HYBRID OPPORTUNISTIC SPECTRUM ACCESS AND POWER ALLOCATION FOR SECONDARY LINK COMMUNICATION

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Abstract—In cognitive radio (CR) system, the spectrum band licensed to the primary link (PL) is usually divided into multiple sub-channels. Traditionally, the secondary link (SL) accesses the primary spectrum by either opportunistically selecting the idle sub-channels, called opportunistic spectrum access (OSA), or coexisting with the PL in the same band, called spectrum sharing (SS). In this paper, we propose a hybrid opportunistic spectrum access (H-OSA) scheme, in which the SL has the opportunity to access all the sub-channels based on the activity and interference threshold of the PL. Based on the lagrange dual-decomposition theory, the optimal power solution on each sub-channel is derived to maximize the ergodic capacity of the SL. Two cases are studied, i) perfect channel state information (CSI) is available at the SL, and ii) the SL has only imperfect CSI of the interference channels. The SL performance is analysed versus different activity and interference threshold of the PL. Our results indicate that the proposed scheme indeed improves the ergodic capacity of secondary link.

I. INTRODUCTION

As the rapid development of the wireless communications and networks, more and more wireless terminals require accessible spectrum bands to supply different services, such as voice, image and video, etc. However, the unlicensed bands have been occupied heavily and the licensed bands have an inefficient utilization according to the report of the Federal Communication Commission (FCC) [1]. Cognitive radio (CR), which was first proposed by Mitola [2], has been considered as the key technology to relieve this contradiction. It enables the unlicensed user to establish link in the licensed bands without any interruption to the link of the licensed user. The links established by the unlicensed and licensed users are named as secondary link (SL) and primary link (PL), respectively.

In the CR environment, the licensed band is usually divided into multiple sub-channels, and there are two conventional schemes for the SL to access these sub-channels: opportunistic spectrum access (OSA) [3] and spectrum sharing (SS) [4]. In the OSA scheme, spectrum sensing is operated to detect the state of the PL before the SL is established. Then, the unlicensed user opportunistically accesses the idle sub-channels to establish the link based on the sensing information. But this scheme ignores the fact that the PL can tolerate certain co-channel interference. The SS scheme takes advantage of this tolerance of PL and allows the SL to be established in the spectrum bands which have been occupied by the PL. In this way, the SS scheme indeed improves the efficiency of the spectrum, but it loses the chance to increase the data rate of the SL when the PL is inactive.

The capacity of SL with the interference power constraint was first studied by Gastpar in [5], considering different additive white Gaussian noise (AWGN) channels and average received power at the licensed receiver. Then, Ghasemi in [4], derived the ergodic capacity of SL with average interference power constraint in different fading channels. The performance of the SL ergodic capacity with peak/average transmit power constraint and peak/average interference power constraint was analysed and compared in [6]. Zhang in [7], derived the ergodic sum capacity of multiple SLs with peak/average transmit power constraints and average total interference power constraints in multiple-access or broadcasting fading channels. However, the capacities studied in [4]-[7] are all analysed in the SS scheme without considering the state of PL. Kang in [9], derived the SL ergodic capacity considering the perfect or imperfect sensing information of the PL. But it is assumed one single channel for the SL. In [8], the SL ergodic capacity with average total interference power constraint in the OSA scheme was studied without considering the total transmit power constraint and the PL’s tolerance.

In this paper, we study the ergodic capacity of SL with the average transmit power and interference power constraints. The unlicensed user establishes link in the licensed sub-channels, no matter whether they are idle or not. This scheme joint considers the optimal channel selection in the OSA scheme and interference restriction in the SS scheme, named as hybrid opportunistic spectrum access (H-OSA). Then, we analyse the performance of the proposed scheme with perfect or imperfect interference channel state information (CSI).

The remainder of this paper is organized as follows. In section II, we describe the system model and related assumptions. The ergodic capacity of SL with perfect or imperfect CSI in the H-OSA scheme will be studied in section III and IV, respectively. The numerical results are presented for performance evaluation in section V. We conclude the whole paper in section VI.

