

# Light scattering by irregular interplanetary dust particles

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We review recent progresses in light scattering for non-spherical particles. Special attention is paid to cluster of spheres in order to improve our understanding of interplanetary dust particles. For scattering by non-spherical particles the discrete-dipole approximation (DDA) has widely been used in many scientific fields. However mainly due to the requirements of large computing memory and long computing time, the applicability of this theory is practically limited for rather small particle compared with wavelength. In order to overcome this practical problem, i.e., the large particle can not be calculated by the DDA, we recently developed the  $a_1$ -term method, which is a modification version of the DDA where the dipole polarizability is determined by the first term of scattering coefficient in Mie theory. Accuracy of this method is tested by comparing the solutions by the  $a_1$ -term method with those by modal analysis, which gives the analytical solutions for cluster of spherical monomers. According to the error analysis mentioned above, the applicabilities of the  $a_1$ -term method are established as follows. The maximum size parameter of the monomer in the cluster is 1 and the total size parameter of the cluster can exceed  $X \sim 100$  when the  $N \sim 10^6$  dipoles are used. We show the extinction efficiencies and asymmetry factors for cluster of spheres whose size parameter is larger than the wavelength, e.g., the volume equivalent size parameter  $X$  is larger than 30. Finally we summarize the applicabilities of DDA, T-Matrix, modal analysis and the  $a_1$ -term method. The  $a_1$ -term method can partly fulfill a gap where both DDA and the ray tracing technique based on geometrical optics can not be applied when the target is cluster. However for the target which has edges remains to be problematic. This would be the topic which should be focused on future research.

## 1. Introduction

For homogeneous sphere, there exists analytical method and this, Mie theory, has been widely used in many applications. By this method, it is possible to cover wide range of size distribution of particles. Although this theory is quite useful because of the short computing time for calculations, its applicability is questionable in many astrophysical situations. For example it becomes recognized that the interplanetary dust particle does not have a spherical shape. This has been proved by air-craft experiments in the stratosphere (e.g., Brownlee, 1978). Actually any particle in nature is not a perfect sphere. Therefore it is highly demanded to develop light scattering theory for non-spherical particle. In 1975 Asano and Yamamoto developed a theory for homogeneous spheroidal particle. In this method the Maxwell's equations are expressed in terms of the spheroidal coordinate. This method is called separation of variables method. It is also known that the scattering problem for infinite cylinder and sphere with core-mantle structure are also analytically solved by the similar separation of variables techniques. T-Matrix method is very attractive for the particle which has rotationally symmetric structure (Mishchenko *et al.*, 1997). Unfortunately it seems there are not many hopes to find analytical

solutions for particles which do not have any symmetry. If this is true, we have to improve approximate theories for non-spherical particle.

The discrete-dipole approximation (DDA) was originally developed by Purcell and Pennypacker (1973). The DDA has been used in many fields, e.g., astronomy (e.g., Draine, 1988; Kozasa *et al.*, 1992, 1993), planetary sciences (West, 1991; Lumme and Rahola, 1994; Okamoto *et al.*, 1994) and atmospheric sciences (e.g., Flatau *et al.*, 1990). The DDA seems to be promising for non-spherical particles since this technique is easily applicable to any target geometries as long as the size parameter of the particle  $X$  is less than 10–15. In Section 2, we describe the theoretical basis of the  $a_1$ -term method which was developed to overcome the practical problem in the DDA (Okamoto, 1995). Explicit expression of the prescription to obtain the dipole polarizability, which is a crucial parameter to determine accuracy of the DDA, is given. Since these methods are approximate theories, the accuracy of them should be verified by comparing their solutions with those obtained by an analytical method or with light scattering experiments. Recently substantial efforts have been made to derive rigorous analytic solutions for cluster of spherical monomers (Fuller, 1991; Mackowski, 1991; Xu, 1995). By using this so called modal analysis, we can have a unique opportunity to test the accuracy of the  $a_1$ -term method. The results in error analysis are given in Section 3. In Section 4, to see the effect of non-sphericity in the wide range of sizes, we carry out the calculations for randomly oriented clusters by the  $a_1$ -term method where the size parameter  $X$  range

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from 1 to 35. In Section 5, applicabilities of several scattering theories for three types of non-spherical particles such as cluster, rotationally symmetric particle and particle with edges are summarized. Difficulties in the calculations of the scattering by the solid target with sharp edge are addressed.

## 2. Method

### 2.1 Theoretical basis of the $a_1$ -term method and other types of DDA

The discrete-dipole approximation (DDA) is originally developed by Purcell and Pennypacker in 1973 for the study of the interstellar dust grains and is further developed by Draine (1988). In the DDA the target is approximated by  $N$  sub-volume elements and each element is replaced by a polarizable element. Each polarizable element is characterized by the dipole polarizability  $\alpha$  where each of the  $N$  dipole elements is exposed to the field of incident wave plus the field generated by the other elements. In the estimation of the latter fields, the retardation effects are fully taken into account. Theoretical basis of this method is given in Draine (1988). The key parameter in the DDA is the prescription of the dipole polarizability which actually determines the accuracy of DDA. Draine (1988) proposed a Clausius-Mossotti plus radiative reaction correction term (CMRR) to determine the dipole polarizability. The aim of introducing the radiative reaction correction term is to avoid a violation of the optical theorem. Liversay and Chen (1974), Goedecke and O'Brien (1988) and Hage and Greenberg (1990) proposed "Digitized Green's Function" method or "Volume Integral Equation Formulation" (hereafter DGF/VIEF). In their derivation of the dipole polarizability, electric field inside sub-volume element is assumed to be uniform. Finally Draine and Goodman (1993) proposed the Lattice Dispersion Relation (LDR) and this prescription turns to be the best one for the cubic sub-volume element. They numerically showed that LDR is superior to others such as CMRR and DGF/VIEF when the same number of dipoles is used. Note that in the zero frequency limit, all prescriptions are equal to the Clausius-Mossotti relation. Therefore when the sub-volume elements are taken to be very small compared to the wavelength of interest, the DDAs with different prescriptions generally give the same accuracy.

