop controls the time dimensions for subsequent use in inner loop. Maximum convergence criteria \( (\varepsilon) \) for inner and outer loops can be given as follows:

\[
E_{\text{max},1} = \text{Max} |H_{ij}^m+1 - H_{ij}^m| \quad (6)
\]

\[
E_{\text{max},2} = \text{Max} |H_{ij}^{m+1,n} - H_{ij}^{m,n}| \quad (7)
\]
Fig. 3 shows the solution domain represented in spreadsheet. This sheet is called ‘Solution’. Six additional sheets are indicated by the ‘tabs’ in Fig. 3.

Storage describes the inputs for the storage capacity as a separate sheet, conductivity describes the inputs for the hydraulic conductivity as a separate sheet, and source and recharge_sources describe the inputs for the sinks and/or sources, which computed using the pumping rate, and recharge values in the pumping and recharge sheets. Note that the sinks are computed using negative (−) pumping rates, and sources are computed using positive (+) pumping rates.

Iteration number and/or maximum convergence criteria (ε) are given in the TGMSS as follows:

Select ‘Tools’ pull-down menu of the spreadsheet and select ‘Options’ in it. Then, select the calculation tab. The iteration option is not selected when the spreadsheet is loaded by default. There are two important options given in this window: one is maximum iterations and the other is maximum allowable convergence (maximum change) criteria (ε) as can be seen in Fig. 4. When both criteria are met, the iterative calculation is stopped.

Solution of TGMSS model is carried out based on Eq. (5) in the following spreadsheet format:

\[
\begin{align*}
C_4 &= ((Storage!C4/Solution! $ AA $ 11)\times Solution!C4 \\
&\quad + $ AA $ 9*(Conductivity!C4 \\
&\quad + Conductivity!D4)\times Solution!D4 \\
&\quad + $ AA $ 9*(Conductivity!B4 \\
&\quad + Conductivity!C4)\times Solution!B4 \\
&\quad + $ AA $ 10*(Conductivity!C4 \\
&\quad + Conductivity!C3)\times Solution!C3 \\
&\quad + Solution! $ AA $ 10*(Conductivity!C4 \\
&\quad + Conductivity!C5)\times Solution!C5 \\
&\quad + Source!C4)/((Storage!C4/Solution! $ AA $ 11) \\
&\quad + $ AA $ 9*(Conductivity!C4 \\
&\quad + Conductivity!D4) \\
&\quad + $ AA $ 9*(Conductivity!B4 \\
&\quad + Conductivity!C4) \\
&\quad + $ AA $ 10*(Conductivity!C4 \\
&\quad + Conductivity!C3) \\
&\quad + $ AA $ 10*(Conductivity!C4 \\
&\quad + Conductivity!C5))
\end{align*}
\]
where \( C_4 \), which is an intersection of the third column \((C)\) and fourth row \((4)\), represents the hydraulic head in cell. Related sheets are used in solving Eq. (5). Cell names consist of column and row numbers and follow the sheet names ending with the ‘!’ indicator (e.g. Conductivity!C5 or Solution!B5).

The output of TGMSS can be seen visually using graphical interface of spreadsheet simultaneously.

5. Numerical applications

The application of the TGMSS model for groundwater systems is tested using two hypothetical examples. The first hypothetical example has homogeneous aquifer parameters and is further divided into two cases. In addition, the second hypothetical example has heterogeneous aquifer parameters and is examined under two scenarios.

5.1. First hypothetical example

Grid spacing, \( \Delta x = \Delta y = 100 \text{ m} \)
Number of grid spacing, \( N = M = 23 \)
Hydraulic conductivity, \( K = 15.0 \text{ m/day} \) (fix in all cells)
Storage capacity, \( S = 1.0 \text{ m} \) \( K \) (fix in all cells)
Time step, \( \Delta t = 1 \text{ day} \)

5.1.1. Case I

Three hypothetical wells having constant pumping rates are located as sinks and/or sources. The wells are located symmetrically in the solution domain. Pumping rates, \( Q_1 = Q_3 = 432.0 \text{ m}^3/\text{day} \) (sinks), \( Q_2 = 864.0 \text{ m}^3/\text{day} \) (source).

