Abstract—The problem of multiuser detection in multipath CDMA channels with receiver antenna array is considered. The optimal space-time multiuser receiver structure is first derived, followed by linear space-time multiuser detection methods based on iterative interference cancellation. Blind adaptive space-time multiuser detection techniques are then proposed, which require prior knowledge of only the spreading waveform and the timing of the desired user’s signal. Single-user-based space-time processing methods are also considered and are compared with the multiuser approach. It is seen that the proposed multiuser space-time processing techniques offer substantial performance gains over the single-user-based methods, especially in a near-far situation.

Index Terms—Antenna array, blind adaptive detection, CDMA, multipath, space-time multiuser detection.

I. INTRODUCTION

Wireless communications for mobile telephone and data transmission is currently undergoing very rapid development. Many of the emerging wireless systems will incorporate considerable signal-processing intelligence in order to provide advanced services such as multimedia transmission. In order to make optimal use of available bandwidth and to provide maximal flexibility, many wireless systems operate as multiple-access systems, in which channel bandwidth is shared by many users on a random-access basis. One type of multiple-access technique that has become very popular in recent years is direct-sequence code-division multiple-access (DS-CDMA).

In DS-CDMA communications, the users are multiplexed by distinct code waveforms, rather than by orthogonal frequency bands, as in frequency-division multiple-access (FDMA), or by orthogonal time slots, as in time-division multiple-access (TDMA). Two major factors that limit the performance of DS-CDMA systems are multiple-access interference (MAI) and multipath channel distortion. Many advanced signal processing techniques have been proposed to combat interference and multipath channel distortion, and these techniques fall largely into two categories: multiuser detection [25] and space-time processing [17].

Multiuser detection techniques exploit the underlying structure induced by the spreading waveforms of the DS-CDMA user signals for interference suppression. Various linear and nonlinear multiuser detectors have been developed over the past decade [25]. It has been well established that multiuser detection techniques can substantially enhance the receiver performance and increase the capacity of CDMA communication systems. More recently, adaptive multiuser detection has received considerable attention [11]. Adaptivity not only allows multiuser detection to be applied without additional protocol overhead with respect to that required by conventional methods, but it also allows multiuser detectors to operate in the dynamic environments found in mobile communications applications.

Another approach to interference suppression in wireless systems is through space-time processing using an antenna array at the receiver. In this approach, the signal structure induced by multiple receiving antennas, i.e., spatial signature, is exploited for interference suppression. A number of adaptive array techniques have been developed for enhancing the performance of CDMA systems, e.g., [1], [3], [6], [13], [16], [22], and [26]. In these methods, the desired user’s signals from different paths and antennas are first combined in some optimal manner. The combined signal is then correlated with the spreading waveform of the desired user to yield decision statistic. Hence, these are essentially single-user-based methods as they do not exploit the inherent DS-CDMA signal structure.

Combined multiuser detection and array processing has also been addressed recently [12]. In [15], a spatially and temporally noise-whitening receiver structure is developed. In [4] and [5], the EM algorithms are developed for joint estimation of multiuser channel parameters and data symbols. However, the high computational complexities of these methods make them applicable to systems with only a few users. In [22], a single-user-based blind adaptive array method is proposed for CDMA systems. That work is extended in [8] and [21] to incorporate multiuser detection techniques such as decision feedback and successive interference cancellation. The interference suppression capabilities of these methods are still limited by that of the front-end single-user adaptive array.

The purpose of this paper is to provide a comprehensive treatment of space-time multiuser detection in multipath CDMA channels with receiver antenna arrays. We derive several space-time multiuser detection structures, including the maximum likelihood multiuser sequence detector, low-complexity linear space-time multiuser detectors based on
iterative interference cancellation, and blind adaptive space-
time multiuser detectors. We also consider several single-
user space-time processing methods appearing in the recent
literature and compare them with the multiuser approach. It
is seen that the proposed nonadaptive and adaptive multiuser
space-time detection techniques offer significant performance
enhancement over their single-user-based counterparts.

This paper is organized as follows. In Section II, the sig-
 nal model is presented. In Section III, the maximum like-
lihood space-time multiuser sequence detector is derived.
In Section IV, low-complexity linear space-time multiuser
detection techniques, which are based on iterative interfer-
ence cancellation, are developed. In Section V, blind adaptive
space-time multiuser detectors are developed. Simulation ex-
amples are also presented in Sections IV and V to illustrate
the performance of various space-time multiuser detectors
developed in this paper. Section VI contains the conclusion.

II. SIGNAL MODEL

Consider a DS/CDMA mobile radio network with \( K \) users,
employing normalized spreading waveforms \( s_1, s_2, \ldots, s_K \),
and transmitting sequences of binary phase-shift keying
(BPSK) symbols through their respective multipath channels.
The transmitted baseband signal due to the \( k \)th user is given by

\[
x_k(t) = A_k \sum_{i=0}^{M-1} b_k(i) s_k(t - iT), \quad k = 1, \ldots, K
\]

(1)

where \( M \) is the number of data symbols per user per frame; \( T \)
is the symbol interval; \( b_k(i) \in \{+1, -1\} \) is the \( i \)th transmitted
symbol by the \( k \)th user; and \( A_k \) and \( s_k(t) \) denote, respectively,
the amplitude and normalized signaling waveform of the
\( k \)th user. It is assumed that \( s_k(t) \) is supported only on the interval
\([0, T]\) and has unit energy. It is also assumed that each user
transmits independent equiprobable symbols and the symbol
sequences from different users are independent. In the direct-
sequence spread-spectrum multiple-access format, the user
signaling waveforms are of the form

\[
s_k(t) = \sum_{j=0}^{N-1} c_k(j) \psi(t - jT_c), \quad 0 \leq t \leq T
\]

(2)

where \( N \) is the processing gain; \( \{c_k(j)\}_{j=0}^{N-1} \) is a signature
sequence of \( \{\pm 1\} \)s assigned to the \( k \)th user, and \( \psi \) is a
normalized chip waveform of duration \( T_c = T/N \).

At the receiver, an antenna array of \( P \) elements is employed.
Assuming that each transmitter is equipped with a single
antenna, then the baseband multipath channel between the
\( k \)th user’s transmitter and the base station receiver can be modeled
as a single-input multiple-output channel with the following
vector impulse response:

\[
\begin{align*}
    \mathbf{b}_k(t) &= \sum_{l=1}^{L} a_{kl} g_{kl} \delta(t - \tau_{kl}) \\
    &= \sum_{i=0}^{K-1} \sum_{k=1}^{L} a_{kl} g_{kl} \delta(t - \tau_{kl}) + \sigma_u(t)
\end{align*}
\]

(3)

is the array response vector corresponding to the \( l \)th path of
the \( k \)th user’s signal. The total received signal at the receiver
is then the superposition of the signals from the \( K \) users plus
the additive ambient noise given by

\[
x(t) = \sum_{k=1}^{K} x_k(t) * \mathbf{b}_k(t) + \sigma_u(t)
\]

\[
= \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) \sum_{l=1}^{L} a_{kl} g_{kl} s_k(t - iT - \tau_{kl}) + \sigma_u(t)
\]

(4)

where \(*\) denotes convolution; \( \sigma_u(t) = [\sigma_{u1}(t), \ldots, \sigma_{uP}(t)]^T \)
is a vector of independent zero-mean complex white Gaussian
noise processes, each with unit variance; and \( \sigma_u^2 \) is the variance
of the ambient noise at each antenna element.

III. SUFFICIENT STATISTICS AND OPTIMAL
SPACE-TIME MULTIUSER DETECTION

In this section, we first derive the sufficient statistic for
demodulating the multiuser symbols from the space-time sig-
nal (4). We then outline the maximum likelihood space-time
multiuser sequence detector based on the sufficient statistic.

A. Sufficient Statistic

Denote \( \mathbf{b}(i) = [b_k(i), \ldots, b_K(i)]^T \) and \( \mathbf{b} = [\mathbf{b}(0)^T, \ldots, \mathbf{b}(M-1)^T]^T \). Define

\[
S(t; \mathbf{b}) \triangleq \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) \sum_{l=1}^{L} a_{kl} g_{kl} s_k(t - iT - \tau_{kl}).
\]

(5)

Using the Cameron–Martin formula [18], the likelihood func-
tion of the received waveform \( x(t) \) in (4) conditioned on all
the transmitted symbols of all users \( \mathbf{b} \) can be written as

\[
\ell\left(\mathbf{x}(t), \mathbf{b}\right) = \begin{cases} 
C C \exp\left[\Omega(\mathbf{b})/\sigma^2\right] & \text{if } 0 < t < \infty \\
0 & \text{otherwise}
\end{cases}
\]

(6)

where \( C \) is some positive scalar constant, and

\[
\Omega(\mathbf{b}) \triangleq 2 \Re\left\{ \int_{-\infty}^{\infty} \left[ S(t; \mathbf{b})^{*} x(t) \right] dt - \frac{1}{2} \int_{-\infty}^{\infty} |S(t; \mathbf{b})|^2 dt \right\}.
\]

(7)

The first integral in (7) can be expressed as

\[
\int_{-\infty}^{\infty} S(t; \mathbf{b})^{*} x(t) dt = \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) \sum_{l=1}^{L} a_{kl} g_{kl} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} x(t) s_k(t - iT - \tau_{kl}) dt \right] dt.
\]

(8)

Since the second integral in (7) does not depend on the
received signal \( x(t) \), by (8), the sufficient statistic for detecting
the multiuser symbols \( \mathbf{b} \) is \( \{y_k(i): 1 \leq k \leq K, 0 \leq i \leq M-1\} \).
From (8), it is seen that the sufficient statistic is obtained by passing the received signal vector \( \mathbf{r}(t) \) through \((KL)\) beamformers directed at each path of each user's signal, followed by a bank of \( K \) maximum-ratio multipath combiners (i.e., RAKE receivers). Since this beamformer is a spatial matched filter for the array antenna receiver, and a RAKE receiver is a temporal matched-filter for multipath channels, the sufficient statistic \( \{y_k(t)\} \) is, thus, simply the output of a space-time matched filter. Next, we derive an explicit expression for this sufficient statistic in terms of the multiuser channel parameters and transmitted symbols, which is instrumental to developing various space-time multiuser receivers in the subsequent sections.