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II. SYSTEM MODEL

In this paper, a broadband frequency-selected channel which is licensed to the PL is divided into \( N \) orthogonal flat fading sub-channels based on the coherence bandwidth \( B_c \). The SL will be established by opportunistically accessing these sub-channels. Each sub-channel is assumed to be block fading, or in other words, the channel power gains in each sub-channel keep stationary in the same block and vary from block to block. The instantaneous channel power gains in all the sub-channels (including transmission channels of the SL and interference channels to the PL) are independent and identically distributed (i.i.d), following Rayleigh fading distribution with unit mean, denoted by \( h = [h_1, h_2, \ldots, h_N] \) and \( g = [g_1, g_2, \ldots, g_N] \), respectively. The noise in all the sub-channels is assumed to be independent circularly symmetric complex Gaussian variable with mean zero and variance \( N_0 \), i.e. \( CN(0, N_0) \).

In order to describe the protection for the PL from the SL’s interruption, two PL models are usually analysed. One is that the primary receiver announces its interference threshold in advance and the SL can transmit data without any detecting the state of the PL. The other is that the PL uses the sub-channels randomly without announcing its interference threshold. The SL will sense the spectrum and detect the state of PL. In this paper, the PL model is the combination of the two models above. So the state of PL and the interference threshold of primary receiver are assumed to be known accurately in advance. The instantaneous state of PL in any sub-channel \( j \) is a random variable (r.v.), denoted by \( \rho_j = \{0, 1\} \), \( j = 1, \ldots, N \). \( \rho_j = 0 \) and \( \rho_j = 1 \) stand for the sub-channel \( j \) is idle and busy state, respectively. And the probability of the sub-channel \( j \) in busy state is \( \eta_j \in [0, 1] \), \( j = 1, \ldots, N \), which is also called primary activity factor.

Due to the non-collaboration between the PL and SL, it is usually hard for the unlicensed user to gather full information of channel power gain \( g \) from the secondary transmitter to the primary receiver. So the SL usually gathers partial information of \( g \) instead, denoted by \( \hat{g} \). Since the channel is assumed to be block fading, \( g \) and \( \hat{g} \) can be rewritten with index as \( g(n) \) and \( \hat{g}(n) \), \( n = 1, 2, \ldots \), respectively. The SL estimates \( g(n) \) by the gathered \( \hat{g}(n) \) using the minimum mean square error (MMSE) method, i.e. \( \hat{g}(n) = E[g(n) | \hat{g}(n), \hat{g}(n-1), \ldots] \), where \( g(n) \) and \( \hat{g}(n) \) are uncorrelated. So the estimation error can be calculated by \( \tilde{g}(n) = g(n) - \hat{g}(n) \), and \( \tilde{g}(n) \) and \( \hat{g}(n) \) follow Rayleigh fading distribution with means \( 1 - \sigma^2_x \) and \( \sigma^2_x \), respectively.

III. H-OSA WITH PERFECT INTERFERENCE CSI

In this section, we study the H-OSA scheme. The SL selects sub-channels and allocates power in each block to achieve the maximum ergodic capacity, subject to the limitation that the average transmit power and interference power must be within some thresholds. The perfect information of instantaneous \( h_j \) and \( g_j \) are assumed to be available at the secondary transmitter.

Unlike the OSA scheme, the SL in the proposed scheme selects not only the idle sub-channels but also the busy sub-channels with under-interference-threshold guarantee. So the interference to the PL in each sub-channel is not valid all the time and the instantaneous interference to the PL in sub-channel \( j \) is \( \rho_j g_j P(h_j, g_j, \rho_j) \). Then, subject to the average transmit power constraint and interference power constraint, the problem of maximizing the secondary ergodic capacity can be formulated as follows:

\[
\max \sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ B_s \log \left(1 + \frac{h_j P_j(h_j, g_j, \rho_j)}{N_0 B_s}\right)\right],
\]

s.t.