The pumping rates in Fig. 5 are converted to sink and/or source values using the procedure in Fig. 6 to avoid the problem of zero division. Note that the value of \( H \) in Fig. 8 (i.e. in Eq. (5f)) is equal to initial hydraulic head before
the iteration is started. For this example, the initial hydraulic head is given 20 m in each cell.

To start the iterative calculation, cell of C4 is copied and pasted from C4 to W24. Column and row numbers of Eq. (8) will be changed in each cell among the range of C4:W24.

Results of the TGMSS can be seen in Fig. 7. The boldface cells indicate the well locations. Note that the values of hydraulic heads in two symmetrically located wells are equal.

Three-dimensional representation of hydraulic heads can be seen in Fig. 8. This figure shows the level of hydraulic heads over the solution domain. Each contour of Fig. 8 indicates the various levels of hydraulic heads.

The TGMSS model has been verified with MODFLOW. The results of TGMSS model and MODFLOW showed good agreement. The contour of TGMSS is given in Fig. 9(a), and the contour of MODFLOW is given in Fig. 9(b).

The contours of the TGMSS model can be seen in Fig. 10(a) for $t=1$ day, Fig. 10(b) for $t=5$ days and Fig. 10(c) for $t=10$ days. Variation of groundwater table (GWT) at any node becomes steady-state as the time increases. Fig. 11 shows the variation of GWT where

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**Fig. 7.** Values of $H_{ij}$ after computations ($t=3600$ days).

**Fig. 8.** Three-dimensional variation of groundwater level ($t=3600$ days).
Fig. 9. Contours of the TGMSS and MODFLOW for $t=3600$ days. (a) Graphical output of the TGMSS. (b) Graphical output of the MODFLOW.

Fig. 10. Variation of groundwater level for (a) $t=1$ day, (b) $t=5$ days and (c) $t=10$ days.
the time indicator \((m)\) is changed from 1 to 3600 in the middle well. For this example, after 100 days GWT becomes steady-state.

5.1.2. Case 2

Three hypothetical wells having variable pumping rates are considered in this case. The following pumping rate function may be given as sinks and/or sources.

\[
Q_{ij} = Q_{\text{max}} \sin \left( \frac{2\pi m \Delta t}{360} \right)
\]

where \(m\) is the time indicator and \(Q_{\text{max}}\) is 432.0 m\(^3\)/day (sinks) for Well 1 and 3 and 864.0 m\(^3\)/day (source) for Well 2.

After describing the model, iterative calculations can be started using the procedure previously described. Fig. 12 shows the variation of hydraulic heads for the well locations while the time indicator \((m)\) is changing from 1 to 3600.

In the TGMSS model, volumetric rates of sinks and/or sources need to be checked in each time interval to control entered- and exit-water to the solution domain. Total mass error can be written as:

\[
\text{Total mass error} \left( \frac{\%}{\text{}} \right) = \frac{\sum \text{Mass before calculation} - \sum \text{Mass after calculation}}{\sum \text{Mass before calculation}} \times 100
\]

(10)

The results of the total mass errors and CPU times for the Case 1 and 2 can be seen in Table 1. Note that this example has been solved on a personal PC (P4 2.80 GHz processor and 512 MB RAM) and screen updating feature of the spreadsheet was closed during the calculation.

5.2. Second hypothetical example

In this example, application of TGMSS model has been carried out using heterogeneous aquifer parameters and variable boundary conditions in the complex solution domain. The data used in this example are given as follows:

- Grid spacing, \(\Delta x = \Delta y = 100\) m
- Number of grid spacing, \(N = 23, M = 39\)
- Hydraulic conductivities, \(K_1 = 15, K_2 = 10, K_3 = 35, K_4 = 30, K_5 = 20, K_6 = 25\) m/day
- Storage capacity, \(S = 1.0\) m\(^{-1}\) (fix in all cells)
- Six hypothetical wells are defined as sinks. Their rates are:
  - \(Q_1 = Q_4 = Q_5 = 864.0\), \(Q_2 = Q_6 = 432.0\), \(Q_3 = 1728.0\) m\(^3\)/day
- Recharge, \(R_{\text{max}} = 35.265\) mm/month (fix in all cells as source),

\[
R_{ij} = R_{\text{max}} \left| \sin \left( \frac{\pi m \Delta t}{360} \right) \right|
\]

(11)

The input parameters of hydraulic conductivities and pumping rates are shown in Fig. 13. This example has been examined under two scenarios. Details of both scenarios are given in Table 2.