Assume that the multipath spread of any user signal is limited to at most \( \Delta \) symbol intervals, where \( \Delta \) is a positive integer. That is,

\[
\tau_{kd} \leq \Delta T, \quad 1 \leq k \leq K; \quad 1 \leq l \leq L.
\]

Define the following correlation of the delayed user signaling waveforms:

\[
\rho_{kl}(\nu) = \frac{1}{\Delta} \int_{-\Delta}^{\Delta} \mathbf{s}_k(t - \tau_{kd}) \mathbf{s}_l(t - jT - \nu k \nu') \, dt - \Delta \leq j \leq \Delta; \quad 1 \leq k, k' \leq K; \quad 1 \leq l, \nu \leq L.
\]

Since \( \tau_{kd} \leq \Delta T \) and \( \mathbf{s}_k(t) \) is nonzero only for \( t \in [0, T] \), it then follows that \( \rho_{kl}(\nu) = 0 \), for \( |\nu| > \Delta \). Now, substituting (4) into (8), we have

\[
a_{kl}^H \mathbf{y}(i) = \sum_{j=-\Delta}^{\Delta} \mathbf{H} \mathbf{y}(i) + \mathbf{n}(i)
\]

where \( \mathbf{y}(i) \) is a sequence of zero-mean complex Gaussian vectors with covariance matrix

\[
E\{\mathbf{y}(i)\mathbf{y}^H(i+1)\} = \mathbf{H} + \mathbf{I}_M.
\]

Substituting (13) into (8), we obtain the expression for the sufficient statistic \( \mathbf{y}(i) \), which is given by

\[
\mathbf{y}(i) = \mathbf{C}^H \zeta(i)
\]

where \( \zeta(i) \) is a sequence of zero-mean complex Gaussian vectors with covariance matrix

\[
E\{\zeta(i)\zeta^H(i+j)\} = \mathbf{C}^H \mathbf{y}(i+1) + \mathbf{C}^H \mathbf{y}(i).
\]

B. Maximum Likelihood Multiuser Sequence Detector

Define the \((MK \times MK)\) block Jacobi matrix (17), shown at the bottom of the next page. In addition, define \( \mathbf{A} \triangleq \mathbf{L}_M \otimes \mathbf{A} \), where \( \otimes \) denotes the Kronecker matrix product; \( \bar{\mathbf{y}} \triangleq [\mathbf{y}(0)^T \cdots \mathbf{y}(M-1)^T]^T \), and recall that \( \mathbf{b} \triangleq [\mathbf{y}(0)^T \cdots \mathbf{y}(M-1)^T]^T \).

The maximum likelihood sequence decision rule chooses \( \mathbf{b} \) that maximize the log-likelihood function (7). Using (5), the second integral in (7) can be computed as

\[
\int_{-\infty}^{\infty} |\mathbf{s}(t; \mathbf{b})|^2 \, dt = \sum_{i=0}^{M-1} \sum_{\nu=0}^{L-1} \sum_{k=1}^{K} \mathbf{A}_k \mathbf{b}_k(\nu) \mathbf{b}_k(\nu)^T
\]

\[
= \mathbf{b}^T \mathbf{A} \mathbf{H} \mathbf{A} \mathbf{b}.
\]
Substituting (8) and (18) into (7), the log-likelihood function \( \Omega(\mathbf{b}) \) can then be written as

\[
\Omega(\mathbf{b}) = 2\Re\{\mathbf{b}^T \mathbf{A} \mathbf{y}\} - \mathbf{b}^T \mathbf{A} \mathbf{H} \mathbf{A}^H \mathbf{b}.
\]  

(19)

For any \( i \), denote its modulo-\( K \) decomposition with remainder \( \kappa(i) = 1, \ldots, K \) by \( \iota = \eta(i)K + \kappa(i) \) [24]. Then, we have\(^1\)

\[
\mathbf{b}^T \mathbf{A} \mathbf{y} = \sum_{i=1}^{MK} \mathbf{A}[i, \iota] \mathbf{b}[\iota] = \sum_{i=1}^{MK} A_{\kappa(i)}b_{\kappa(i)}(\eta(i))y_{\kappa(i)}(\eta(i))
\]

(20)

\[
\mathbf{b}^T \mathbf{A} \mathbf{H} \mathbf{A}^H \mathbf{b}
\]

\[
= \sum_{i=1}^{MK} A_{\kappa(i)}b_{\kappa(i)}(\eta(i)) \cdot \left[\mathbf{H}[i, \iota] \mathbf{A}[i, \iota] \mathbf{b}[\iota] + 2\Re\left\{\sum_{j=1}^{\iota-1} \mathbf{H}[i, j] \mathbf{A}[j, j] \mathbf{b}[j]\right\}\right]
\]

\[
= \sum_{i=1}^{MK} A_{\kappa(i)}b_{\kappa(i)}(\eta(i)) \cdot \left[A_{\kappa(i)}b_{\kappa(i)}(\eta(i)) + 2\Re\{f_i^H \mathbf{x}_i\}\right]
\]

(21)

where \( f_i \) is given, respectively, by

\[
x_i \triangleq \begin{bmatrix}
    1 \\
    0 \\
    \vdots \\
    0 \\
    b_{\kappa(i)}(\eta(i)) \\
    \vdots \\
    b_{\kappa(i)-1}(\eta(i)) \\
\end{bmatrix}^T
\]

\[
f_i \triangleq \begin{bmatrix}
    K - \kappa(i) \\
    0 \\
    \vdots \\
    0 \\
    (\mathbf{H}^{-1})_{\kappa(i)}(\eta(i)) \\
    \vdots \\
    (\mathbf{H}^{-1})_{\kappa(i)-1}(\eta(i)) \\
\end{bmatrix}^T
\]

where \( \mathbf{H}_{ik} \) denotes the \( k \)th element of matrix \( \mathbf{H} \), and \( (i, k') \)th entry of \( \mathbf{H} \). Substituting (20) and (21) into (19), the log-likelihood function can then be decomposed as follows:

\[
\Omega(\mathbf{b}) = \sum_{i=1}^{MK} \lambda_i(\xi_i, \mathbf{x}_i)
\]

(22)

\(^1\) Notation: \( \mathbf{A}[i, j] \) denotes the \( (i, j) \)th element of matrix \( \mathbf{A} \); \( \mathbf{b}[i] \) denotes the \( i \)th element of vector \( \mathbf{b} \).

\[\begin{aligned}
\mathbf{H} &\triangleq \begin{bmatrix}
    \mathbf{H}[0] & \mathbf{H}[1] & \cdots & \mathbf{H}[\Delta] \\
    \mathbf{H}[\Delta] & \mathbf{H}[0] & \cdots & \mathbf{H}[\Delta] \\
    \vdots & \vdots & \ddots & \vdots \\
    \mathbf{H}[\Delta] & \mathbf{H}[\Delta] & \cdots & \mathbf{H}[0]
\end{bmatrix} \\
\Phi &\triangleq \begin{bmatrix}
    \mathbf{a}_{11} & \mathbf{a}_{1L} & \cdots & \mathbf{a}_{K1} & \cdots & \mathbf{a}_{KL}
\end{bmatrix}^T \\
\zeta(i) &\triangleq \begin{bmatrix}
    \mathbf{a}_{1L}^H z_{1L}(i) & \mathbf{a}_{1L}^H z_{1L}(i) & \cdots & \mathbf{a}_{1L}^H z_{0L}(i) & \cdots & \mathbf{a}_{KL}^H z_{KL}(i)
\end{bmatrix} \\
\mathbf{u}(i) &\triangleq \begin{bmatrix}
    \mathbf{u}_{1L}(i) & \cdots & \mathbf{u}_{1L}(i) & \cdots & \mathbf{u}_{KL}(i)
\end{bmatrix} \\
\varphi_k &\triangleq \begin{bmatrix}
    \varphi_{k1} & \cdots & \varphi_{kL}
\end{bmatrix}^T \\
\mathbf{G} &\triangleq \begin{bmatrix}
    \mathbf{g}_{1L} & \cdots & \mathbf{g}_{KL}
\end{bmatrix} \\
\mathbf{A} &\triangleq \begin{bmatrix}
    \mathbf{A}_{1L} & \cdots & \mathbf{A}_{KL}
\end{bmatrix} \\
\mathbf{y}(i) &\triangleq \begin{bmatrix}
    \mathbf{y}_{1L}(i) & \cdots & \mathbf{y}_{1L}(i)
\end{bmatrix}^T \\
\mathbf{b}(i) &\triangleq \begin{bmatrix}
    \mathbf{b}_{1L}(i) & \cdots & \mathbf{b}_{KL}(i)
\end{bmatrix}^T
\end{aligned}\]  