One of the problem in the DDA calculations is that it is only applicable for the target with the size parameter  $< 10$ – $15$ . It means that the target should be smaller than or comparable to the wavelength of interest. This restriction is mainly from the computing time and memory requirements. In order to overcome the practical problem, we recently introduced the first scattering coefficient  $a_1$ -term in Mie theory for the prescription of the dipole polarizability. Here we explain the theoretical basis of this method. Scattering target is taken to be the cluster of spherical monomers. Then each monomer is replaced by one dipole and the dipole polarizability  $\alpha$  is determined by the  $a_1$ -term (Eq. (1)). This approach is called the  $a_1$ -term method.

$$a_1 = \frac{m\psi_1(mx_0)\psi_1'(x_0) - \psi_1(x_0)\psi_1'(mx_0)}{m\psi_1(mx_0)\xi_1'(x_0) - \xi_1(x_0)\psi_1'(mx_0)}, \quad (1)$$

where  $\psi_1$  and  $\xi_1$  are called the first order of Riccati-Bessel function and represented in Eqs. (2) and (4), respectively.

The size parameter of the monomer  $x_0$  is  $2\pi r_0/\lambda$ , where  $r_0$  is the radius of the spherical monomer,  $\lambda$  is the vacuum wavelength and  $m$  denotes the refractive index of the material. The explicit forms for  $\psi_1'(x_0)$  and  $\xi_1'(x_0)$  are given in Eqs. (3) and (5) respectively.  $\psi_1(mx_0)$ ,  $\xi_1(mx_0)$ ,  $\psi_1'(mx_0)$  and  $\xi_1'(mx_0)$  can simply be obtained by replacing  $x_0$  with  $mx_0$  in Eqs. (2), (3), (4), in the order mentioned since the prime in the Eq. (1) indicates differentiation with respect to the argument in parentheses.

$$\psi_1(x_0) = \frac{\sin x_0}{x_0} - \cos x_0, \quad (2)$$

$$\psi_1'(x_0) = \frac{\cos x_0}{x_0} - \frac{\sin x_0}{x_0^2} + \sin x_0, \quad (3)$$

$$\xi_1(x_0) = \frac{\sin x_0}{x_0} - \cos x_0 - i \left( \frac{\cos x_0}{x_0} + \sin x_0 \right), \quad (4)$$

$$\xi_1'(x_0) = \frac{\cos x_0}{x_0} - \frac{\sin x_0}{x_0^2} + \sin x_0 + i \left( \frac{\sin x_0}{x_0} + \frac{\cos x_0}{x_0^2} - \cos x_0 \right), \quad (5)$$

$$C_{\text{ext}} = 4\pi k \text{Im}\{\alpha\}, \quad (6)$$

$$C_{\text{ext}} = \frac{4\pi}{k^2} \text{Re}\{S_1\}, \quad (7)$$

$$S_1 = \frac{3}{2}a_1. \quad (8)$$

From the optical theorem, extinction cross section  $C_{\text{ext}}$  is expressed by using the dipole polarizability  $\alpha$  in Eq. (6). On the other hand, according to Mie theory,  $C_{\text{ext}}$  is also given by the first element of the amplitude scattering matrix  $S_1$  is in Eq. (7) (see Bohren and Huffman, 1983). The dipole approximation in Mie theory for sphere leads to the expression of  $S_1$  in terms of  $a_1$  in Eq. (8).

From Eqs. (6)–(8), the dipole polarizability can be derived (Eq. (9)) which is expected to be the best prescription for the spherical monomer in the cluster since the boundary condition of the sphere is already taken into account. The relationship between the first scattering coefficient  $a_1$ -term and electric dipole polarizability is first found in Doyle (1985).

$$\alpha = i \frac{3}{2k_0^3} a_1, \quad (9)$$

where the wave number  $k_0$  is  $2\pi/\lambda$ .

After the dipole polarizability  $\alpha_j$  is obtained, electromagnetic scattering problem can be solved for the cluster of monomers where each monomer ( $j = 1, \dots, N$ ) is replaced by a single dipole with polarizability  $\alpha_j$  located at position  $\mathbf{r}_j$ . A polarization  $\mathbf{P}_j$  is given by  $\alpha_j \mathbf{E}_j$  where  $\mathbf{E}_j$  can be calculated by the sum of incident field at  $\mathbf{r}_j$  and the contribution of each of the other  $N - 1$  monomers which is represented with  $N - 1$  polarizations. Therefore the scattering problem is converted to the one for finding the  $N$  polarizations which satisfy  $3N$  linear equations according to the usual DDA procedure. Once  $\mathbf{P}_j$  is found, the extinction cross section can easily be estimated (see Draine, 1988).

$$C_{\text{ext}} = \frac{4\pi}{|E_0|^2} \sum_{j=1}^N \text{Im}(E_{\text{inc},j}^* \cdot \mathbf{P}_j). \quad (10)$$

It is noted in the  $a_1$ -term method the target should be the cluster composed of spherical monomers.