Iterative calculations for Scenario A and B can be started using the procedure previously described. The initial hydraulic heads are given 15 m. Before the calculation is started, this value needs to be pasted to all cells in the solution domain. The results of the total mass errors and CPU times for Scenario A and B is given in Table 3.

As can be seen in Table 3, total mass errors of Scenario A and B are in good agreement with accuracy of result, but CPU times are very different. It may be concluded that Scenario A is better for obtaining the hydraulic heads at
a desired time than Scenario B. However, Scenario B is better for obtaining hydraulic heads at sequential times than Scenario A. Fig. 14 shows the variation of hydraulic heads for the wells locations while the time indicator \((m)\) is varied between 1 and 3600. The hydraulic heads at the well locations vary depending on recharge values although pumping rates of them are constant as in Fig. 14. Moreover, total mass conversation corresponding to the Scenario A and B is given in Fig. 15.

![Fig. 13. Hydraulic conductivities and pumping rates.](image)

### Table 2
Scenario A and B used in Example 2

<table>
<thead>
<tr>
<th>(m)</th>
<th>(\Delta t) (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario A</td>
<td>1, 3, 5, 10, 30, 50, 100, 1000, 3600</td>
</tr>
<tr>
<td>Scenario B</td>
<td>1, 2, 3, ..., 3600</td>
</tr>
</tbody>
</table>

![Fig. 14. Variation of hydraulic heads against time for the well locations.](image)

### Table 3
Total mass errors and CPU times for Scenario A and B

<table>
<thead>
<tr>
<th>Time (day)</th>
<th>Scenario A</th>
<th>Scenario B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total mass</td>
<td>CPU time</td>
</tr>
<tr>
<td></td>
<td>error (%)</td>
<td>(s)</td>
</tr>
<tr>
<td>1</td>
<td>0.0123</td>
<td>172</td>
</tr>
<tr>
<td>3</td>
<td>0.0143</td>
<td>83</td>
</tr>
<tr>
<td>5</td>
<td>0.0146</td>
<td>57</td>
</tr>
<tr>
<td>10</td>
<td>0.0146</td>
<td>36</td>
</tr>
<tr>
<td>30</td>
<td>0.0128</td>
<td>17</td>
</tr>
<tr>
<td>50</td>
<td>0.0109</td>
<td>11</td>
</tr>
<tr>
<td>100</td>
<td>0.0069</td>
<td>7</td>
</tr>
<tr>
<td>1000</td>
<td>0.0084</td>
<td>3</td>
</tr>
<tr>
<td>3600</td>
<td>0.0161</td>
<td>2</td>
</tr>
</tbody>
</table>

![Fig. 15. Total mass conversation for Scenario A and B.](image)
Outputs of the TGMSS model for both Scenario A and B can be seen in Fig. 16 for $t = 3600$ days.

Fig. 17 shows contours of hydraulic heads. While the effect of pumping can be seen around the wells of 1, 2, and 3, and inclination of water surface can be seen around the wells of 4, 5, and 6.

### 6. Conclusions and limitations

This study proposes the TGMSS model to solve groundwater-modeling problems. Each cell in the spreadsheet was matched with middle point of each element with FDM. Copying–pasting, iterative calculation and graphical representation of spreadsheets has been illustrated. The TGMSS model was tested with two hypothetical examples. The following conclusions can be drawn from study.

The TGMSS model may be adapted to learn for basic groundwater problems, which may be considered as a flexible structure. There are no difference between the numerical solutions of homogeneous-isotropic systems and heterogeneous-anisotropic systems in terms of solution techniques and data processing for this problem.

There may be no need to use macros and subroutines in the TGMSS model. The writing of the input values to the related sheets and the generations are easy in terms of solution procedure.

The present method may not be applied for solving large systems of equations due to the limitation in the size of...
spreadsheets. Another limitation of spreadsheet is that finite difference grids, which are mapped into spreadsheet cells, can only be rectangular or triangular. Hence, some geometry problems may be occurred at the excluding curved boundaries in complex geometries. Future study should be on this issue.

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References