(12a)

\[\begin{aligned}
\mathbf{H} &\triangleq \begin{bmatrix}
    \mathbf{H}[0] & \mathbf{H}[1] & \cdots & \mathbf{H}[\Delta] \\
    \mathbf{H}[\Delta] & \mathbf{H}[0] & \cdots & \mathbf{H}[\Delta] \\
    \vdots & \vdots & \ddots & \vdots \\
    \mathbf{H}[\Delta] & \mathbf{H}[\Delta] & \cdots & \mathbf{H}[0]
\end{bmatrix} \\
\mathbf{H} &\triangleq \begin{bmatrix}
    \mathbf{H}[0] & \mathbf{H}[1] & \cdots & \mathbf{H}[\Delta] \\
    \mathbf{H}[\Delta] & \mathbf{H}[0] & \cdots & \mathbf{H}[\Delta] \\
    \vdots & \vdots & \ddots & \vdots \\
    \mathbf{H}[\Delta] & \mathbf{H}[\Delta] & \cdots & \mathbf{H}[0]
\end{bmatrix}
\end{aligned}\]  

(17)
where \( \xi_i \triangleq b_{\phi(i)}(\gamma(i)) \), and

\[
\lambda_i(\xi, x) = \xi \cdot A_{\phi(i)} \cdot [2x(\gamma(i)) - 2f(i)HH^T x - A_{\phi(i)}H_{\phi(i),\gamma(i)} x] \tag{23}
\]

with the state vector recursively defined according to

\[
x_{i+1} = [x_1[2] \cdots x_1[\Delta K + K - 1]]^T, \quad x_1 = \Omega_k \Delta K + K - 1, \quad \Omega_k \text{ denotes a zero vector of dimension } m.
\]

Given the additive decomposition (22) of the log-likelihood function, it is straightforward to apply the Viterbi algorithm to compute the sequence \( \hat{b} \) that maximizes \( \Omega(b) \), i.e., the maximum likelihood estimate of the transmitted multiuser symbol sequences. Since the dimensionality of the state vector is \( (\Delta K + K - 1) \), the computational complexity of the maximum likelihood sequence detector is on the order of \( O(2^{\Delta K + K}) \). Note that in the absence of multipath (i.e., \( L = 1 \) and \( \Delta = 1 \), if the users are numbered according to their relative delays in an ascending order (i.e., \( 0 \leq \tau_1 \leq \cdots \leq \tau_{K_1} < T \)), then the matrix \( H_{[-1]} \) becomes strictly upper triangular. In this case, the dimension of the state vector is reduced to \( (K - 1) \), and the computational complexity of the corresponding maximum likelihood sequence detection algorithm is \( O(2^K) \) \cite{14, 24}. However, in the presence of multipath, even if the multipath delays are still within one symbol interval (i.e., \( \Delta = 1 \)), the matrix \( H_{[-1]} \) no longer has an upper triangular structure in general. Hence, the dimension of the state vector in this case is \( (2K - 1) \), and the complexity of the Viterbi algorithm is \( O(2^{2K}) \).

IV. Linear Space-Time Multiuser Detection

As seen from the previous section, the optimal space-time multiuser detection algorithm has a prohibitive computational complexity. In this section, we develop linear space-time multiuser detection techniques. It is assumed that the receiver has the knowledge of the spreading waveforms and the channel parameters of all users. The proposed method is based on iterative interference cancellation and has a low computational complexity. We also discuss the single-user-based linear space-time processing methods.

A. Linear Multiuser Detection via Iterative Interference Cancellation

From (15), we can write the expression for the sufficient statistic vector \( y \) in a matrix form as

\[
y = HAb + \sigma \nu \tag{24}
\]

where by (16) \( \nu \sim N_c(0, H) \). In linear multiuser detection, a linear transformation is applied to the sufficient statistic vector \( y \) followed by local decisions for each user, that is, the multiuser data bits are demodulated according to

\[
\hat{b} = \text{sign} [\text{Re} (Wy)] \tag{25}
\]

where \( W \) is an \((MK \times MK) \) complex matrix. Two popular linear multiuser detectors \cite{25} are the linear decorrelating (i.e., zero-forcing) detector, which chooses the weight matrix \( W \) to completely eliminate the interference (at the expense of enhancing the noise), and the linear MMSE detector, which chooses the weight matrix \( W \) to minimize the mean-square error (MSE) between the transmitted user signals and the outputs of the linear transformation, i.e., \( E[\| Ab - Wy \|^2] \). The corresponding weight matrices for these two linear multiuser detectors are given, respectively, by

\[
W_d = H^{-1}, \quad \text{[linear decorrelating detector]} \tag{26}
\]

\[
W_m = (H + \sigma^2 A^{-2})^{-1} \quad \text{[linear MMSE detector]}, \tag{27}
\]

Since the frame length \( M \) is usually large, direct inversion of the above matrices is too costly for practical purposes. We next consider applying the Gauss–Seidel iteration to obtain the linear multiuser detector output. This method effectively performs serial interference cancellation on the sufficient statistic vector \( y \) and recursively refines the estimates of the multiuser signals \( \{x_k(i) \triangleq A_k b_k(i) : 1 \leq k \leq K, 0 \leq i < M\} \). Denote such an estimate at the \( m \)th iteration as \( x_m^n(i) \).

Consider the following linear system equation:

\[
Hx = y. \tag{28}
\]

The Gauss–Seidel procedure \cite{28} for iteratively solving \( x \) from (28) is given by

\[
x_m^n[i'] = \frac{1}{H[i', \cdot']} \left( y[i'] - \sum_{j', j' < i'} H[i', j'] x_m^{n-1}[j'] \right)
\]

\[
- \sum_{j' > i'} H[i', j'] x_m^{n-1}[j'], \quad i' = 1, \ldots, MK; \quad m = 1, 2, \ldots \tag{29}
\]

Substituting in (29) the notations \( x_m^n[Ki + k] = x_m^n(i) \), \( y[Ki + k] = y_k(i) \), for \( k = 1, \ldots, K, i = 0, \ldots, M - 1 \), and the elements of the matrix \( H \) given in (17), then by the Ostrowski–Reich Theorem \cite{28}, a sufficient condition for the Gauss–Seidel iteration (29) to converge to the solution of (28), i.e., the output of the linear decorrelating detector \( x_m^n \rightarrow H^{-1}y \triangleq W_n x \) is that \( H \) is positive definite.

Similarly, consider the linear system equation

\[
(H + \sigma^2 A^{-2})x = y. \tag{30}
\]

The corresponding Gauss–Seidel iteration is given by

\[
x_m^n[i'] = \frac{1}{H[i', \cdot'] + \sigma^2 A[i', j']^2 \{ y[i'] - \sum_{j' < i'} H[i', j'] x_m^{n-1}[j'] \}
\]

\[
- \sum_{j' > i'} H[i', j'] x_m^{n-1}[j'], \quad i' = 1, \ldots, MK
\]

\[
m = 1, 2, \ldots. \tag{31}
\]

It is seen from (18) that the matrix \( H \) is positive semidefinite, as \( \int_0^\infty \|S(t; b)\|^2 dt = x^H H x \geq 0 \), where \( x \triangleq Ab \). Therefore, \( (H + \sigma^2 A^{-2}) \) is positive definite, and iteration...
The computational complexity of the above iterative serial interference cancellation algorithm per user per bit is\( \overline{m}M(2\Delta + 1)K^{2}/KM = O(\overline{m}K\Delta), \) where \( \overline{m} \) is the total number of iterations. The complexity per user per bit of direct inversion of the matrices in (26) or (27) is\( O(K^{3}M^{3}/KM) = O(K^{2}M^{2}). \) By exploiting the Hermitian \((2\Delta + 1)\)-block Toeplitz structure of the matrix \( \mathbf{H} \), this complexity can be reduced to \( O(K^{2}M\Delta) \) [14]. Since, in practice, the number of iterations is a small number, e.g., \( \overline{m} \leq 5 \), the above iterative method for linear multiuser detection achieves significant complexity reduction over the direct matrix inversion method.

Remark: A natural alternative to the serial interference cancellation method is the parallel interference cancellation method. Unlike the serial method, in which the new estimate \( x_{k}^{(i)}(\hat{\mathbf{z}}_{k}^{(i)}) \) is used to update the subsequent estimates as soon as it is available; in the parallel method, at the \( m \)th iteration, \( x_{k}^{(i)}(\hat{\mathbf{z}}_{k}^{(i)}) \) is updated using the estimates only from the previous iteration. The parallel interference cancellation corresponds to the Jacobi’s method [28] for solving the linear system (28) or (30), i.e.,

\[
x_{k}^{(i)}(\hat{\mathbf{z}}_{k}^{(i)}) = \frac{1}{\gamma(\hat{\mathbf{z}}_{k}^{(i)})} \left( \mathbf{y}[\hat{\mathbf{z}}_{k}^{(i)}] - \sum_{j \neq \hat{\mathbf{z}}_{k}^{(i)}} \mathbf{H}[\hat{\mathbf{z}}_{k}^{(i)}, j \hat{\mathbf{z}}_{k}^{(i)}] \mathbf{x}^{(i)}[\hat{\mathbf{z}}_{k}^{(i)}] \right)
\]

\( i = 1, \ldots, MK; \ m = 1, 2, \ldots \) (32)

with \( \gamma(\hat{\mathbf{z}}_{k}^{(i)}) = \mathbf{H}[\hat{\mathbf{z}}_{k}^{(i)}, \hat{\mathbf{z}}_{k}^{(i)}] \) or \( \gamma(\hat{\mathbf{z}}_{k}^{(i)}) = \mathbf{H}[\hat{\mathbf{z}}_{k}^{(i)}, j \hat{\mathbf{z}}_{k}^{(i)}] + \sigma^{2}/\mathbf{A}[\hat{\mathbf{z}}_{k}^{(i)}, \hat{\mathbf{z}}_{k}^{(i)}]. \) However, the convergence of the Jacobi’s method (32) is not guaranteed in general. To see this, for example, let \( \mathbf{D} \) be the diagonal matrix containing the diagonal elements of \( \mathbf{H} \), and let \( \mathbf{H} = \mathbf{D} + \mathbf{B} \) be the splitting of \( \mathbf{H} \) into its diagonal and off-diagonal elements. Suppose that \( \mathbf{H} = \mathbf{D} + \mathbf{B} \) is positive definite; then, the necessary and sufficient for the convergence of the Jacobi’s iteration is that \( \mathbf{D} - \mathbf{B} \) is positive definite [28]. In general, such a condition may not be satisfied, and hence, the parallel interference cancellation method (32) is not guaranteed to produce the linear multiuser detector output.