Dungey and Bohren (1991) used this Doyle's expression as the dipole polarizability in the DDA expressed in Eqs. (1) and (9) but with different definition of complex refractive index. It might be instructive to explain the differences between our approach and theirs. In their approach, scattering target, such as spheroid or hexagonal column, is divided into many sub-volume elements which are spherical dipolar unit and spaced on a cubic lattice. For a given volume of the target, the volume equivalent radius of the target  $r_{\text{eff}}$  is defined by Eq. (11). The radius of the spherical dipolar unit  $r_d$  is defined by Eq. (12) and the lattice spacing corresponding to the size of the cubic cell,  $d$  is defined by Eq. (13).

$$r_{\text{eff}} = \left( \frac{3V}{4\pi} \right)^{1/3}, \quad (11)$$

$$r_d = \left( \frac{1}{N} \right)^{1/3} r_{\text{eff}}, \quad (12)$$

$$d = 2r_d. \quad (13)$$

To obtain the dipole polarizability for this cubic cell, at first the spherical dipolar sub unit is embedded in the cubic cell. Thus the volume fraction of the dipolar unit  $f$  is  $\pi/6$  which can be estimated by Eqs. (11)–(13). Then Maxwell-Garnett effective medium theory (Eq. (14)) is applied to estimate the average complex refractive index for this cubic cell (e.g., Bohren and Huffman, 1983),

$$\epsilon_{\text{av}} = \epsilon_m \left[ 1 + \frac{3f \left( \frac{\epsilon - \epsilon_m}{\epsilon + 2\epsilon_m} \right)}{1 - f \left( \frac{\epsilon - \epsilon_m}{\epsilon + 2\epsilon_m} \right)} \right], \quad (14)$$

where  $\epsilon$  is the dielectric function of the material and is equal to  $m^2$ .  $\epsilon_m$  is that for the surrounded media, i.e., 1 for vacuum.  $\epsilon_{\text{av}}$  denotes the average dielectric function. Finally average refractive index  $m_{\text{av}}$ , is calculated from  $m_{\text{av}} = \epsilon_{\text{av}}^{1/2}$  and then inserted into Eq. (1) instead of  $m$  to derive a dipole polarizability of a cubic volume element. Contrary to the approach of Dungey and Bohren, there is no need to use effective medium theory in our approach. We would like to distinguish between the two.

## 2.2 Modal analysis

Recently there has been substantial progresses to derive rigorous analytical expressions for the cross sections as well as the all elements of scattering matrix by aggregates of an arbitrary number of arbitrarily configured spheres (Fuller, 1991; Mackowski, 1991; Xu, 1995). These spherical monomers should be isotropic and homogeneous but can have different sizes and chemical compositions. In this approach, the Mie theory is generalized, that is, the incident field, scattered field and the internal field at the center of an individual sphere in the cluster are expanded in terms of infinite-series of the spherical vector wave functions. Boundary conditions for each sphere is incorporated. This leads to a set of coupled linear equations with the unknown interactive coefficients. To seek the solutions, translational addition theorem for spherical vector wave functions and asymptotic iteration technique are developed. We call this method modal analysis in this paper. In this approach, multipole interactions between spheres in the cluster are fully taken into account. Note for the case of  $N = 1$ , all the formulations

derived by modal analysis are in complete agreement with the Mie theory (Xu, 1995). Since this method provides quite accurate solutions for the cluster of monomers, this can give us a unique opportunity to test the accuracy of the  $a_1$ -term method.

## 3. Results for Error Analysis: The $a_1$ -Term Method versus Modal Analysis for the Cluster of Spheres

In this section the accuracy of the  $a_1$ -term method is numerically tested. To calculate the extinction by non-spherical particles, we introduce different definition of extinction efficiency from the usual one. Efficiency factor  $Q_{\text{ext}}$  is usually defined as

$$Q_{\text{ext}} = \frac{C_{\text{ext}}}{G}. \quad (15)$$

$G$  denotes geometrical cross section of the target particle. In following discussions, volume equivalent extinction efficiency  $Q_{\text{ext},v}$  is introduced in order to allow the comparison between extinction for non-spherical particle and spherical one for the same volume.

$$Q_{\text{ext},v} = \frac{C_{\text{ext}}}{\pi r_{\text{eff}}^2}, \quad (16)$$

where  $r_{\text{eff}}$  is the volume equivalent effective radius and is defined in Eq. (11).

In Fig. 1(a) we use the modal analysis to calculate  $Q_{\text{ext},v}$  for cluster of spherical monomers. The complex refractive index used in this paper is  $m = 1.33 + 0.01i$ . The shape of the cluster is taken to be a linear-chain, e.g., each monomer is located on a straight line. The incident electromagnetic field is taken to be parallel to the longest axis, i.e. the longest axis of the target. And the size of the monomer  $X_o$  ranges from 0.2 to 2.0. The corresponding volume equivalent size parameter of the cluster  $X$  is calculated from  $X_o$ . We considered four different clusters, i.e., the number of monomers  $N = 2, 5, 10$  and  $20$ . It is found that extinction by the cluster is smaller than that by sphere for this target geometry. As the number of monomers increases,  $Q_{\text{ext},v}$  is decreased for the same target volume. The differences between  $Q_{\text{ext},v}$  for sphere and that for the clusters are sometimes several factors. This is due to electromagnetic interactions between monomers. By using these results as reference solutions, we performed similar calculations for the same target but by the  $a_1$ -term method described above. Then fractional error in  $Q_{\text{ext},v}$  defined in Eq. (17) is shown in Fig. 1(b).

$$\text{Relative Error} \equiv \frac{Q_{\text{ext},v}(a_1\text{-term}) - Q_{\text{ext},v}(\text{modal})}{Q_{\text{ext},v}(\text{modal})} \times 100[\%]. \quad (17)$$

The errors do not change for different number of dipoles when the errors at the same  $X_o$  are considered. As long as  $X_o < 1$ , the error is less than 10%, e.g., the errors are 4% for  $X_o = 0.6$ , 6% for  $X_o = 0.8$  and 8% for  $X_o = 1$ . The  $a_1$ -term method always underestimates the efficiencies because in this method the higher order of multipole moments than electric dipole one, such as magnetic dipole and electric quadrupole one, is not taken into account.