\[
\sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ P_j(h_j, g_j, \rho_j)\right] \leq P_{\text{avg}},
\]

\[
\sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ \rho_j g_j P_j(h_j, g_j, \rho_j)\right] \leq I_{\text{avg}},
\]

\[
P_j(h_j, g_j, \rho_j) \geq 0, \quad \forall j,
\]

where \( P_{\text{avg}} \) and \( I_{\text{avg}} \) are the thresholds of maximal average transmit power and average interference power, respectively. \( \mathbb{E}_{h_j, g_j, \rho_j} [\cdot] \) denotes the joint expectation of \( h_j, g_j \) and \( \rho_j \).

Based on the dual decomposition method [11], we design an efficient algorithm to decompose the original problem into several subproblems. First, the constraints (2) and (3) are relaxed. And the Lagrangian function is

\[
N \sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ B_s \log \left(1 + \frac{h_j P_j(h_j, g_j, \rho_j)}{N_0 B_s}\right)\right] + \lambda \left(P_{\text{avg}} - \sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ P_j(h_j, g_j, \rho_j)\right]\right) + \mu \left(I_{\text{avg}} - \sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ \rho_j g_j P_j(h_j, g_j, \rho_j)\right]\right),
\]

where \( \lambda \) and \( \mu \) are nonnegative dual variables associated with constraints (2) and (3), respectively. Then, the dual function can be rewritten as

\[
D(\lambda, \mu) = \max \left\{ L \left( \{P_j(h_j, g_j, \rho_j)\}, \lambda, \mu \right) \right\},
\]

s.t. \( P_j(h_j, g_j, \rho_j) \geq 0, \quad \forall j.\)

The above maximum optimization problem can be decomposed into two layers of optimization. At the lower layer, the dual function separates into several sub-dual functions, each of which can be written as

\[
D_j(\lambda, \mu) = \max \mathbb{E}_{h_j, g_j, \rho_j} \left[ B_s \log \left(1 + \frac{h_j P_j(h_j, g_j, \rho_j)}{N_0 B_s}\right)\right] - \left(\lambda P_j(h_j, g_j, \rho_j) - \mu \rho_j g_j P_j(h_j, g_j, \rho_j)\right),
\]

s.t. \( P_j(h_j, g_j, \rho_j) \geq 0, \quad \forall j.\)
and at the higher layer, the dual problem is optimized subject to the dual variable $\lambda$ and $\mu$.

$$
\min \ D(\lambda, \mu) = \sum_{j=1}^{N} D_j(\lambda, \mu) + \lambda P_{avg} + \mu I_{avg}, \quad (10)
$$

subject to

$$
\lambda \geq 0, \ \mu \geq 0. \quad (11)
$$

Denote $r^*$ as the optimal value of the original problem defined in (1). Then, given $\lambda$ and $\mu$, the value of the dual function in (10) provides an upper bound on $r^*$, i.e. $r^* \leq D(\lambda, \mu)$. Denote the minimum of $\min D(\lambda, \mu)$ as $d^*$, which can be obtained under the optimal dual variables $\lambda^*$ and $\mu^*$, i.e. $d^* = D(\lambda^*, \mu^*)$. Then, the duality gap can be written as $r^* - d^*$, which is nonpositive. But due to the strict convexity of the proposed problem [8], the Slater’s condition [12] is satisfied and the duality gap is indeed zero, i.e. $r^* = d^*$. That is to say, we can solve the original problem equivalently by solving the dual problem.

Then, the dual problem can be solved using the subgradient method to update $\lambda$ and $\mu$ to their optimal values

$$
\lambda(n + 1) = \left[ \lambda(n) - \alpha(n) \left( P_{avg} - \sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ P_j^*(h_j, g_j, \rho_j) \right] \right) \right]^+, \quad (12)
$$

$$
\mu(n + 1) = \left[ \mu(n) - \beta(n) \left( I_{avg} - \sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ \rho_j g_j P_j^*(h_j, g_j, \rho_j) \right] \right) \right]^+, \quad (13)
$$

where $\alpha$ and $\beta$ are step sizes and $\{ P_j^*(h_j, g_j, \rho_j) \}$ is the optimal solution of transmit power. $[X]^+$ means $\max\{0, X\}$.