**B. Single User Space-Time Detection**

In this section, we consider several single-user-based linear space-time processing methods. These methods have been advocated in the recent literature as they lead to several space-time adaptive receiver structures [1], [13], [16], [26]. In what follows, we derive the exact forms of these single-user detectors in terms of multiuser channel parameters. In Section IV-D, the performance of these single-user detectors is compared with that of the multiuser detectors developed in Section IV-A, and it is seen that substantial performance gain can be achieved by using multiuser detection, especially in a near-far situation.

Denote \( r_{p}(t) \) as the received signal at the \( p \)th antenna element, i.e., the \( p \)th element of the received vector signal \( \mathbf{z}(t) \) in (4),

\[
\mathbf{r}_{p}(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_{k} h_{k}(i) \sum_{l=1}^{L} a_{kl,p} \vartheta_{kl}(t - \tau_{kl}) + \sigma_{n_{p}}(t), \quad p = 1, \ldots, P.
\] (33)

Suppose that the user of interest is the \( k \)th user. In the single-user approach, in order to demodulate the \( p \)th symbol of the \( k \)th user, that user’s matched filter output corresponding to each path at each antenna element is first computed, i.e.,

\[
z_{kd,p}(i) = \int_{-\infty}^{\infty} r_{p}(t) s_{k}(t - iT - \tau_{kl}) dt
\]

\[
= \sum_{j=0}^{\Delta} \sum_{k'=1}^{K} A_{k} h_{k'}(i + j) \sum_{l=1}^{L} a_{kl,p} g_{l}\left(\vartheta_{kl}(t), \vartheta_{kl}^{p}(t)\right)
\]

\[
+ n_{kd,p}(i), \quad l = 1, 2, \ldots, L; \ p = 1, 2, \ldots, P.
\] (34)

where \( \{n_{kd,p}(i)\} \) are zero-mean complex Gaussian random sequences with covariance

\[
E\{n_{kd,p}(i)n_{kd,p}^{*}(i')\} = \begin{cases} 0, & \text{if } p \neq p' \text{ or } |i - i'| > \Delta \\ \sigma^{2}/\mathbf{B}[i, j]'s[i, j], & \text{otherwise}. \end{cases}
\] (35)

Note that \( z_{kd,p}(i) \) is the \( p \)th element of the vector \( \hat{\mathbf{z}}_{kd}(i) \) defined in (8). Denote\(^2\)

\[
\hat{\mathbf{z}}_{kd}(i) = \left[ z_{11,p}(i) \cdots z_{1L,p}(i) \right]^{T} \quad [L \text{-vector}]
\]

\[
\hat{\mathbf{u}}_{kd}(i) = \left[ u_{11,p}(i) \cdots u_{1L,p}(i) \right]^{T} \quad [L \text{-vector}]
\]

\[
\Theta_{kd} = \left[ u_{1L,p} \cdots a_{kL,p} \right]^{T} \quad [L \text{-vector}]
\]

\[
\Theta_{kd}^{(i)} = \text{diag}(\Theta_{kd}, \ldots, \Theta_{kd}) \quad [(K \times L) \text{-matrix}]
\]

\[
\tilde{\mathbf{U}}_{kd} = \left[ \tilde{\mathbf{U}}_{kd} \right]^{(i)} \quad [(L \times KL) \text{-matrix}]
\]

\[
\tilde{\mathbf{U}}_{kd}^{(i)} = \left[ \tilde{\mathbf{U}}_{kd} \right]^{(i)} \quad [(L \times KL) \text{-matrix}]
\]

Then, we can write (34) in the following matrix form:

\[
\hat{\mathbf{z}}_{kd}(i) = \sum_{j=-\Delta}^{\Delta} \tilde{\mathbf{U}}_{kd}^{(i)} (\Theta_{kd}^{(i)}, \Theta_{kd}^{(i)}) + \sigma_{\hat{\mathbf{z}}_{kd}}^{(i)}
\]

\( p = 1, \ldots, P \) (36)

where, by (35), the complex Gaussian vector sequence \( \{\hat{\mathbf{u}}_{kd}(i)\} \) has the following covariance matrix:

\[
E\{\hat{\mathbf{z}}_{kd}(i)\hat{\mathbf{u}}_{kd}^{*}(i')\} = \begin{cases} \Theta_{kd}^{(i)}, & \text{if } p \neq p' \text{ or } |i - i'| > \Delta \\ \Theta_{kd}^{(i)} & \text{otherwise}. \end{cases}
\] (37)

\(^2\)Notation: \( \mathbf{B}[i_{0}, i_{1}, j_{0}, j_{1}] \) denotes the submatrix of \( \mathbf{B} \) consisting of rows \( i_{0} \) to \( i_{1} \) and columns \( j_{0} \) to \( j_{1} \).
where \( O_L \) denotes a \((L \times L)\) zero matrix. From (36), we then have
\[
\begin{bmatrix}
\hat{z}_{kk}(i) \\
\vdots \\
\hat{z}_{KF}(i)
\end{bmatrix} = \sum_{j=-\Delta}^{\Delta} \begin{bmatrix}
\hat{H}_k^{LF}(O_F \circ O_L) \\
\vdots \\
\hat{H}_k^{LF}(O_F \circ O_L)
\end{bmatrix} \Delta(k(i + j)) + \sigma \begin{bmatrix}
\hat{z}_{kF}(i) \\
\vdots \\
\hat{z}_{kF}(i)
\end{bmatrix}
\]
(38)

where, by (37), \( \hat{z}_{kF}(i) \sim N_c(0_{N_p - L_p} \otimes \tilde{h}_k^{[0]}) \).

In the single-user-based space-time processing methods, the \( k \)th user’s \( i \)th bit is demodulated according the following rule:
\[
\hat{b}_k(i) = \text{sign}[\Re\{\hat{w}_k^H \hat{z}_k(i)\}]
\]
(39)

where \( \hat{w}_k \in \mathbb{C}^{L_p} \). We next consider three choices of the weight vector \( \hat{w}_k \) according to different criteria.

1) Space-Time Matched Filter (MF): The simplest linear combining strategy is the space-time matched filter, which chooses the weight vector as
\[
\hat{w}_k = h_k = [(\theta_{k1} \circ g_k)^T \cdots (\theta_{kP} \circ g_k)^T]^T.
\]
(40)

Note that the output of this space-time matched filter is \( \hat{y}_k(i) = \hat{h}_k^H \hat{z}_k(i) \); the quantity first appeared in (8).

2) Minimum Mean-Squared Error (MMSE): In MMSE combining, the weight vector is chosen to minimize the mean-squared error between the \( k \)th user’s transmitted signal and the output of the linear combiner, i.e.,
\[
\hat{w}_k = \arg \min_{w \in \mathbb{C}^{L_p}} E\{|\hat{y}_k(i) - w^H \hat{z}_k(i)|^2\} = \Sigma_k^{-1} p_k
\]
(41)

where using (38), we have
\[
\Sigma_k \triangleq E\{|\hat{z}_k(i) \hat{z}_k(i)^H\} = \sum_{j=-\Delta}^{\Delta} \begin{bmatrix}
\hat{H}_k^{LF}(O_F \circ O_L) \\
\vdots \\
\hat{H}_k^{LF}(O_F \circ O_L)
\end{bmatrix} \Delta(k(i + j)) + \sigma^2 I_{L_p} \otimes \tilde{h}_k^{[0]}
\]
(42)
\[
p_k \triangleq E\{|\hat{z}_k(i) \hat{h}_k(i)|\} = \|\hat{h}_k\|_2
\]
(43)

where \( 1_k \) is a \( K \) vector of all zeros except for the \( k \)th entry, which is 1.

3) Maximum Signal-to-Interference Ratio (MSIR): In MSIR combining, the weight vector \( \hat{w}_k \) is chosen to maximize the signal-to-interference ratio
\[
\hat{w}_k = \arg \min_{w \in \mathbb{C}^{L_p}} \frac{\|E\{w^H \hat{z}_k\}\|^2}{\|E\{|w^H \hat{z}_k|^2\} - E\{|w^H \hat{z}_k|^2\}|^2}
\]
\[
= \arg \min_{w \in \mathbb{C}^{L_p}} \frac{w^H p_k \hat{p}_k^H w}{w^H \Sigma_k w}
\]
\[
= \arg \min_{w \in \mathbb{C}^{L_p}} \frac{w^H p_k \Sigma_k^{-1} p_k^H w}{w^H \Sigma_k w}.
\]
(44)

The solution to (44) is then given by the generalized eigenvector associated with the largest generalized eigenvalue of the matrix pencil \((p_k \Sigma_k^{-1}, \Sigma_k)\), i.e.,
\[
p_k \Sigma_k^{-1} p_k = \lambda \Sigma_k w.
\]
(45)

From (45) it is immediate that the largest generalized eigenvalue is \( p_k \Sigma_k^{-1} p_k \), and the corresponding generalized eigenvector is \( \hat{w}_k = \alpha \Sigma_k^{-1} p_k \), with some scalar constant \( \alpha \). Since scaling the combining weight by a positive constant does not affect the decision (39), the MSIR weight vector is the same as the MMSE weight vector (41).

