# Seasonality in Outliers of Daily Stock Returns:

A Tail that Wags the Dog?

Dan Galai<sup>a</sup>

Haim Kedar-Levy<sup>b</sup>

Ben Z. Schreiber<sup>c</sup>

<sup>&</sup>lt;sup>a</sup> Abe Gray Professor of Banking and Finance at the School of Business Administration, Hebrew University, Jerusalem, Israel. <u>msgalai@mscc.huji.ac.il</u>. Partial financial support of the Zagagi Center is acknowledged. Corresponding author. T-(972) 2-5617234.

<sup>&</sup>lt;sup>b</sup> Assistant Professor of Finance at the School of Management, Ben Gurion University of the Negev, Beer Sheva, 84105, Israel. T-(972)8-6472569 <u>hlevy@som.bgu.ac.il</u> and Fox School of Business, Temple University Philadelphia, PA.

 <sup>&</sup>lt;sup>c</sup> Head of Research Unit, Department of Foreign Currency Activity, Bank of Israel Jerusalem, 91007, Israel. T-(972) 2-6552595. <u>schreibe@bankisrael.gov.il</u>

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### ABSTRACT

We document significant intra-year and less significant intra-week seasonality in outliers of S&P500 daily returns. Controlling for outliers in dummy regressions reveals that 1) Monday's mean returns turn from insignificantly to significantly positive and insignificantly higher than all weekdays; 2) January return doubles and turns significantly higher than all other months while June, August, and September turn out to be months with remarkably low rates of returns; 3) the recently documented Halloween effect turns significant only after controlling for outliers. Our findings indicate that while outliers cannot be instrumental variables, they severely affect empirical measures of seasonal anomalies documented in the literature.

<u>Key Words:</u> Outliers, Monday Effect, January Effect, Halloween Effect, Month-of-the-Year. JEL Classification: G10, G14

### 1. Introduction

Seasonality in periodic rates of return would defy market efficiency if investors can consistently implement profitable trading strategies. The January and Monday (or Day of the Week) effects attract much research attention over the past 25 years (see a thorough review of the latter in Schwert, 2003.) A new anomaly was recently reported and denoted "Sell in May and go Away," (or the "Halloween effect") (Bouman and Jacobsen, 2002) whereby average returns between May-October are significantly lower than returns between November-April. While these anomalies are about the mean rate of return, our note reports a new, significant seasonal in *outliers* of daily returns across Months-of-the-Year (MOY) and Days-of-the-Week (DOW). Apparently, this anomaly has an important impact on empirical estimates, i.e., ex-post measures of the January, Monday and Halloween effects. Its relevance for the ex-ante design of profitable investment strategies is questionable since outliers are random on one hand, but typically clustered on the other hand.

Our data-set is daily rates of return on the S&P500 index over the period 1/1980-8/2002, out of which 2.03% are classified as outliers based on the Huber *M*-Estimator (described below.) We analyze outliers across MOY and DOW, where the null hypothesis is that outliers are uniformly distributed across each. Chi-square test for this null across MOY is rejected (*p*-value=0.00), but cannot be rejected for DOW (*p*=0.71) though it strongly affects the Monday anomaly as described below.

Our findings suggest that there may be a relationship between the outlier seasonality and the January, Monday and Halloween effects. January effect (Keim 1983, Reinganum 1983, Blume and Stambaugh 1983, and Booth and Keim 2000) refers to the finding whereby the mean rate of return in January tends to be significantly higher than that of any other month of the year,

predominantly in small firms. Tax considerations are often cited as the underlying rationale. We find that once outliers are controlled for, the mean daily return in January increases from less than twice the year-long average (0.070% vs. 0.037%) to almost three times the average (0.141% vs. 0.050%). With respect to the Monday effect, French (1980) showed that Monday's average return (Friday to Monday closing prices) is negative, contrary to one of two hypotheses: that returns are generated over calendar time or through trade. These findings were replicated over numerous national markets. Based on our sample, we report that Monday's mean return turns *significantly positive* and not different from weekdays only after controlling for outliers, as the trading-day hypothesis suggests. We also find that negative outliers tend to concentrate before and after the weekend (Friday and Monday) whereas positive outliers tend to concentrate in mid-week days. Finally, the Halloween effect is highly significant in daily return data only after controlling outliers by regression dummies, and that it spans over the period May-September, rather than May-October.