In Fig. 2(a), the propagation direction of the incident electromagnetic field is taken to be perpendicular to the longest

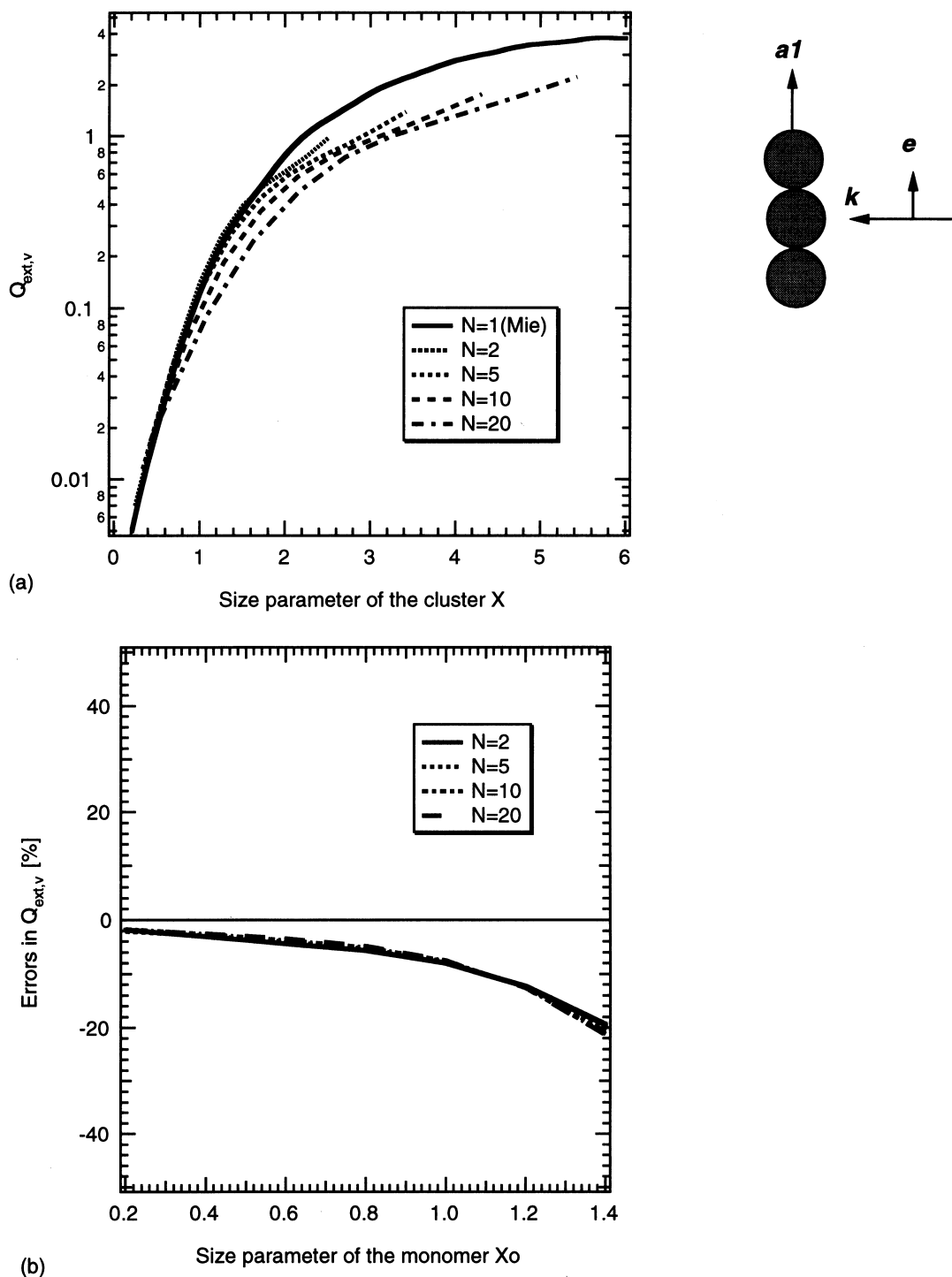


Fig. 1. (a) Volume equivalent extinction efficiency  $Q_{\text{ext},v}$  of cluster of monomers calculated by the modal analysis. Horizontal axis denotes the size parameter of the cluster. The refractive index  $m$  is  $1.33 + 0.01i$ .  $N$  denotes the number of monomers in the cluster. The  $a_1$ ,  $k$ , and  $e$  represent the longest axis of the cluster, the propagation direction of the incident wave and the direction of the electric field, respectively. (b) The relative errors in computed values of  $Q_{\text{ext},v}$  by the  $a_1$ -term method. The results are shown as a function of size parameter of the monomer  $X_0$ .

axis of the target as in Fig. 1(a) but electric field is also perpendicular to the longest axis of the target. Again,  $Q_{\text{ext},v}$  for cluster is smaller than that for sphere and this effect is more pronounced for larger  $N$  (in Fig. 2(b)). As long as  $X_0 < 1$ , the errors is less than 10%, e.g., the errors are 4% for  $X_0 = 0.6$ , 6% for  $X_0 = 0.8$  and 10% for  $X_0 = 1$ . When

the propagation direction of the incident wave is taken to be parallel,  $Q_{\text{ext},v}$  for cluster is larger than that for sphere for large size parameter (in Fig. 3(a)). For small  $X$ ,  $Q_{\text{ext},v}$  is smaller than that of sphere as in the previous cases. For this geometry, the error for the  $a_1$ -term method is larger compared to those for the previous cases (in Fig. 3(b)). The