Remark: The space-time matched-filter is data independent (assuming that the array responses and the multipath gains are known), and the single-user MMSE (MSIR) method is data dependent. Hence, in general, the latter outperforms the former. In essence the single-user MMSE method exploits the “spatial signatures” to suppress the interference. [For example, such a “spatial signature” for the \( k \)th user is given by (40)]. The \( k \)th user’s MSIR receiver then correlates the signal vector \( \hat{z}_k(i) \) along a direction in the space spanned by such “spatial signatures” of all users, such that the SIR of the \( k \)th user is maximized. Moreover, this approach admits several interesting blind adaptive implementations, even for systems that employ aperiodic spreading sequences [13], [26].

However, the interference suppression capability of such a single-user approach is limited since it does not exploit the inherent signal structure induced by the multiuser spreading waveforms. This method can still suffer from the near-far problem, as in matched filter detection, because the degree of freedom provided by the spatial signature is limited. Furthermore, since the user signals are originally designed to separate from each other by their spreading waveforms, the multiuser space-time approach, which exploits the structure of the user signals in terms of both spreading waveforms and spatial signatures, can significantly outperform the single-user approach. This is illustrated in Section IV-D by simulation examples.

C. Combined Single-User/Multiuser Linear Detection

The linear space-time multiuser detection methods discussed in Section IV-A are based on the assumption that the receiver has the knowledge of the spreading signatures and channel parameters (multipath delays and gains, array responses) of all users. In a practical cellular system, however, there might be a few external interfering signals (e.g., signals from other cells), whose spreading waveforms and channel parameters are not known to the receiver. In this section, we consider space-time processing in such a scenario by combining the single-user and multiuser approaches. The basic strategy is to first suppress the known interferers’ signals through the iterative interference cancellation technique discussed in Section IV-A and then apply the single-user MMSE method discussed in Section IV-B to the residual signal to further suppress the unknown interfering signals.

Consider the received signal model (4). Assume that the users of interest are users \( k = 1, \cdots, K_0 < K \), and the
spreading waveforms as well as the channel parameters of these users are known to the receiver. Users \( k = K_0 + 1, \cdots, K \) are unknown external interferers whose data are not to be demodulated. For each user of interest, the receiver first computes the \((LP)\)-vectors of the matched-filter outputs \( \mathbf{z}_k(i) \), \( 1 \leq k \leq K_0, i = 0, \cdots, M - 1 \) [cf. (38)]. The space-time matched-filter outputs \( y_k(i) \) [cf. (8)] are then computed by correlating \( \mathbf{z}_k(i) \) with the space-time matched filter given in (40).

Next, the iterative serial interference cancellation algorithm discussed in Section IV-A (here, the total number of users \( K \) is replaced by the total number of users of interest \( K_0 \) ) is applied to the data \( \{y_k(i)\} : 1 \leq k \leq K_0; i = 0, \cdots, M - 1 \) to suppress the interference from the known users. This is equivalent to implementing a linear multiuser detector assuming only \( K_0 \) (instead of \( K \)) users present. As a result, only the known interferers’ signals are suppressed at the detector output. Upon convergence, denote \( \hat{\mathbf{z}}_k(i) \) as the estimate of the user signals, i.e., \( \hat{\mathbf{z}}_k(i) \overset{\Delta}{=} \lim_{m \to \infty} x_m(i) \). \( 1 \leq k \leq K_0, i = 0, \cdots, M - 1 \). Note that \( \hat{\mathbf{z}}_k(i) \) contains the desired user’s signal, the unknown interferers’ signals, and the ambient noise. Using these estimates and based on the signal model (38), we next cancel the known interferers’ signal from the vector \( \mathbf{z}_k(i) \) to obtain

\[
\hat{\mathbf{z}}_k(i) \overset{\Delta}{=} \mathbf{z}_k(i) - \sum_{j=-\Delta}^{\Delta} \sum_{k' \neq 0, k}^{K_0} \mathbf{h}_{k,k'}^{[j]} \mathbf{e}_{k'} \hat{x}_{k'}(i+j)
\]

\( k = 1, \cdots, K_0, i = 0, \cdots, M - 1 \). (46)

Finally, a single-user combining weight \( \mathbf{w}_k \) is applied to the vector \( \hat{\mathbf{z}}_k(i) \), and the decision rule is given by

\[
\hat{b}_k(i) = \text{sign}[\Re(\mathbf{w}_k^H \hat{\mathbf{z}}_k(i))].
\]

If the weight vector \( \mathbf{w}_k \) is chosen to be a scaled version of the matched filter (40), i.e., \( \mathbf{w}_k = (1/\gamma_k) \mathbf{h}_k \), then the output of this matched filter is simply \( \hat{x}_k(i) \), i.e., \( (1/\gamma_k) \mathbf{h}_k^H \hat{\mathbf{z}}_k(i) = \hat{x}_k(i) \). To see this, first using (38) and (40), we have the following identitity:

\[
\mathbf{w}_k^H \mathbf{z}_k(i) = \frac{1}{\gamma_k} \sum_{l=1}^{P} \sum_{l'=1}^{P} \sum_{l''=1}^{P} \mathbf{a}_{l,l',l''} g_{l,l'} g_{l'',l''} \delta_{l,l'} \delta_{l'',l'} \mathbf{u}_{l,l'}. \mathbf{u}_{l'',l''} \mathbf{F}_{l,l'}(k, l', l''), \mathbf{F}_{l',l''}(k, l', l'')
\]

\[
= \frac{1}{\gamma_k} \sum_{l=1}^{P} \sum_{l'=1}^{P} \sum_{l''=1}^{P} \sum_{l'=1}^{P} \mathbf{a}_{l,l',l''} g_{l,l'} g_{l'',l''} \mathbf{u}_{l,l'}. \mathbf{u}_{l'',l''} \mathbf{F}_{l,l'}(k, l', l''), \mathbf{F}_{l',l''}(k, l', l'')
\]

\[
= \frac{1}{\gamma_k} \sum_{l=1}^{P} \sum_{l'=1}^{P} \sum_{l''=1}^{P} \sum_{l'=1}^{P} \mathbf{a}_{l,l',l''} g_{l,l'} g_{l'',l''} \mathbf{u}_{l,l'}. \mathbf{u}_{l'',l''} \mathbf{F}_{l,l'}(k, l', l''), \mathbf{F}_{l',l''}(k, l', l'')
\]

\[
= \frac{1}{\gamma_k} \sum_{l=1}^{P} \sum_{l'=1}^{P} \sum_{l''=1}^{P} \sum_{l'=1}^{P} \mathbf{a}_{l,l',l''} g_{l,l'} g_{l'',l''} \mathbf{u}_{l,l'}. \mathbf{u}_{l'',l''} \mathbf{F}_{l,l'}(k, l', l''), \mathbf{F}_{l',l''}(k, l', l'')
\]

Now, applying the matched filter (40) to both sides of (46), we have

\[
\mathbf{w}_k^H \hat{\mathbf{z}}_k(i) = \frac{1}{\gamma_k} \sum_{j=-\Delta}^{\Delta} \sum_{k' \neq 0, k}^{K_0} \mathbf{h}_{k,k'}^{[j]} \hat{\mathbf{x}}_{k'}(i+j)
\]

\[
= \hat{\mathbf{x}}_k(i), \quad k = 1, \cdots, K_0, \quad i = 0, \cdots, M - 1
\]

(50)

where (49) follows from (48), \( y_k(i) = \gamma_k \mathbf{h}_k^H \hat{\mathbf{z}}_k(i) \). (50) follows from the fact that \( \{\hat{\mathbf{x}}_k(i)\} \) are the converged outputs of the iterative serial interference cancellation algorithm.

On the other hand, if the combining weight is chosen according to the MMSE criterion, then it is given by

\[
w_k = \arg \min_{w \in C^2} \mathbb{E} \{ ||b_k(i) - w_k^H \mathbf{z}_k(i)||^2 \}
\]

\[
= [\mathbb{E} \{ \mathbf{z}_k(i) \mathbf{z}^H_k(i) \}]^{-1} \cdot \mathbb{E} \{ \mathbf{z}_k(i) b_k(i) \}
\]

\[
= \left( \sum_{i=0}^{M-1} \mathbf{z}_k(i) \mathbf{z}^H_k(i) \right)^{-1} \mathbf{p}_k.
\]

(51)

It is clear from the above discussion that in this combined approach, the interference due to the known users is suppressed by serial multiuser interference cancellation, whereas the residual interference due to the unknown users is suppressed by the single-user MMSE combiner.

D. Simulation Examples

In what follows, we assess the performance of the various multiuser and single-user space-time processing methods discussed in this section by computer simulations. We first outline the simulated system in Examples 1–3. It consists of eight users \( (K = 8) \) with a spreading gain 16 \((N = 16)\). Each user’s propagation channel consists of three paths \((L = 3)\). The receiver employs a linear antenna array with three elements \((M = 3)\) and half-wavelength spacing. Let the direction of arrival (DOA) of the \( k \)th user’s signal along the \( l \)th path with respect to the antenna array be \( \phi_{kl} \); then, the array response is given by

\[
a_{kl} = \exp[j(p-1)\pi \sin(\phi_{kl})].
\]

(52)

The spreading sequences, multipath delays and complex gains, and the DOA’s of all user signals in the simulated system are tabulated in Table I. These parameters are randomly generated and kept fixed for all the simulations. All users have equal transmitted power, i.e., \( A_1 = \cdots = A_K \). However, the received signal powers are unequal due to the unequal strength of the multipath gain for each user. The total strength of each user’s multipath channel, measured by the norm of the channel gain vector \( ||\mathbf{g}_k|| \), is also listed in Table I. Note that this system has a near-far situation, i.e., user 3 is the weakest user and user 6 is the strongest.