#### 2. Outliers: Definition and Descriptive Statistics

### 2.1. A Definition of Outliers

What turns an observation into an outlier? In this paper, we use *M*-Estimators to distinguish between outliers and the body of the distribution. *M*-Estimators are iterative procedures designed to find a robust estimate for the first moment (location) and can be extended to estimate the second moment (scale.) These procedures generalize a Maximum Likelihood location estimate for a given distribution thus more robust as the sample increases. Out of the several existing robust estimation procedures, the *Huber M-Estimator* appears to be suitable for the analysis of financial data. Most other *M*-Estimators entirely eliminate outliers *and* modify some non-extreme observations, making the body of the distribution non informative for the purpose of rates of

return analysis. The Huber *M*-Estimator discerns positive and negative outliers without distorting the body of the distribution, thus chosen for our research. A formal description of the Huber *M*-Estimator procedure is presented in Hoaglin, Mosteller and Tukey (1983.) We used a Huber standardized k-value of 2.496 in order to account for the large sample.

### 2.2. Summary Statistics

Our sample comprises 5,364 observations of the S&P500 index over the period January 3, 1980 to August 21, 2002. Returns on non-consecutive trading days, other than weekend returns, are excluded, as they do not represent daily returns. The Huber M-Estimator procedure revealed 109 outliers for the entire test period, 48 positive and 61 negative, constituting 2.03% of the sample. The lowest remaining observation in the sample is -2.57% and the highest remaining observation is 2.67%. The cutoff for positive and negative outliers need not be symmetric. The mean positive outlier is 3.35% and the mean negative outlier is -3.62%. After controlling for outliers, the mean, median and standard deviations have changed significantly in several months, and not at all in other months, as illustrated in Table 1.

Mean daily return of the entire sample increased from 0.037% to 0.050% because of outliers trimming while the median changed slightly from 0.043% to 0.045%. Mean daily returns in January, March, August and October more than doubled while those of April, May and September declined. The differential effect is due to count and magnitude of outliers.

# Table 1<sup>\*</sup>

		S&P	Trimmed			S&P	Trimmed
	Statistic	Return	Return		Statistic	Return	Return
January	Mean	0.070	0.141	July	Mean	0.021	0.034
	Median	0.085	0.100		Median	0.100	0.110
	Std. Deviation	1.029	0.839		Std. Deviation	0.967	0.832
	N	438	425		Ν	444	437
February	Mean	0.030	0.038	August	Mean	0.005	0.012
	Median	0.032	0.033		Median	0.034	0.034
	Std. Deviation	0.918	0.906		Std. Deviation	1.072	0.840
	N	398	397		N	501	483
March	Mean	0.035	0.070	September	Mean	(0.040)	(0.057)
	Median	0.007	0.026		Median	(0.023)	(0.023)
	Std. Deviation	0.975	0.882		Std. Deviation	0.996	0.827
	N	499	491		N	402	390
April	Mean	0.058	0.051	October	Mean	0.038	0.094
	Median	0.095	0.090		Median	0.024	0.026
	Std. Deviation	1.038	0.869		Std. Deviation	1.704	0.894
	N	433			N	487	462
May	Mean	0.042	0.028	November	Mean	0.090	0.090
	Median	0.060	0.058		Median	0.159	0.159
	Std. Deviation	0.863	0.837		Std. Deviation	0.954	0.877
	N	450			N	409	
June	Mean	0.023	0.017	December	Mean	0.077	0.078
	Median	0.008	0.006		Median	0.025	0.025
	Std. Deviation	0.834	0.825		Std. Deviation	0.921	0.846
	N	483	482		N	420	414
Total	Mean	0.037	0.050				
	Median	0.043	0.045				
	Std. Deviation	1.052	0.857				
	N	5364	5255				

## **Daily Rates of Return Statistics across MOY**

\* Trimmed return refers to the daily rate of return excluding all outliers.