If the step sizes satisfy the following conditions, the subgradient method can guarantee that $\lambda$ and $\mu$ converge to the optimal dual solutions [12]

$$
\alpha(n) \to 0, \ \beta(n) \to 0, \ \text{as} \ t \to \infty,
$$

and

$$
\sum_{n} \alpha(n) = \infty, \ \sum_{n} \beta(n) = \infty.
$$

Otherwise, if the first condition isn’t satisfied, the $\lambda$ and $\mu$ will also coverage to a small neighborhood around the optimal dual solutions when the step sizes are small enough.

Finally, each of the optimal transmit power solutions, $P_j^*(h_j, g_j, \rho_j)$, can be obtained by the Karush-Kuhn-Tucker (KKT) [12] conditions in the optimization problem (8) [6].

$$
P_j^*(h_j, g_j, \rho_j) =
\begin{cases}
\frac{h_j K}{\lambda^* \rho_j g_j - N_0 B_s h_j} \geq \frac{N_0 B_s}{K} (\lambda + \mu \rho_j g_j), \\
0, \quad \text{otherwise},
\end{cases}
\quad (14)
$$

where $K = B_s / \ln 2$. Notice that this solution only gives us the optimal transmit power under the given $\lambda$ and $\mu$. And the optimal $P_j^*(h_j, g_j, \rho_j)$ will be obtained under the optimal dual variables $\lambda^*$ and $\mu^*$ iteratively.

From the expression of $P_j^*(h_j, g_j, \rho_j)$, we can find that, the busy sub-channels with good transmit channel gains $h$ and weak interference channel gains $g$ would be allocated more power than the idle sub-channels with weak transmit channel gains $h$. So the unlicensed user in the H-OSA scheme has more options to establish the link than the one in the OSA scheme. And consequently, the ergodic capacity of SL can be increased by using some busy sub-channels.

Then, the optimal achievable ergodic capacity of the SL can be obtained by substituting (14) into (1)

$$
C_{er} = \sum_{j=1}^{N} \mathbb{E}_{h_j, g_j, \rho_j} \left[ B_s \log \left( \frac{h_j K}{N_0 B_s (\lambda^* + \mu^* \rho_j g_j)} \right) \right]. \quad (15)
$$

As the distribution of $h_j, g_j$ and $\rho_j$ are independent, the above expectation over the joint probability density function (PDF) of $h_j, g_j$ and $\rho_j$ can be transformed into an expectation over joint variable $x_j = \frac{h_j}{\lambda^* + \mu^* \rho_j g_j}$. Then, the ergodic capacity can be rewritten as:

$$
C_{er} = \sum_{j=1}^{N} \int_{0}^{\infty} B_s \log \left( \frac{K x_j}{N_0 B_s} \right) f_X(x_j) dx_j, \quad (16)
$$

where

$$
f_X(x_j) = e^{-x_j} \left( \frac{\eta_j \lambda^* x_j}{(1 + \mu^* x_j)^2} \right) \left( \frac{\eta_j \mu^*}{(1 + \mu^* x_j)^2} \right) \frac{1}{x_j}, \quad (17)
$$

is the PDF of $x_j$, which is evaluated in Appendix A.

IV. H-OSA WITH IMPERFECT INTERFERENCE CSI

As the SL accesses the sub-channels without any license, the interference channels from secondary transmitter to primary receiver have no direct feedback mechanism to estimate the instantaneous channel power gain $g$. So the secondary transmitter usually gathers partial information of the interference channels and evaluates the estimation value $\hat{g}$. In this section, we study the optimal achievable ergodic capacity of the SL in the H-OSA scheme with imperfect interference CSI. This maximum optimization problem can be formulated as follows:

$$
\max \sum_{j=1}^{N} \mathbb{E}_{h_j, \hat{g}_j, \rho_j} \left[ B_s \log \left( 1 + \frac{h_j P_j(h_j, \hat{g}_j, \rho_j)}{N_0 B_s} \right) \right], \quad (18)
$$

subject to

$$
\sum_{j=1}^{N} \mathbb{E}_{h_j, \hat{g}_j, \rho_j} \left[ P_j(h_j, \hat{g}_j, \rho_j) \right] \leq P_{avg}, \quad (19)
$$

$$
\sum_{j=1}^{N} \mathbb{E}_{h_j, \hat{g}_j, g_j, \rho_j} \left[ \rho_j g_j P_j(h_j, \hat{g}_j, \rho_j) \right] \leq I_{avg}, \quad (20)
$$

$$
P_j(h_j, \hat{g}_j, \rho_j) \geq 0, \ \forall j. \quad (21)
$$

The interference power constraint (20) means that the actual interference caused by the secondary transmit power which is related to $h$ and $\hat{g}$ can not exceed the interference threshold. As the perfect information of $g$ is unavailable, the constraint
where \( \bar{P} \) denotes the average transmit power.

The formulation in this section has a similar expression to that in section III, except for the interference power constraint. In this formulation, the sum of average interference power and weighted average transmit power must be restricted under the interference threshold, while in the formulation in above section, only average interference power has this restriction. That is to say, the average interference power constraint is stricter in this formulation.

Then, the optimal power solution of this problem can also be obtained through the dual decomposition and KKT conditions.

\[
 P_j^*(h_j, \hat{g}_j, \rho_j) = \left[ \frac{K}{\lambda^* + \mu^* \rho_j (\hat{g}_j + \sigma^2_n)} - \frac{N_0 B_s}{h_j} \right]^+, \tag{23}
\]

where \( \lambda \) and \( \mu \) will converge to the optimal solutions \( \lambda^* \) and \( \mu^* \) by iteration functions similar to (12) and (13).

By substituting (23) into (18), the maximum ergodic capacity of SL with imperfect interference CSI can be expressed as:

\[
 C_{er}^{lm} = \sum_{j=1}^{N} \mathbb{E}_{h_j, \hat{g}_j, \rho_j} \left[ B_s \log \left( \frac{h_j K}{N_0 B_s (\lambda^* + \mu^* \rho_j (\hat{g}_j + \sigma^2_n))} \right) \right],
\]

which can be recalculated using a joint variable \( y_j = \frac{h_j}{\lambda^* + \mu^* \rho_j (\hat{g}_j + \sigma^2_n)} \) and its PDF \( f_{Y}(y_j) \)

\[
 C_{er}^{lm} = \sum_{j=1}^{N} \int_{0}^{\infty} B_s \log \left( \frac{K y_j}{N_0 B_s} \right) f_{Y}(y_j) dy_j, \tag{25}
\]

where the expression of \( f_{Y}(y_j) \) can be evaluated similarly in Appendix A for (17).

V. NUMERICAL RESULTS

In this section, we evaluate the proposed H-OSA scheme in some numerical results. The spectrum licensed to the PL is divided into 16 sub-channels and the channel power gains in all the sub-channels follow the independent rayleigh distribution with unit mean. The primary activity factor in each sub-channel is assumed to have the same value, i.e. \( \eta_j = \eta, \ j = 1, \ldots, N \). We adopt Monte-Carlo simulations to approximate the actual ergodic process of the channel power gains and the results are obtained by averaging over 10000 blocks. Both the bandwidth of sub-channel \( B_s \) and AWGN noise \( N_0 \) are assumed to be 1.

Figure 1 shows the comparison between the H-OSA scheme and OSA scheme in the normalized ergodic capacity versus the primary activity factor \( \eta \). From the figure we can see that the ergodic capacity in the H-OSA scheme indeed has an improvement compared to the OSA scheme, attribute to the opportunistic busy sub-channels access in the H-OSA scheme. It is worth pointing out that the improvement in ergodic capacity achieved by the H-OSA scheme increases as \( \eta \) increases. This is because more busy sub-channels are available and selected by the SL, and consequently, available sub-channels become less in the OSA scheme. But the ergodic capacities in the two schemes all decrease as \( \eta \) increases due to that more and more sub-channels occupies by the PL.