Example 1—Performance Comparison: Multiuser Versus Single-User Space-Time Processing: We first compare the performance of the multiuser linear space-time detection and that of the single-user linear space-time detection. The figure
of merit for comparison is the probability of bit detection error.
Three receivers are considered:

1) the single-user space-time matched filter given by (40);
2) the single-user space-time MMSE receiver given by (41);
3) the multiuser MMSE receiver implemented by the iterative interference cancellation algorithm (29) (five
   iterations are used).

Fig. 1 shows the performance of the weak users (users 1, 3, 4, and 8). It is seen that in general, the single-user MMSE

<table>
<thead>
<tr>
<th>#</th>
<th>Signature ((s_k(j)))</th>
<th>Delay ((T_c))</th>
<th>DOA (°)</th>
<th>Multiuser gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0011011100000111</td>
<td>0 2 3</td>
<td>34 -16</td>
<td>0.193 - j0.714</td>
</tr>
<tr>
<td>2</td>
<td>11010111100011101</td>
<td>1 4 5</td>
<td>2 42</td>
<td>-0.508 - j0.113</td>
</tr>
<tr>
<td>3</td>
<td>1110110000110001</td>
<td>2 3 6</td>
<td>-33 -13</td>
<td>0.125 - j0.064</td>
</tr>
<tr>
<td>4</td>
<td>01000111011111001</td>
<td>2 4 5</td>
<td>58 13 61</td>
<td>0.354 - j0.121</td>
</tr>
<tr>
<td>5</td>
<td>00111011101000000</td>
<td>4 6 7</td>
<td>72 69 1</td>
<td>0.597 + j0.395</td>
</tr>
<tr>
<td>6</td>
<td>1111111100111001</td>
<td>5 7 8</td>
<td>3 18 -55</td>
<td>0.084 + j1.205</td>
</tr>
<tr>
<td>7</td>
<td>11100100000010010</td>
<td>6 8 9</td>
<td>-79 -53 70</td>
<td>-0.428 + j0.166</td>
</tr>
<tr>
<td>8</td>
<td>11011010011011000</td>
<td>8 9 12</td>
<td>53 25 -20</td>
<td>-0.575 + j0.018</td>
</tr>
</tbody>
</table>
receiver outperforms the matched filter receiver. Interestingly though, for user 1, the matched filter actually slightly outperforms the single-user MMSE receiver. This is not surprising, since due to the interference, the detector output distribution is not Gaussian, and minimizing the mean-square error does not necessarily lead to minimum bit error probability.] It is also evident that the multiuser approach offers substantial performance improvement over the single-user methods.

**Example 2—Convergence of the Iterative Interference Cancellation Method:** This example serves to illustrate the convergence behavior of the iterative interference cancellation method (29). The bit error rate performance corresponding to the first four iterations for users 4 and 8 are shown in Fig. 2. It is seen that the algorithm converges within four to five iterations. It is also seen that the biggest performance improvement occurs at the second iteration.

**Example 3—Performance of the Combined Multiuser/Single-User Space-Time Processing:** In this example, it is assumed that users 7 and 8 are external interferers, and their signature waveforms and channel parameters are not known to the receiver. Therefore, the combined multiuser/single-user space-time processing method discussed in Section IV-C is employed at the receiver. Fig. 3 illustrates the bit error rate performance of users 3 and 4. Four methods are considered here:

1) the single-user matched-filter (40);
2) the single-user MMSE receiver (41);
3) the partial interference cancellation;
4) a matched-filter or a single-user MMSE receiver.

It is seen that the combined multiuser/single-user space-time processing achieves the best performance among the four methods.
Example 4—Performance Versus Number of Antennas/Number of Users: In the next example, we illustrate how performance varies with the number of users and receive antennas for both the multiuser space-time detector and the single-user space-time detector. The simulated system has $K = 16$ users and processing gain $N = 16$. The number of paths for each user is $L = 3$. The performance of User 5 in this system using the single-user MMSE receiver and the multiuser MMSE receiver is plotted, respectively, in Fig. 4, with the number of antennas $P = 2, 4$, and 6. It is seen that while in the single-user approach, the performance improvement due to the increasing number of receive antennas is only marginal, such performance improvement in the multiuser approach is of orders of magnitude. Next, we fix the number of antennas as $P = 4$ and vary the number of users in the system. The processing gain is still $N = 16$, and the number of paths for each user is $L = 3$. The performance of the single-user MMSE receiver and the multiuser receiver for User 2 is plotted, respectively, in Fig. 5, with the number of users $N = 10, 20$, and 30. Again, we see that in all these cases, the multiuser method significantly outperforms the single-user method.

Example 5—Performance Versus Spreading Gain/Number of Antennas: In this example, we consider the performance of single-user method and multiuser method by varying the processing gain $N$ and the number of receive antennas $P$ while keeping their product $(NP)$ fixed. The simulated system has $K = 16$ users, and the number of paths for each user is $L = 3$. Three cases are simulated:

1) $N = 64, P = 1$
2) $N = 32, P = 2$
3) $N = 16, P = 4$
Fig. 6. Single-user and multiuser receiver performance under different space-time gains \((K = 16)\). Left: single-user MMSE receiver; Right: multiuser MMSE receiver.

The performance for User 2 is shown in Fig. 6. It is seen that in this case, this user’s signal is best separated from others when \(N = 16, P = 4\) for both the single-user and multiuser methods. Moreover, the multiuser approach offers orders of magnitude performance improvement over the single-user method.

V. BLIND SPACE-TIME MULTIUSER DETECTION

As noted in Section IV-B, one of the attractive features of the single-user-based space-time methods is that they admit several blind adaptive implementations. However, these single-user detectors can suffer substantial performance loss compared with multiuser detectors, as illustrated in Section IV-D. So far, in developing the space-time multiuser detectors, we have assumed that the receiver has the knowledge of the signature sequences as well as the channel parameters of all users in the channel. Such an assumption is valid for base station receivers at the uplink of a wireless network, where data from all mobile users are to be demodulated. However, this assumption may not be realistic for the mobile receiver at the downlink of a wireless network. In a downlink scenario, the receiver of a mobile user usually knows only its own signature sequence and its channel but not the signature sequences or channel parameters of other users in the network. In this section, we develop blind space-time multiuser detection techniques to cope with such a downlink situation. The blind receiver developed in this section assumes only the knowledge of the desired user’s signature sequence and the timing information. This information requirement is even less than the conventional RAKE receiver, which, in addition, needs information on the multipath channel gains and array response of the desired user. However, the sensitivity of the blind receiver to the delay estimation error is not considered in this paper and is subject to future research.

A. Blind Space-Time Multiuser Detection Algorithm

Throughout this section, we assume that the user of interest is the \(k\)th user and that the receiver knows this user’s spreading waveform \(s_k(t)\) [cf. (2)] and its multipath delays \(\tau_{k1}, \ldots, \tau_{kL}\). In what follows, we consider demodulating the \(k\)th user’s \(n\)th transmitted symbol. The received signal at the \(p\)th antenna element \([r_p(t)\) given in (33)] is first passed through a chip-matched filter and then sampled at the chip rate to obtain a \(N\)-vector of signal samples

\[
\mathbf{r}_p(i) = [r_{p,0}(i) \ r_{p,1}(i) \ldots r_{p,N-1}(i)]^T
\]

where \(N = N + [(\tau_{kL} - \tau_{k1})/T_c]\), and the samples are given by

\[
r_{p,n}(i) = \int_{iT + (\tau_{k1} + nT_c)}^{(i+1)T + (\tau_{k1} + nT_c)} r_p(t) e^{-j2\pi f(t - \tau_{k1} - nT_c)} dt.
\]

Note that the choice of \(N\) is to capture the desired user’s signal from all paths. Using (33), the signal vector \(\mathbf{r}_p(i)\) can then be expressed as

\[
\mathbf{r}_p(i) = A_k b_k(i) \sum_{l=1}^{L} a_{kl} p_g h_k s_{kl}^{[0]} + \mathbf{i}_p(i) + \mathbf{n}_p(i)
\]

where the first term on the right-hand side of (55) contains the desired symbol \(b_k(i)\), the \(\mathbf{n}_p(i)\) is the ambient noise vector \(\sim \mathcal{N}(0, \mathbf{I}_N)\), and \(\mathbf{i}_p(i)\) consists of the interfering signals given by

\[
\mathbf{i}_p(i) = \sum_{j=\Delta}^{\Delta} A_k b_k(i + j) \sum_{l=1}^{L} a_{kl} p_g h_k s_{kl}^{[j]}
\]

\[
+ \sum_{j=\Delta}^{\Delta} \sum_{k' \neq k} A_k b_{k'}(i + j) \sum_{l=1}^{L} a_{k'l} p_g h_{k'} s_{k'l}^{[j]},
\]

The V. BLIND SPACE-TIME MULTIUSER DETECTION section discusses the development of blind space-time multiuser detection algorithms for wireless networks. The algorithm is designed to operate under conditions where the receiver has only partial knowledge of the channel and interference signals, specifically the knowledge of the desired user’s signature sequence and timing information. The approach aims to improve performance over single-user methods, which can suffer from substantial losses compared to multiuser detectors.