In Table 2 we show the impact of outliers on daily returns by days of the week. The mean rate of return on Monday increased from 0.011% to 0.069%. Some 0.022% of it can be attributed to the October 1987 crash. Friday's return is also more than doubled, whereas Tuesday's and Wednesday's decreased. As expected, standard deviations of daily returns across both months and weekdays declined because of the trimming, as depicted in Tables 1 and 2. The most dramatic declines were in October (48%) and Monday (32%).

# Table 2<sup>\*</sup>

		S&P	Trimmed
	Statistic	Return	Return
Monday	Mean	0.011	0.069
	Median	0.074	0.088
	Std. Deviation	1.296	0.882
	Ν	1019	993
Tuesday	Mean	0.055	0.041
	Median	-	(0.001)
	Std. Deviation	0.981	0.851
	Ν	1041	1018
Wednesday	Mean	0.084	0.065
	Median	0.078	0.075
	Std. Deviation	0.963	0.830
	Ν	1076	1059
Thursday	Mean	0.014	0.019
	Median	0.008	0.011
	Std. Deviation	0.985	0.848
	Ν	1129	1106
Friday	Mean	0.023	0.058
	Median	0.048	0.054
	Std. Deviation	1.014	0.874
	Ν	1099	1079
Total	Mean	0.037	0.050
	Median	0.043	0.045
	Std. Deviation	1.052	0.857
	Ν	5364	5255

# Daily Rates of Return Statistics across DOW

\* "Trimmed return" refers to the daily rate of return excluding all outliers.

# 3. Analysis of Outliers' Seasonality

## 3.1. The Distribution of Outliers across MOY and DOW

Table 3 reports the number of outliers as distributed across MOY and DOW in two subperiods (1/1980-12/1989, 1/1990-8/2002) and over the entire period. Eye-balling reveals the nonuniformity of outliers across MOY and the less pronounced differences across DOW, though negative outliers tend to cluster before and after the weekend. In addition, the number of outliers during the 1990's is larger than in the 1980's even after controlling for period length – possibly a support for the claim that return variability increased over the 1990's.

# Table 3<sup>\*</sup>

3.A	1980's Only			1990's-8/02 Only			Entire Sample		
	Pos.	Neg.	Total	Pos.	Neg.	Total	Pos.	Neg.	Total
Jan	1	4	5	1	7	8	2	11	13
Feb					1	1		1	1
Mar		2	2	1	5	6	1	7	8
Apr	1	1	2	5	3	8	6	4	10
May				2		2	2		2
Jun				1		1	1		1
Jul				2	5	7	2	5	7
Aug	4	1	5	5	8	13	9	9	18
Sep	1	1	2	6	4	10	7	5	12
Oct	6	8	14	6	5	11	12	13	25
Nov	3	2	5		1	1	3	3	6
Dec	2	2	4	1	1	2	3	3	6
Total	18	21	39	30	40	70	48	61	109
3.B	1980's Only			1990's-8/02 Only			Entire Sample		
	Pos.	Neg.	Total	Pos.	Neg.	Total	Pos.	Neg.	Total
Monday	3	9	12	6	8	14	9	17	26
Tuesday	6	2	8	7	8	15	13	10	23
Wednesday	3	3	6	7	4	11	10	7	17
Thursday	3	4	7	7	9	16	10	13	23
Friday	3	3	6	3	11	14	6	14	20
Sum	18	21	39	30	40	70	48	61	109

## Number of Outliers across MOY and DOW

\* Outliers were calculated by Huber *M*-Estimator procedure.