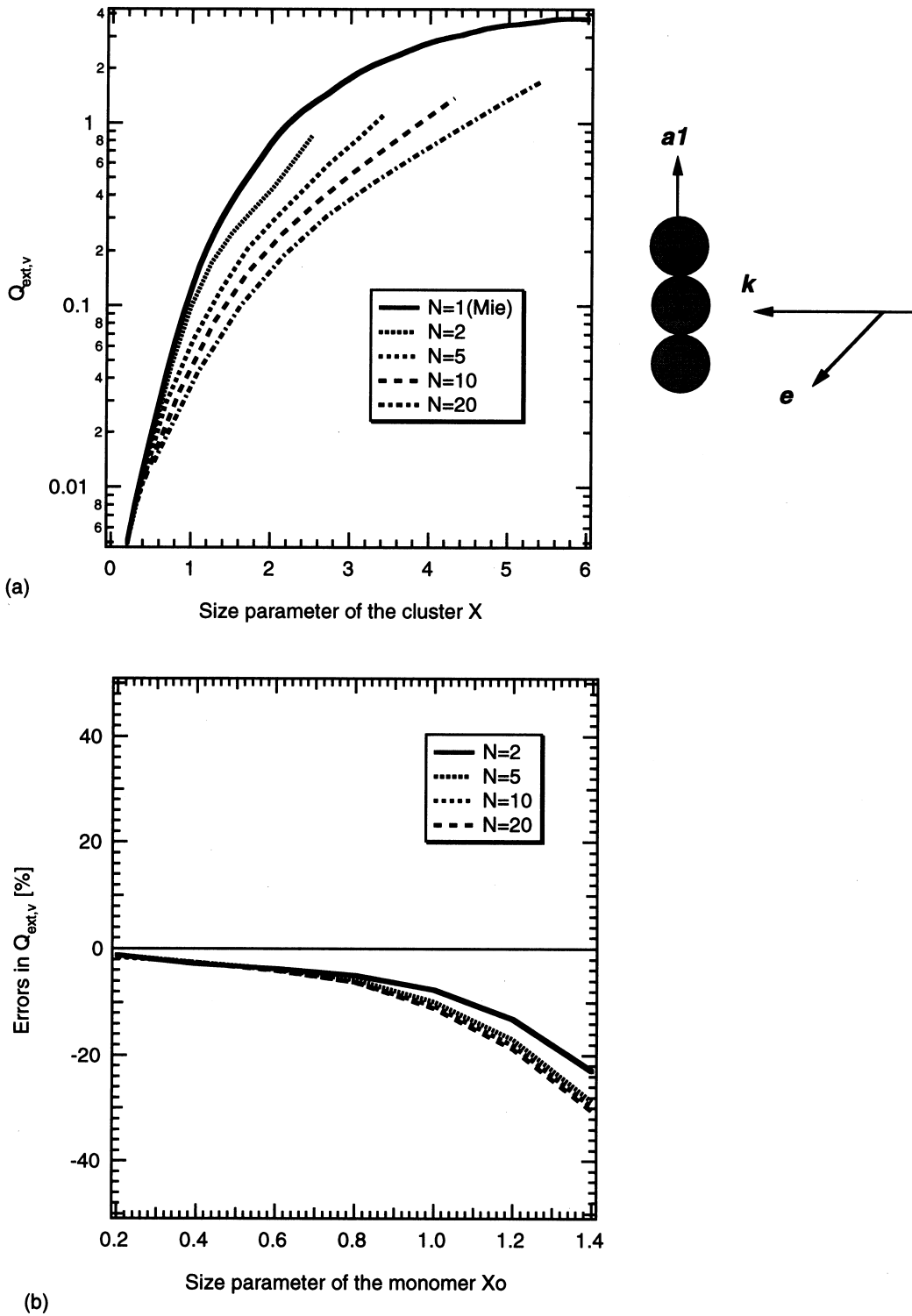


Fig. 2. (a), (b) The same as Fig. 1. (a) except for the direction of the electric field  $e$  being perpendicular to the  $a_1$  axis.

contribution of multipole interactions for this case is more important than for other geometries. The error for  $N = 20$  with  $X_o = 0.6$ , that with  $X_o = 0.8$  and that with  $X_o = 1.0$  are 10%, 20% and 35%, respectively. As  $N$  increases, the error slightly increases. However, the difference between the error for  $N = 20$  and that for  $N = 10$  is much smaller than those between  $N = 2$  and  $N = 5$ . That is, the er-

ror for  $N$  is converging. Therefore we expect the error for  $N > 20$  is not significantly different from that for  $N = 20$ . In conclusion, this method allows treatment of a relatively large sub volume element which is replaced by a dipole, i.e.,  $X_o \sim 1.0$ . Consequently, when we use  $N \sim 10^6$  dipoles, the DDA calculations based on the  $a_1$ -term method are feasible for the cluster with the size parameter  $X \sim 100$ . Therefore