The blind detection algorithm is outlined in the section, focusing on the use of a chip-matched filter and sampling at the chip rate to obtain the signal samples. The received signal is represented in a vector form, capturing the signal from all paths within a specified delay range. The desired signal is expressed as a linear combination of the desired user’s spreading sequence and channel coefficients, along with ambient noise and interfering signals. This formulation allows for the separation and extraction of the desired user’s signal from the total received signal, thereby improving the overall system performance.

The algorithm’s effectiveness is demonstrated through performance analysis and comparison with traditional single-user and multiuser receivers, highlighting the advantages of the blind detection approach in real-world wireless communication scenarios, particularly in downlink settings where users have limited knowledge of the network’s infrastructure.

The development of such algorithms is crucial for enhancing the capacity and reliability of wireless communication systems, enabling more efficient use of spectrum resources and improved user experience in scenarios with limited network information and challenging channel conditions.
The first term on the right-hand side of (56) represents the interference caused by the previous and the subsequent symbols of the desired user, i.e., intersymbol interference (ISI); and the second term represents the interference caused by the other user's signals, i.e., multiple-access interference (MAI). In (55) and (56), the $\mathbf{N}$-vector $\mathbf{s}_k[n]$ is the discretized version of the delayed signature waveform of user $k'$, $s_k(t-jT-\tau_{kl})$ with its $n$th element given by

$$
\mathbf{s}_k[n] = \int_{\tau_{l1}+(n-1)T_c}^{\tau_{l1}+(n+1)T_c} s_k(t-jT-\tau_{kl}) \psi(t-\tau_{kl}-nT_c) \, dt
$$

$k' = 1, \ldots, K; l = 1, \ldots, L; j = -\Delta, \ldots, \Delta$. (57)

Since the spreading waveform $s_k(t)$, as well as the multipath delays $\tau_{kl}$, of the desired user are assumed to be known to the receiver, so are the vectors $\{\mathbf{s}_k[n]\}_{n=1}^{\mathbf{N}}$. In what follows, we denote $\mathbf{s}_k \triangleq \mathbf{s}_k[0]$ for notational convenience.

The proposed blind space-time multiuser receiver operates in the following way. At each antenna element $p$, for each path $l$, a linear filter $\mathbf{w}_{pl} \in \mathbb{C}^N$ is applied to signal $\mathbf{r}_l(i)$ to extract the desired user's signal from the $l$th path and to suppress the signals from other paths, as well as the interfering signals. Denote the multipath filter bank at the $p$th antenna as $\mathbf{W}_p = [\mathbf{w}_{pl}, \ldots, \mathbf{w}_{pl,L}]$, $p = 1, \ldots, P$. In addition, denote $\mathbf{S} = [\mathbf{s}_k, \ldots, \mathbf{s}_{k,L}]$. The multipath filter banks $\mathbf{W}_p$ are chosen according to the linear constrained minimum variance (LCMV) criterion [7], [9], [10], [23]

$$
\mathbf{W}_p = \arg \min_{\mathbf{W} \in \mathbb{C}^{N \times L}} E\{||\mathbf{W}_p^H \mathbf{r}_l(i)||^2\} = \arg \min_{\mathbf{W} \in \mathbb{C}^{N \times L}} \text{tr}((\mathbf{W}_p^H \mathbf{C}_p \mathbf{W}))(58)
$$

subject to $\mathbf{W}_p^H \mathbf{S} = \mathbf{I}_L$. (59)

where $\text{tr}(\cdot)$ denote the matrix trace operator, and $\mathbf{C}_p \triangleq E\{\mathbf{r}_l(i) \mathbf{r}_l(i)^H\}$. The constraints (59) ensure that the desired user's signal from the $l$th path is held constant at the filter output, whereas the desired user's signals from other paths are nulled out. The suppression of MAI and ISI is through the minimization of the objective function, i.e., the mean-squared value of the filter output. The solution to the above-constrained optimization problem is given by

$$
\mathbf{W}_p = \mathbf{C}_p^{-1} \mathbf{S}(\mathbf{S}^T \mathbf{C}_p^{-1} \mathbf{S})^{-1}.
$$

(60)

The outputs of all linear multipath filters are concatenated to form the following $(LP)$-vector:

$$
\mathbf{y}(i) = [(\mathbf{W}_1^H \mathbf{r}_1(i))^T \cdots (\mathbf{W}_P^H \mathbf{r}_P(i))^T]^T
$$

Finally, a linear combining vector $\mathbf{g} \in \mathbb{C}^{LP}$ is applied to $\mathbf{y}(i)$ to yield the decision statistic for the $i$th symbol of the desired user

$$
z(i) = \mathbf{g}^H \mathbf{y}(i).
$$

(62)

Ideally, the linear filter $\mathbf{w}_{pl}$ can eliminate the signals from other paths (ISI and MAI) to yield

$$
\mathbf{w}_{pl}^H \mathbf{r}_l(i) = A_k b_k(i) \mathbf{g}_{kl} + \sigma \mathbf{n}_l(i)
$$

$l = 1, \ldots, L; p = 1, \ldots, P$. (63)

Substituting (63) into (61), we have

$$
\mathbf{y}(i) = A_k b_k(i) \mathbf{h}_k + \sigma \mathbf{n}(i)
$$

(64)

TABLE II

<table>
<thead>
<tr>
<th>B A T C H A LGOR ITHM FOR B LIND S PACE -T IME M ULTIUSER D ETECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Compute the sample autocorrelation matrix of $\mathbf{r}_l$</td>
</tr>
<tr>
<td>$\hat{\mathbf{C}}<em>l = \frac{1}{M-1} \sum</em>{i=1}^{M} \mathbf{r}_l(i) \mathbf{r}_l(i)^H$</td>
</tr>
<tr>
<td>$p = 1, \ldots, P$</td>
</tr>
<tr>
<td>2. Compute the multipath filter bank</td>
</tr>
<tr>
<td>$\mathbf{w}_p = \mathbf{C}_p^{-1} (\mathbf{S}^H \mathbf{C}_p^{-1} \mathbf{S})^{-1}$</td>
</tr>
<tr>
<td>$p = 1, \ldots, P$</td>
</tr>
<tr>
<td>3. Apply the multipath filters to the signals</td>
</tr>
<tr>
<td>$\mathbf{y}(i) = [(\mathbf{W}_1^H \mathbf{r}_1(i))^T \cdots (\mathbf{W}_P^H \mathbf{r}_P(i))^T]^T$</td>
</tr>
<tr>
<td>$i = 0, \ldots, M-1$</td>
</tr>
<tr>
<td>4. Compute the sample autocorrelation matrix of $\mathbf{y}(i)$</td>
</tr>
<tr>
<td>$\hat{\mathbf{R}}<em>n = \frac{1}{M} \sum</em>{i=0}^{M-1} \mathbf{y}(i) \mathbf{y}(i)^H$</td>
</tr>
<tr>
<td>5. Compute the principal eigenvector $\hat{\mathbf{g}}$ of $\hat{\mathbf{R}}_n$</td>
</tr>
<tr>
<td>6. Compute the decision statistic for demodulation</td>
</tr>
<tr>
<td>$z(i) = \hat{\mathbf{g}}^H \mathbf{y}(i)$</td>
</tr>
<tr>
<td>$i = 0, \ldots, M-1$</td>
</tr>
</tbody>
</table>

with $\mathbf{h}_k$ defined in (40), and $\mathbf{n}(i) = [\mathbf{W}_1^H \mathbf{n}_1(i)^T \cdots (\mathbf{W}_P^H \mathbf{n}_L(i)^T]^T \sim \mathcal{N}(0, \mathbf{Q})$, where $\mathbf{Q} \triangleq \text{diag}[(\mathbf{W}_1^H \mathbf{W}_1), \ldots, (\mathbf{W}_P^H \mathbf{W}_P)]$. Denote $\mathbf{Q}^{-1/2}$ as the inverse of the Hermitian square-root of $\mathbf{Q}$; then, $\mathbf{Q}^{-1/2} = \text{diag}[(\mathbf{W}_1^H \mathbf{W}_1)^{-1/2}, \ldots, (\mathbf{W}_P^H \mathbf{W}_P)^{-1/2}]$. Premultiplying both sides of (64), we obtain

$$
\hat{\mathbf{y}}(i) \triangleq \mathbf{Q}^{-1/2} \mathbf{y}(i) = A_k b_k(i) \mathbf{h}_k + \sigma \mathbf{n}(i)
$$

(65)

where $\mathbf{n} \sim \mathcal{N}(0, \mathbf{I}_{LP})$. It then follows that the optimal combining weight in (62) for detecting $b_k(i)$ is given by $\mathbf{g} = \hat{\mathbf{h}}_k$. Using (65), the autocorrelation matrix of $\hat{\mathbf{y}}(i)$ is given by

$$
E\{\hat{\mathbf{y}}(i) \hat{\mathbf{y}}(i)^H\} = A_k^2 \mathbf{h}_k^H \mathbf{h}_k + \sigma^2 \mathbf{I}_{LP}.
$$

(66)

Therefore, the combining weight $\hat{\mathbf{h}}_k$ is given by the principal eigenvector of autocorrelation matrix of the signal $\hat{\mathbf{y}}(i)$. In practice, there is residual interference at the outputs of the linear filters, and therefore, the output interference-plus-noise terms are weakly correlated both temporally and spatially. Nevertheless, we adopt a simple combining strategy by choosing the combining weight $\mathbf{g}$ in (62) as the principal eigenvector of the autocorrelation matrix of the signal $\mathbf{y}(i)$ in (61).

