Preface to special issue on high-dimensional dependence and copulas

1. Background

One of the biggest advances in recent years for high-dimensional copula models and applications has been the development of the vine pair-copula construction that covers continuous and discrete variables, and its extensions to include latent variables. Software has been made available in the VineCopula R package and the package that is companion to the book by Joe [6].

This special issue of the Journal of Multivariate Analysis is based on presentations by leading copula researchers at the International Workshop on High-Dimensional Dependence and Copulas: Theory, Modeling, and Applications, held at Central University of Finance & Economics (CUFE), Beijing China during January 3–5, 2014. Earlier vine copula workshops were held in Delft (Netherlands), Oslo (Norway) and Munich (Germany) in the years 2007, 2008, 2009 and 2011.

Simple parametric multivariate copula families do not have flexible dependence but there are plenty of parametric bivariate copula families that can cover a range of dependence (from countermonotonicity or independence to comonotonicity) and a range of tail behavior for the joint lower and upper tails. The vine pair-copula construction is flexible because it is built from a tree sequence of bivariate copulas, with these bivariate copulas applied to conditional distribution after the first tree (details are given in several of the articles in this issue). There is a graphical representation of vines as a sequence of trees where edges of trees indicate pairing of variables. The pair-copula construction assigns (algebraically independent) bivariate copulas to the edges of the trees. When each bivariate copula is Gaussian, the corresponding parameters are correlation parameters for the first tree and they are interpreted as partial correlation parameters for trees 2 and higher. Hence vines also provide another parametrization for the Gaussian correlation matrix. The partial correlation parametrization is exploited in order to get the copula extension of Gaussian-based factor models and structural equation models.

Applications of copula models to joint risks involve tail inference where there is more probability in joint tails than would be implied by multivariate Gaussian. Hence tail properties of copulas and tail inference based on copulas have developed along with construction of copula models. Other multivariate dependence concepts have also been studied with motivation from risk analysis in finance and insurance.

The word ‘vine’ for the structure was coined by Roger Cooke, and he also developed one of its graphical representations. Early research on vines was developed by Cooke, Bedford and Kurowicka at the Technische Universitêt Delft [2,3,7]. Statistical applications have been growing since Aas et al. [1] and the statistical computing for vines has simplified with the VineCopula R package, which has been developing at the Technische Universität München since 2010.

2. Contents of this issue

The 12 articles in this special issue are ordered by topic and cover some of the latest research developments in (a) algorithms, inference and applications of vine copulas; (b) tail properties of copulas and (c) multivariate dependence concepts relevant to finance and risk analysis.

The first paper in this issue is by Cooke, Kurowicka and Wilson. This paper has a brief history of the development of vines and their applications. It provides another view of sampling from vine copulas based on different orderings of variables; this is important for generating variables sequentially in a possibly different order than the variable order used in the creation of the vine.

The second paper is by Brechmann and Joe. Fit indices, that appear in the psychometrics literature, are applied to the correlation matrix of normal scores, and a non-greedy algorithm for deciding on the truncation level of parsimonious truncated vine dependence structures is studied. The decision of assigning bivariate copulas to edges of the vine is done in a second stage. The algorithm is shown to work well compared with the first sequential algorithms in [5,4].

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The third paper is by Schepsmeier. It compares different approaches to obtain goodness-of-fit statistics for vine copula models. Development of useful goodness-of-fit statistics is not easy because the null distributions are not simple and sparsity in high dimensions can weaken the power of some approaches. Some of the best methods found to date are based on the information matrix ratio.

The fourth paper by Krupskii and Joe is on structured factor copula models that are useful when variables can be divided into non-overlapping groups and dependence can be explained by latent variables of which some may be linked to specific groups. These models include the copula counterparts of Gaussian parsimonious dependence structures used for high-dimensional data. The copula densities involve two-dimensional integrals, and included are details of how to efficiently carry out likelihood inference with these models.

The fifth paper by Erhardt, Czado and Schepsmeier is on the use of vine copulas for spatial data with covariates when the common assumption of multivariate Gaussianity is inappropriate. In order to get locally-based predictions and avoid a high-dimensional density, a composite likelihood is constructed from local C-vine densities for subsets of locations that are spatially close; an isotropic assumption is not required.

The sixth paper by Jaworski studies the tail behavior of conditional joint distributions given that one variable is extreme. There are motivations from some risk scenarios in finance. Some interpretable results are obtained in the special class of vine copulas where there is a selected variable at the root of the vine.

The seventh paper by Bernard and Czado studies how tail properties of copulas can affect conditional quantiles. It is shown, in general, that linear conditional quantile estimation can be too strong an assumption, and one can use copulas to get nonlinear conditional quantile functions. The behavior of conditional quantile functions in the tails of the conditioning variables is related to the strength of dependence in joint tails.

The eighth paper is by Kurowicka and van Horssen. An interaction function for copulas is studied. Some characterizations are given for copulas where the interaction function has some special structure. Some examples show that the interaction function might be useful for assessing strengths of dependence in joint tails.

The ninth paper is by Li and Hua. It has results for determining the strength of dependence of multivariate distributions when the joint density has a simple form and the joint cumulative distribution function might be more complex in form. The main theorems have results on the form of the tail density function when univariate marginal densities have tails that match the Fréchet, Gumbel or Weibull cases. Links are also made with hidden regular variation.

The tenth paper is by Cai and Wei. Notions of multivariate dependence based on the concept of weakly stochastic arrangement increasing are studied, with applications to dependent risks. Characterizations and links to existing dependence structures are given, and it is shown how the concepts are interpreted for multivariate normal distributions and for Archimedean copulas.

The eleventh paper is by Mao and Wang. New results are obtained on the set of distributions of sum of random variables with given marginal distributions that have finite means. A concept of convex order is key to derivation of the results. The set is relevant for understanding acceptable risks with marginal constraints in quantitative risk management.

The twelfth paper is by Grothe and Hofert. It covers Lévy copulas which can have a nested Archimedean form. Lévy copulas have domain consisting of a Euclidean product space and are not copulas (having domain of a hypercube); and comparisons are made with between nested Archimedean for the two different domains. Mention is made for financial applications of Lévy copulas, and why sampling algorithms are important.

We summarize with some common themes in these articles.

1. Vines are prominently featured in the first six papers. Novel models for high-dimensional data are shown in papers 4 and 5 in that (i) factor copulas can handle dimensions exceeding 100 and they have the property of closure under margins, and (ii) the local specification for non-Gaussian spatial dependence can accommodate covariates and avoid the need to model non-local spatial dependence.
2. Assessment of adequacy of fit is done through formal test statistics in paper 3, but also by diagnostic methods in papers 2 and 4 with comparison of empirical and model-based measures of dependence, possibly tail-weighted.
3. Financial risk applications are covered as examples or motivation in papers 1–4, 6–7, 10–12. But the theory in all of the papers are applicable more generally.
4. Dependence concepts helpful for determining tail properties of multivariate models are covered in papers 6–9.

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