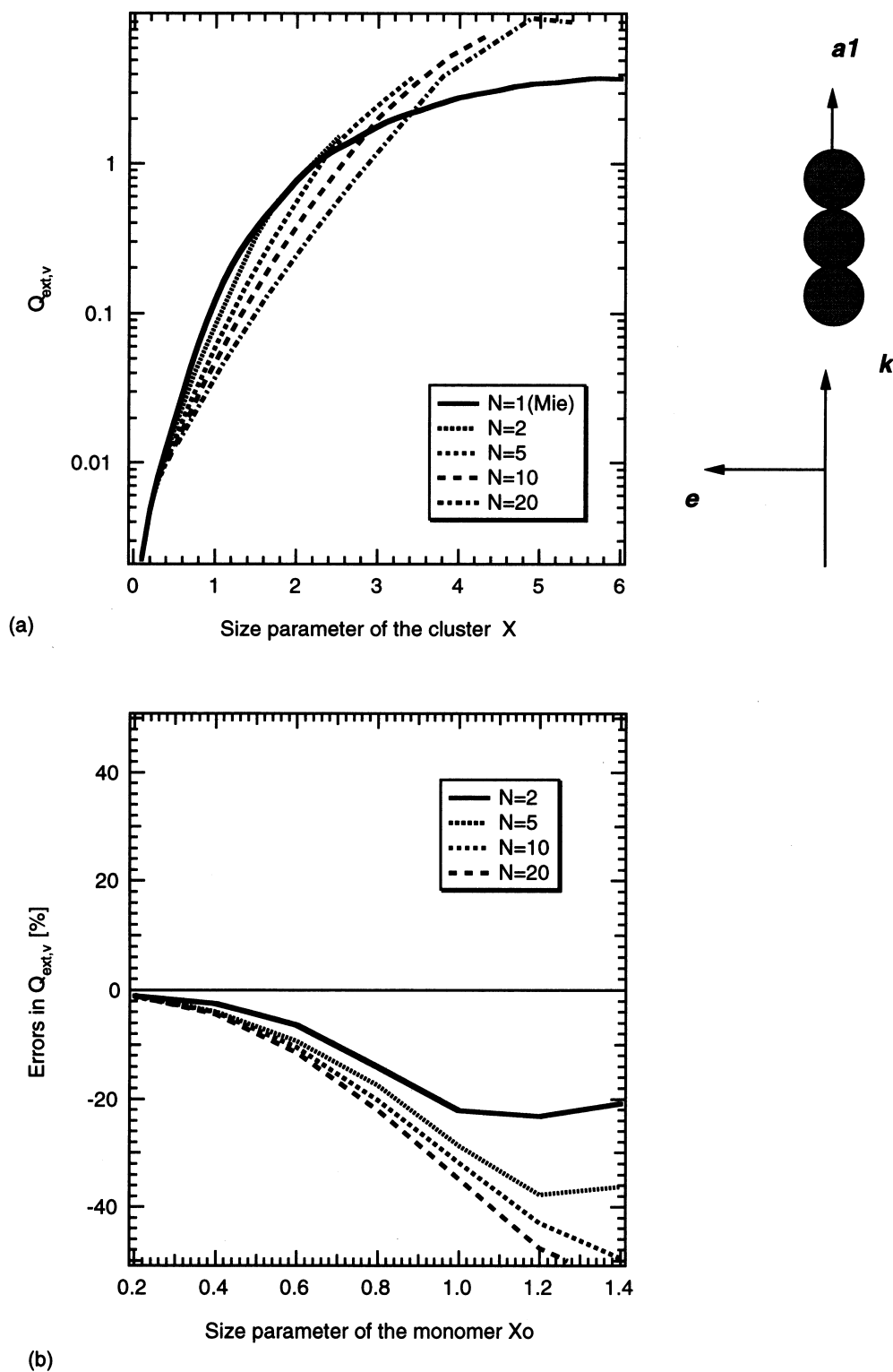


Fig. 3. (a), (b) The same as Fig. 1. (a) but for the propagation direction of the incident wave  $k$  being parallel to the longest axis  $a1$  of the target.

by this method the particles with  $X$  between 10 and 100 can become tractable while the former types of DDA can not be applied. And the great reduction of the computing memory and time can also be achieved. It should be noted that other prescription, e.g., the lattice dispersion relation (LDR) can

not be applied to achieve the same accuracies. The LDR leads to the error of +100% when  $N = 2$  model with its monomer size parameter  $X_0 = 1$  is considered as a target (Okamoto, 1995).

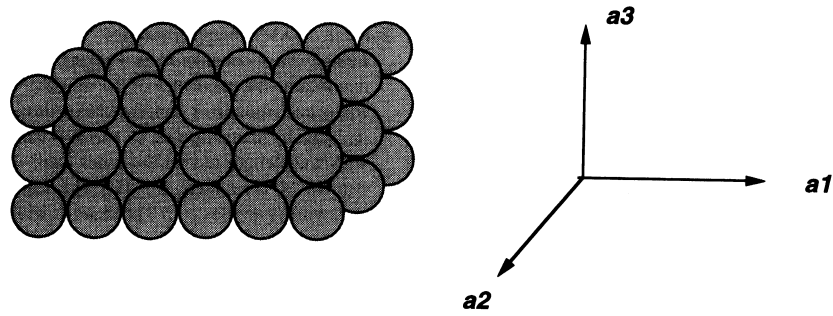


Fig. 4. “Densely packed” rectangular shaped cluster composed of monomers used in the calculations in Figs. 5 and 6. The  $a1$  denotes the longest direction of the target.

Table 1. The target parameters for the calculations.  $N$  denotes the number of monomers in the cluster and  $n(a1)$  denotes the number of monomers along the longest axis  $a1$  of the cluster.

$N$	$n(a1)$	$n(a2)$	$n(a3)$	$X_o$	$X$
2	2	1	1	0.9	1.2
16	4	2	2	0.9	2.3
54	6	3	3	0.9	3.5
128	8	4	4	0.9	4.6
250	10	5	5	0.9	5.8
432	12	6	6	0.9	6.9
2000	20	10	10	0.9	11.5
6750	30	15	15	0.9	17.3
16000	40	20	20	0.9	23.0
54000	60	30	30	0.9	34.5

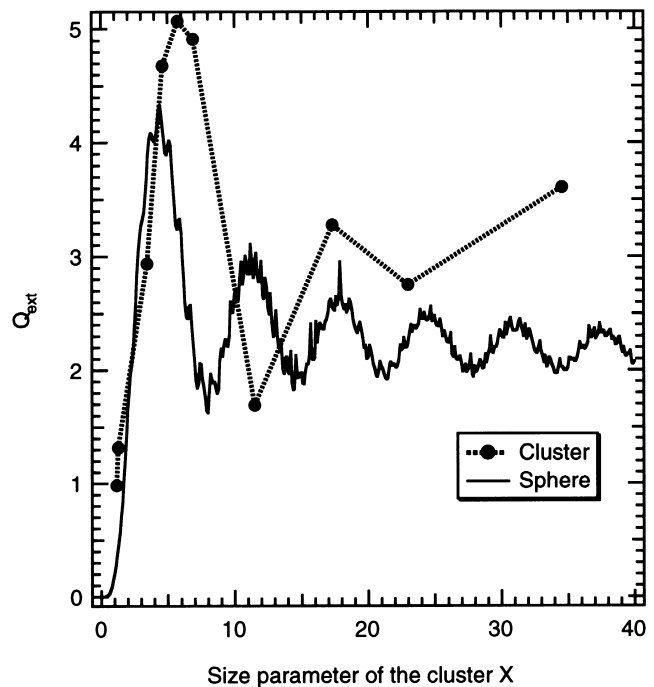


Fig. 5.  $Q_{ext,v}$  of the randomly oriented cluster of monomers computed by the  $a_1$ -term method as a function of  $X$ . For the comparison,  $Q_{ext,v}$  of the sphere is also shown. The target parameters are summarized in Table 1. The complex refractive index  $m$  is taken to be that of silicate at the visible wavelength to investigate the extinction by interplanetary dust particles, i.e.,  $m = 1.48 + 2.6 \times 10^{-5}$ .