Figure 2 shows the variance of the normalized ergodic capacity versus different maximal average interference threshold \( I_{avg} \) for different \( \eta \). When \( I_{avg} \) is low, the ergodic capacity in different \( \eta \) (except for \( \eta \) being zero) has a small value due to...
that the power allocation limitation is based on $I_{avg}$. As $I_{avg}$ increases, the ergodic capacity increases and converges to the same value in $I_{avg}$ higher than 10 dB. Because the power allocation limitation is based on $P_{avg}$ in the high $I_{avg}$.

Figure 3 illustrates the impact of interference channel estimation variance $\sigma_x^2$ to the normalized ergodic capacity versus different $I_{avg}$. From the figure we can see that the ergodic capacity decreases as $\sigma_x^2$ increases. This is because $\sigma_x^2$ makes the interference power constraint much stricter. The ergodic capacities converge to the same value when $I_{avg}$ is high enough, since in the case of high $I_{avg}$, the power allocation is only limited by the transmit power constraint.

VI. CONCLUSION

In this paper, we study the ergodic capacity of SL which shares multiple sub-channels with the PL, considering the average transmit power constraint and average interference power constraint. In order to take advantage of the spectrum holes and the PL’s tolerance, a H-OSA scheme is proposed to allocate power not only on the idle sub-channels but also on the busy sub-channels based on the transmit and interference channel power gains and interference threshold. Numerical results show that the H-OSA scheme indeed improves the ergodic capacity of SL compared to the OSA scheme. We also analyse the secondary ergodic capacity in the H-OSA scheme with only partial information of interference channel power gains. Numerical results show that the estimation errors lead to stricter interference power constraint and result in lower ergodic capacity in the small interference power threshold region.

APPENDIX A

The channel power gains $h_j$ and $g_j$ are independent and identically distributed (i.i.d) variables following the rayleigh distribution with unit mean. So their joint PDF is:

$$f_{H,G}(h_j, g_j) = e^{-(h_j+g_j)}$$ (26)

Then, we set the joint variable $x_j = \frac{h_j}{\lambda^*+\mu^*\rho_j\sigma_j}$ to substitute $h_j$, $g_j$ and $\rho_j$. And the cumulative density function (CDF) of $x_j$ can be calculated as follows:

$$F_X(x_j) = Pr\{X \leq x_j\} = Pr\{\frac{H}{\lambda^* + \mu^*G} \leq x_j\}$$

$$= \eta_j Pr\{\frac{H}{\lambda^* + \mu^*G} \leq x_j\} + (1 - \eta_j)Pr\{\frac{H}{\lambda^*} \leq x_j\}$$

$$= \eta_j \int_{-\infty}^{x_j} \int_{-\infty}^{\infty} e^{-(h_j+g_j)} dh_j dg_j$$

$$+ \eta_j \int_{x_j}^{\infty} \int_{-\infty}^{\infty} e^{-(h_j+g_j)} dh_j dg_j$$

$$+ (1 - \eta_j)\int_{-\infty}^{\infty} e^{-h_j} dh_j.$$ (27)

Due to the nonnegativity of $h_j$ and $g_j$, $F_X(x_j)$ is:

$$F_X(x_j) = \eta_j \int_{0}^{x_j} e^{-g_j} \int_{0}^{\infty} e^{-(\lambda^*+\mu^*)g_j} e^{-h_j} dh_j dg_j$$

$$+ (1 - \eta_j)\int_{0}^{\infty} e^{-h_j} dh_j$$ (28)

$$= 1 - \eta_j e^{-\lambda^*x_j} - \frac{1}{1 + \mu^*x_j} - (1 - \eta_j)e^{-\lambda^*x_j},$$ (29)

and the PDF of $x_j$ in (17) can be obtained by performing the differential operation to $F_X(x_j)$.

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