There is a record of 25 outliers in October, 13 negative and 12 positive. Six of the negative outliers were between October 6<sup>th</sup> to 26<sup>th</sup>, 1987 and 4 positive outliers following the crash, between October 20<sup>th</sup> to 30<sup>th</sup>. On the contrary, February, May and June have only 1-2 outliers over a 23-year period. These findings must be considered together with the fact that under the null hypothesis of uniform distribution we expect an average of  $109/12 \cong 9.1$  observations per month. On a weekly basis, the largest number of outliers, 26, were on Mondays out of which 17

negative and 9 positive. Tuesday and Thursday follow with 23 outliers each. Wednesday is the day with the least number of outliers, 17, out of which 10 are positive and 7 negative. Monday had the highest number of negative outliers in the 1980's, but this uniqueness had faded during the 1990's.

### 3.2. Chi-Square Tests

In order to validate the statistical significance of outlier distribution across MOY and DOW we conduct Chi-Square tests on the number of outliers. We perform a separate analysis for *Positive, Negative,* and *Both* outliers for each sub-period and the entire sample. The null hypothesis is that outliers are uniformly distributed over the different days of the week and months of the year. Thus, we expected 9.1 observations per month and 21.8 per day. Table 4 summarizes the *p*-Values obtained from these tests.

### Table 4

		P-Values		
A. MOY	Positive	Negative	Both	
1/1980-8/2002	0.00	0.00	0.00	
1/1980-12/1989	0.00	0.00	0.00	
1/1990-8/2002	0.01	0.01	0.00	
B. DOW				
1/1980-8/2002	0.62	0.31	0.71	
1/1980-12/1989	0.74	0.12	0.53	
1/1990-8/2002	0.74	0.52	0.91	

#### **Chi-Square Test for Outliers Distribution by MOY or DOW\***

\* Under the null, outliers should be uniformly distributed. P<5% imply rejection of the null.

Panel A analyzes the MOY effect and Panel B the DOW effect. The null hypothesis is rejected at 5% confidence level in all outlier classifications in Panel A. The null is not rejected in

Panel B. In order to lessen potential data mining effects we tested various Huber's *M*-Estimator cut-off values and several sub-periods and report that the Chi-Square results appear to be robust.

## 3.3. Volatility Clustering

Daily returns exhibit alternating episodes of high and low volatilities, which imply that outliers will cluster in periods of high volatility. If outliers cluster arbitrarily across months of the year or days of the week, our results might be subject to data mining biases. In order to explore this possibility, we assigned the value of 1.0 to any month that had at least one outlier, and zero otherwise. We then segmented our dataset to three sub-periods, 1980-1986, 1987-1994 and 1995-2002. A plot of the results is presented in Figure 1. There was a total of 55 months with at least one outlier over the sample period, 14 between 1980-1986, 13 between 1987-1994 and 28 between 1995-2002. As the figure indicates, the last period is more volatile and the summer months May, June, July, as well as February, consistently had the lowest number of months with outliers in any sub-period.

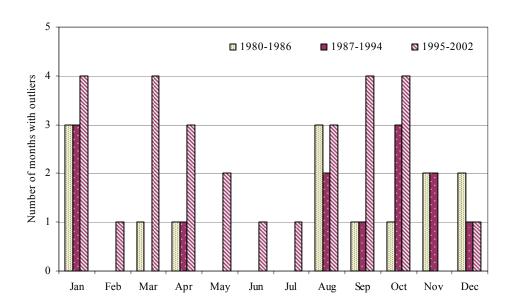


Figure 1 Number of Months with at least one Outlier, by sub-Periods

Although the number of observations is too small to conduct a reliable Chi-square test, the pattern appears to support our prior findings of the non-uniform distribution of outliers in spite of potential volatility clustering effects.

### 4. Correspondence with other Anomalies

In this section, we conduct dummy regression tests aimed to replicate the standard seasonal tests, with and without outliers in the sample. We do so by assigning dummy values to positive and negative outliers as if they were instrumental variables. However, since outliers arrive randomly, an investor cannot condition on them and therefore cannot design such an ex-ante trading rule. Consequently, the economic meaning of regressions with outliers as dummy variables cannot be taken to imply that a trading rule can be designed, but rather to measure the impact of outliers on the coefficients of the original model. By doing so, we actually measure the hypothesized anomaly (DOW, MOY or Halloween effect) on the body of the distribution, controlling for the effect of outliers on the means and variances. Nevertheless, volatility clustering might still be used to design a trading rule that will facilitate avoidance of at least some outliers. If such trading rule can be designed, our regressions with dummy variables of outliers may still bear some ex-ante economic content. We have tested simple trading rules based on the arrival of the first outlier, but found none with economic significance, particularly after the inclusion of transaction costs.