#### 4. Extinction Properties of Cluster of Monomers

On the basis of the above findings, we have carried out calculations for randomly oriented clusters by the  $a_1$ -term method whose size parameter  $X$  ranges from 1 to 35. As far as we know, this is the first time to show extinction of non-spherical particles whose size parameter exceeds 30 by DDA approach. The shape of target considered here is rectangular solids composed of monomers as sketched in Fig. 4. Geometries of the targets such as total number  $N$  in the cluster, the number of monomers located on three orthogonal axes and the size parameter  $X$  used in these calculations are summarized in Table 1. For all calculations we set  $X_o$  to be about 0.9 which corresponds to radius of the monomer of  $0.8 \mu\text{m}$  at visible wavelength. The complex refractive index  $m$  is taken to be  $1.48 + 2.6 \times 10^{-5}$ , which is silicate’s at this wavelength. This value is chosen to mimic the scattering properties of interplanetary dust particles. In Fig. 5,  $Q_{ext,v}$  by these cluster is shown and compared to that for sphere. The position of extinction maximum for the cluster is around  $X = 6$  and is relatively larger than that of sphere, i.e., the maximum is occurred at  $X = 5$ . And the maximum value in  $Q_{ext,v}$  by the cluster is larger than that for sphere. For  $X < 3$ ,

$Q_{ext,v}$  by the cluster is larger than that by sphere for the same  $X$ . For  $3 < X < 4$ , the cluster gives smaller  $Q_{ext,v}$  compared with sphere. For the target with large  $X$ , i.e.  $X > 15$ ,  $Q_{ext,v}$  by the cluster is generally larger than that by sphere for the same volume. This can be explained as follows. In the geometrical optics, i.e., the size of the target is much larger than wavelength, extinction efficiency  $Q_{ext}$  appears to approach the limiting value 2. That is  $C_{ext}/G$  is 2 in this limit, where  $G$  is geometrical cross section. For sphere  $G$  is the same as  $\pi r_{eff}^2$  where  $r_{eff}$  is already defined in Eq. (10). Therefore  $Q_{ext}$  is equal to  $Q_{ext,v}$  when the target is sphere. However  $G$  of cluster is not equal to  $\pi r_{eff}^2$ . Although there is no easy way to estimate the geometrical cross section of particles which has non-convex structure such as cluster of monomers, we can

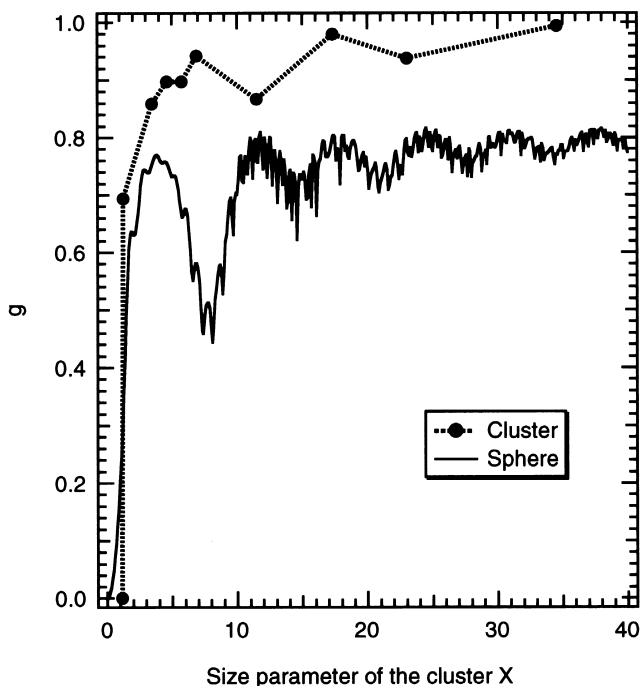


Fig. 6. The asymmetry factor  $g$  for the same targets as in Fig. 5.

expect that the average cross section of the “densely packed” cluster considered here might be similar to that of rectangular solids which can be easily estimated. Van de Hulst (1981) gave a proof that the average geometrical cross section of a convex particle with random orientation is one-fourth its surface area. Therefore  $Q_{\text{ext},v}$  of non-spherical particle, by its definition, tends to be larger than that of sphere when the size is much larger than wavelength. According to this theorem,  $G$  for rectangle whose aspect ratio to be 2, i.e., the longest dimension is twice as large as the minimum dimension, is  $1.3 \times \pi r_{\text{eff}}^2$ . In the case considered here,  $Q_{\text{ext},v}$  for cluster can be expected to approach 2.6, which corresponds to  $Q_{\text{ext}} = 2$ , in the geometrical optics limit according to the above estimates of the average cross section of the cluster. Since  $Q_{\text{ext},v}$  for  $X = 35$  is 3.6 in our calculation,  $X = 35$  is still not large enough to allow geometrical optics approximation for the qualitative estimations of cross sections.

In Fig. 6 the asymmetry factor  $g$  of the cluster is shown. The cluster tends to have larger  $g$  than sphere does. Since this value plays a crucial role for the dynamics of interplanetary dust particles, e.g., the radiation pressure calculations (e.g., Mukai *et al.*, 1992) or for radiative transfer in the planetary atmosphere, it seems necessary to take into account effects of non-sphericity on such scattering calculations.

## 5. Applicabilities of Scattering Theories

In this section we summarize the current status of applicabilities of several different scattering theories (Table 2). To see performance of each theory, we introduce three kinds of particle shape, i.e., cluster, rotationally symmetric particle, and solid particle with edges. For cluster calculation, as described in Section 3, the  $a_1$ -term method would be promising when the size parameter of the monomer fulfills condition  $X_0 < 1$ . For larger  $X_0$ , modal analysis would be the best be-

Table 2. (A) The applicabilities of several methods when the cluster of monomers is considered as a scattering target. For the DDA-LDR, the maximum size parameter of the monomer  $X_0$  is estimated for the model where each monomer is replaced by a single dipole. When the monomer is replaced by many dipoles, DDA-LDR can treat large monomer compared with wavelength but on the other hand, it is not possible to treat large number of monomers  $N$ . (B) The same as in A but for rotationally symmetric particles. (C) The same as in A but for the particle which has edges.

(A) Cluster

Method	$N$	$X$	$X_0$
DDA-LDR	$10^6$	$0 < X < 15$	$0 < X_0 < 0.15$
The $a_1$ -term method	$10^6$	$0 < X < 100$	$0 < X_0 < 1$
Modal analysis	100	$0 < X < 50$	$0 < X_0 < 50$

(B) Rotationally symmetric particle

Method	$X$
T-Matrix	$0 < X < 100$
Separation of variables	$0 < X < 30$
Ray tracing	$100 < X$

(C) Particle with edge

Method	$X$
DDA-LDR	$0 < X < 10$

cause this gives fairly accurate results, though the tractable number of monomer is quite limited, i.e.,  $N \sim 100$ . The DDA can also be used for the cluster where  $X_0 > 1$  (West, 1991; Lumme and Rahola, 1994). Unfortunately, it is not possible to treat the particle with  $X > 15$  by this method because each large monomer in the cluster should be replaced by many dipoles and consequently requirements of memory becomes prohibitively large.

For rotational symmetric particle such as spheroid, T-matrix method would be the most promising since the results are fairly accurate and it can be applied very large particle, i.e.,  $X \sim 100$  (Mishchenko *et al.*, 1997). For the particle with sharp edge, up to now, it is not possible to derive analytical solutions. When the particle with edges is small or comparable to the wavelength, DDA with LDR can be applied. Unfortunately, it is necessary to use more number of dipoles than that for the particle with the same volume but without any sharp edge to achieve the same accuracy and consequently the tractable size parameter would be smaller than 10 (Okamoto *et al.*, 1995).

It has been numerically shown the computations of scattering matrix elements are more difficult than those of the total cross sections such as extinction efficiencies with equal accuracy for DDA (Okamoto *et al.*, 1995) and for T-matrix method (Wielaard *et al.*, 1997). From these two numerical experiments the accuracy of the  $a_1$ -term method for the scattering elements may be worse than that for total cross sections. The quantitative analysis for it is currently under investigation and will be reported elsewhere.



## 6. Summary

The principal findings of this paper are as follows:

1. Theoretical basis of the  $a_1$ -term method is reviewed. This method is suitable for the cluster of spheres. In this method a spherical monomer in the cluster is replaced by a single dipole and the dipole polarizability of the monomer is obtained by the first scattering coefficient  $a_1$ -term in Mie theory. Several prescriptions of the dipole polarizability used in the discrete-dipole approximation (DDA) are also reviewed.

2. The accuracy of the  $a_1$ -term method is examined by computing the extinction cross sections for the cluster of spherical monomers by the  $a_1$ -term method and comparing the results by this method with those by the modal analysis which gives analytical solutions for the cluster of spherical monomers. The error depends on the target geometry. When the incident wave is taken to be perpendicular to the longest axis of the target, the errors are much smaller than that for the geometry where the incident wave is parallel to the longest axis. Consequently the  $a_1$ -term method allows to treat relatively large monomer as a single dipole, i.e. the size parameter of the monomer  $X_0$  is as large as 1. For  $N \sim 10^6$  dipoles,  $X$  can be  $\sim 100$ .

3. The volume equivalent extinction efficiencies  $Q_{\text{ext},v}$  by randomly oriented cluster of spherical monomers are examined to see the effects of non-sphericity. To do so, we chose silicate as a candidate of the interplanetary particles. The size parameter  $X$  of the clusters ranges from about 1 to 35, corresponding to  $N = 2$  touching spheres and  $N = 54000$  touching spheres, respectively.  $X_0$  is fixed to be 0.9. As an example of cluster, the approximate shape of the densely packed cluster composed of monomers is taken to be rectangle with an aspect ratio of 2. The target is randomly oriented in terms of the incident wave. For  $X > 15$ ,  $Q_{\text{ext},v}$  of the cluster is larger than that of sphere with the same volume. This can be explained partly due to the fact that the average geometrical cross section  $G$  for non-spherical particle is larger than that for sphere with the same volume but partly due to the mutual interaction between each monomer in the rectangle. It turns out the geometrical optics approximation can not be applied to calculate extinction by the cluster with  $X = 35$ . This is because the expected  $Q_{\text{ext},v}$  for the rectangle in the geometrical optics limit is 1.3 times larger than the limited value ( $Q_{\text{ext}} = 2$ ) and it is smaller than 3.6 which is the computed  $Q_{\text{ext},v}$  for  $X = 35$ . The asymmetry factor  $g$  is also investigated for the cluster and it turns out to be higher than that of sphere.

4. We summarize the applicabilities of several scattering theories. For cluster with small monomer but with  $N > 100$  monomers, the  $a_1$ -term method is suitable. For cluster with large monomer, modal analysis can be applied. For rotational symmetric particle, T-matrix method is the best. For particle with edge, there is no analytical solutions. DDA can be applied for this shape but its applicability seems to be very limited, i.e.,  $X < 10$ . We should pay attention to this problem in future research.

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