### 4.1. January and Halloween Effects

Following the anomalies literature, we estimate the classic dummy regression

$$R_i = \alpha + \sum_i \beta_i D_{i,t} + \delta_t$$
  $i = 2,3,...,12$  (1)

where  $D_{i,t}$  is a dummy that receives 1 in month *i* and 0 otherwise. In this model  $\alpha$  estimates the average return on January and the  $\beta_i$  coefficients estimate the average difference between mean return on the *i*-th month and the mean return on January. The estimated regression when outliers are included in the data-set is

$$R_{i} = 0.07 - 0.041D_{2} - 0.036D_{3} - 0.012D_{4} - 0.028D_{5} - 0.048D_{6} - 0.049D_{7} - 0.065D_{8} - 0.110D_{9} - 0.032D_{10} + 0.020D_{11} + 0.007D_{12},$$
(1a)

with F=0.462 which is insignificant (p=0.927.) As *t*-values (under the coefficients) indicate, none of the coefficient is significant. However, estimating the model with outlier dummies  $D_{0+}$  and  $D_{0-}$  for positive and negative outliers, respectively, i.e.,

$$R_{t} = \alpha + \sum_{i} \beta_{i} D_{i,t} + \beta_{O+} D_{O+,t} + \beta_{O-} D_{O-,t} + \delta_{t}$$
(2)

yields

$$R_{i} = 0.141 - 0.103D_{2} - 0.071D_{3} - 0.090D_{4} - 0.113D_{5} - 0.124D_{6} - 0.107D_{7} - 0.129D_{8} - 0.197D_{9} - 0.047D_{10} - 0.050D_{11} - 0.063D_{12} + 3.480D_{0+} - 4.019D_{0-}$$
(2a)

This regression is highly significant (p=0.00) mainly, due to the outliers' dummies. January is now significantly positive and all other months have lower means, thus the January effect is revealed from the body of the return distribution. June, August, and September are significantly below January at 5% confidence level, and May and July at 10%. These significant differences imply that the Halloween effect can also be discerned in daily return data by controlling for outliers. Regression (2a) further shows that the effect spans May through September, and not to October, though Bouman and Jacobsen (2002) report that their results do not change whether the ending month is September or October. A direct test for the Halloween effect before and after accounting for outliers is conducted by models (3) and (4), respectively

$$R_i = \alpha + \beta_H D_{H,t} + \delta_t \tag{3}$$

$$R_{i} = \alpha + \beta_{H} D_{H,t} + \beta_{O+} D_{O+,t} + \beta_{O-} D_{O-,t} + \delta_{t}$$
(4)

where  $D_{H,t}$  receives 1 between May to September and 0 otherwise. The results are

$$R_i = \underbrace{0.056}_{2.97} - \underbrace{0.045D}_{-1.54}$$
(5)

with a regression p-value=0.12. i.e., though mean returns between October-April are significantly positive, summer months are insignificantly below them. Controlling for outliers we find than model (4) yields

$$R_{i} = 0.081 - 0.072D_{H} + 3.540D_{O+} - 3.960D_{O-}$$
(6)

with *p*-value=0.00. The control of outliers revealed a higher mean value for the winter months (0.025=0.081-0.056) and a lower mean value for the summer months (0.009 vs. 0.011 with outliers), increasing the difference between the two periods, and turning it highly significant.

#### 4.2. DOW Effect

In order to assess the effect of outliers on the DOW effect we estimate the trading-day hypothesis using French (1980) regression model with and without outliers' dummies. For the standard test, dummy variables  $D_{j,t}$  are 1 if returns  $R_t$  occur on the *j*th day, and 0 otherwise:

$$R_t = \gamma + \sum_j \delta_j D_{j,t} + \varepsilon_t \qquad j = 3,...,6,$$
(7)

where  $\gamma$  captures Monday's return and the coefficients  $\delta_j$  measure the difference between the return on day *j* and Monday's return. The model with outliers' dummies is

$$R_{t} = \gamma + \sum_{j} \delta_{j} D_{j,t} + \beta_{O+} D_{O+,t} + \beta_{O-} D_{O-,t} + \varepsilon_{t} \qquad j = 3,...,6.$$
(8)

The result of model (7) is

$$R_{j} = \underbrace{0.011}_{0.34} + \underbrace{0.044D_{3}}_{0.94} + \underbrace{0.073D_{4}}_{1.59} + \underbrace{0.003D_{5}}_{0.06} + \underbrace{0.012D_{6}}_{0.25}, \qquad (9)$$

with p=0.42. The results in (9) support the common knowledge about the DOW anomaly: Monday's return is insignificantly above zero, and the other daily returns are insignificantly higher than Monday's mean return. However, when testing model (8) we obtain

$$R_{j} = \underbrace{0.069}_{2.41} - \underbrace{0.028D_{3}}_{-0.71} - \underbrace{0.004D_{4}}_{-0.11} - \underbrace{0.050D_{5}}_{-1.25} - \underbrace{0.012D_{6}}_{-0.29} + \underbrace{3.551D_{O+}}_{26.57} - \underbrace{3.948D_{O-}}_{-33.09} - \underbrace{(10)}_{-33.09}$$

Now Monday's return is significantly positive and insignificantly *above* all other days of the week. These results support the trading-day hypothesis.

### 5. Conclusions

The main message of this note is that a few outliers of daily returns (109, 2.03% of the sample) severely affect the empirical estimation of the January, Day-of-the-Week and Halloween effects in daily data over the period 1/1980-8/2002. We find that January's and Monday's mean returns turn from insignificantly to significantly positive after controlling for outliers, and both are above the rest of the months and rest of the days, respectively. The Halloween effect, whereby mean returns between May-September are lower than returns between October-April was insignificant when outliers are included in the sample, but turns significant after controlling for outliers. This study should not be interpreted to imply that outliers could be used as control variables for designing an ex-ante trading strategy that is aimed to generate excess returns, but to report the severe effect a few outliers have on empirical estimation of seasonal anomalies. Further research is needed in order to establish whether ex-ante trading strategies can be constructed to exploit the seasonality in outliers revealed in this note.

### References

Blume, M. E., and R. F., Stambaugh, 1983. Biases in Computed Returns: An Application to the Size Effect. *Journal of Financial Economics*, 387-404.

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Booth, D. G. and D. B. Keim, 2000. Is There Still a January Effect? in: D. B. Keim and W. T.
```

Ziemba (eds.). Security Market Imperfections in Worldwide Equity Markets. Cambridge: Cambridge University Press, pp. 169-178.

Bouman, S. and B. Jacobsen, 2002. The Halloween Indicator, 'Sell in May and Go Away': Another Puzzle. *American Economic Review*, 92, 1618-1630.

French, K. R., 1980. Stock Returns and the Weekend Effect. *Journal of Financial Economics* 8, 55-70.

Hoaglin, D.C., F. Mosteller and J. W. Tukey, eds. 1983. Understanding Robust and Exploratory Data Analysis. Wiley, New York.

Keim, D. B., 1983. Size Related Anomalies and Stock Return Seasonality: Further Empirical Evidence. *Journal of Financial Economics* 12, 13-32.

Reinganum, M., R., 1983. The Anomalous Stock Market Behavior of Small Firms in January: Empirical Tests for Tax-Loss Effects. *Journal of Financial Economics* 12 89-104.

Schwert, W. G., 2003. Anomalies and Market Efficiency. **Handbook of the Economics of Finance**, eds. George Constantinides, Milton Harris, and René Stulz, Elsevier Science, pp. 937-972.