Finally, the batch algorithm for blind space-time multiuser detection discussed above is summarized in Table II. Notice that there is an arbitrary phase ambiguity in this method caused by Step 5, which is the principal eigenvector estimator. This can be resolved by differentially encoding and decoding the data. That is, the demodulator decides $\hat{d}_k(i) = b_k(i+1) b_k(i)$, according to the rule $\hat{d}_k(i) = \text{sign}[\mathbf{h}_k^H \mathbf{z}(i-1)^* \mathbf{z}(i)]$, where $\mathbf{z}(i)$ is the decision statistic yield by Step 6. We next consider adaptive implementations of the blind space-time multiuser detection method developed above. Adaptation rules based on least mean-square (LMS) algorithm and recursive least-squares (RLS) algorithm are considered.
1) LMS Algorithm: We adopt the projection approach in [19] to find the LMS solution to the constraint optimization problem (59). Define the following projection matrix \( P_p \) that projects a vector onto the column space of \( S \):

\[
P_p = S(S^H S)^{-1} S^H.
\]

Decompose the optimal linear filter-bank \( \mathbf{W}_p \) in (60) into two orthogonal components as follows:

\[
\mathbf{W}_p = \mathbf{W}_p^s - M^H \mathbf{W}_p^a \tag{67}
\]

where \( \mathbf{W}_p^s \triangleq P_p \mathbf{W}_p = S(S^H S)^{-1} S^H \) is the projection of the columns of \( \mathbf{W}_p \) onto the column space of \( S \). Note that this projection \( \mathbf{W}_p^s \) is independent of data and represents the nonadaptive portion of the weight matrix \( \mathbf{W}_p \). The matrix \( M \) in (67) is a \((N - L) \times N\) matrix whose row space spans the null space of \( S \), i.e., \( MS = 0 \). \( N \times L \) and \( \mathbf{W}_p^a \) is a \((N - L) \times N\) weight matrix.

Using the orthogonal decomposition (67) of the weight matrix, the constraint optimization problem (59) is then converted into the following unconstrained optimization problem:

\[
\mathbf{W}_p^a = \arg \min_{\mathbf{W}_p^a \in \mathbb{C}^{(N-L)\times L}} E[\| \mathbf{W}_p^a - M^H \mathbf{W}_p^a \|^2_2]. \tag{68}
\]

The LMS algorithm for adapting the weights \( \mathbf{W}_p^a \) is given by

\[
\mathbf{W}_p^a(i + 1) = \mathbf{W}_p^a(i) - \mu g(\mathbf{W}_p^a(i)) \tag{69}
\]

where the stochastic gradient \( g(\mathbf{W}_p^a(i)) \) is given by

\[
g(\mathbf{W}_p^a(i)) \triangleq \frac{d}{d \mathbf{W}_p^a(i)}[\| \mathbf{W}_p^a - M^H \mathbf{W}_p^a \|^2_2] \tag{70}
\]

Hence, the LMS algorithm for updating \( \mathbf{W}_p^a(i) \) is given by

\[
\mathbf{W}_p^a(i + 1) = \mathbf{W}_p^a(i) + \mu \mathbf{W}_p^a(i) \mathbf{W}_p^a(i) \tag{71}
\]

where \( \mathbf{y}_p(i) \) is the output of the multipath filter at the \( i \)th antenna element. The outputs of the filters at all antenna elements are concatenated to obtain \( \mathbf{y}(i) = [\mathbf{y}_p(i)^T \cdots \mathbf{y}_p(i)^T]^T \).

Finally, the principal eigenvector of the autocorrelation matrix of \( \mathbf{y}(i) \) must be estimated. There are numerous algorithms in the literature for adaptive tracking of the principal eigenvector of the autocorrelation matrix of a signal [2], [20]. In this paper, we adopt the following simple algorithm found in [27], which tracks the largest eigenvalue \( \lambda \) and the corresponding eigenvector \( \mathbf{g} \) of the autocorrelation matrix of the signal \( \mathbf{y}(i) \):

\[
z(i) = g(i - 1) \mathbf{y}(i) \tag{73}
\]

\[
\lambda(i) = \beta \lambda(i - 1) + |z(i)|^2 \tag{74}
\]

\[
g(i) = g(i - 1) + [\mathbf{y}(i) - g(i - 1)z(i)]z(i)' / \lambda(i) \tag{75}
\]

where \( 0 < \beta < 1 \) is a forgetting factor. Note that \( z(i) \) in (73) is the decision statistic.

2) QR-RLS Algorithm: We next consider the RLS adaptive version of the multipath filter for interference suppression. In particular, we derive a numerically stable and computationally efficient QR-RLS algorithm for this application. The exponentially windowed RLS algorithm selects the weight matrix \( \mathbf{W}_p(i) \in \mathbb{C}^{N \times L} \) to minimize the sum of exponentially weighted mean-squared output values:

\[
\min_{n=1} \sum \beta^n || \mathbf{W}_p(i)^H \mathbf{r}(n) ||^2, \text{ s.t. } \mathbf{W}_p(i)^H \mathbf{S} = \mathbf{I}_L
\]

where \( 0 < \beta < 1 \) is the forgetting factor \((1 - \beta) \ll 1\). Its purpose is to ensure that the data in the distant past be forgotten in order to provide the tracking capability in a nonstationary environment. The solution of this optimization problem is given by

\[
\mathbf{W}_p(i) = \mathbf{C}_p(i)^{-1} \mathbf{S} \mathbf{g}_p(i)^{-1} \tag{76}
\]

where \( \mathbf{C}_p(i) \triangleq \sum_{n=1}^{i} (\beta^n \mathbf{r}(n)) \mathbf{r}(n)^H, \) and \( \mathbf{g}_p(i) = \mathbf{S}^H \mathbf{C}_p(i)^{-1} \mathbf{S} \).

We next derive a recursive procedure based on the QR-RLS algorithms [9] for calculating the output of the RLS adaptive multipath filter \( \mathbf{y}_p(i) \triangleq \mathbf{W}_p(i - 1) \mathbf{r}(i) \). Assume that \( \mathbf{C}_p(i) \) is positive definite. Let

\[
\mathbf{C}_p(i) = \mathbf{R}_p(i)^H \mathbf{R}_p(i) \tag{77}
\]

be the Cholesky decomposition, i.e., \( \mathbf{R}_p(i) \) is the unique upper triangular Cholesky factor with positive diagonal elements. Define the following quantities:

\[
\mathbf{U}_p(i) = \mathbf{R}_p(i)^{-H} \mathbf{S}, \tag{78}
\]

\[
\mathbf{V}_p(i) = \mathbf{R}_p(i)^{-H} \mathbf{r}(i) \tag{79}
\]

Suppose that \( \mathbf{R}_p(i - 1) \) and \( \mathbf{U}_p(i) \) have been computed at time \((i - 1)\). At time \( i \), new data \( \mathbf{r}(i) \) becomes available. We construct a block matrix consisting of \( \mathbf{R}_p(i - 1), \mathbf{U}_p(i - 1) \), and \( \mathbf{r}(i) \) and apply an orthogonal transformation as follows:

\[
\mathbf{Q}(i) = \begin{bmatrix} \sqrt{\beta} \mathbf{r}_p(i - 1) & \mathbf{U}_p(i - 1) / \beta \\ \mathbf{R}_p(i - 1) & \mathbf{U}_p(i) \end{bmatrix} = \begin{bmatrix} \mathbf{R}_p(i) & \mathbf{U}_p(i) \end{bmatrix} \begin{bmatrix} \mathbf{P}_p(i) & \mathbf{V}_p(i) \end{bmatrix} \tag{80}
\]

In (80), the matrix \( \mathbf{Q}(i) \), which zeros the first \( N \) elements on the last row of the partitioned matrix appearing on the left-hand side of (80), is an orthonormal matrix consisting of \( N \) Givens rotations

\[
\mathbf{Q}(i) = \mathbf{Q}_N(i) \cdots \mathbf{Q}_2(i) \mathbf{Q}_1(i) \tag{81}
\]

where \( \mathbf{Q}_n(i) \) zeros the \( n \)th element in the last row by rotating it with the \( n \)th row. An individual rotation is specified by two scalars \( c_n \) and \( s_n \) and affects only the last row and the \( n \)th row. The effects on these two rows are

\[
\begin{bmatrix}
  c_n & s_n & \cdots & 0 & y_1 & y_{n+1} & \cdots \\
  -s_n & c_n & \cdots & 0 & r_{n+1} & r_{n+1} & \cdots
\end{bmatrix} \leftarrow \text{last row}
\]

\[
\begin{bmatrix}
  0 & \cdots & 0 & y_1' & y_{n+1}' & \cdots \\
  0 & \cdots & 0 & r_{n+1}' & r_{n+1}' & \cdots
\end{bmatrix} \leftarrow \text{last row}
\]

\[
\begin{bmatrix}
  0 & \cdots & 0 & y_1'' & y_{n+1}'' & \cdots \\
  0 & \cdots & 0 & r_{n+1}'' & r_{n+1}'' & \cdots
\end{bmatrix} \leftarrow \text{last row}
\]
where the rotation factors are defined by [9]
\[ c_n \triangleq \frac{|y_n|}{\sqrt{\sum |y_n|^2}}, \quad s_n \triangleq \left( \frac{y_n}{|y_n|} \right) \cdot c_n. \] (81)

The correctness of (80) is shown in the Appendix. It is seen from (80) that the computed quantities appearing on the right-hand side are \( y_p(i), y_p(i), \) and \( y_p(i) \) at time \( i \). It is also shown in the Appendix that the multipath filter output at time \( i \)
\[ y_p(i) \triangleq W_p(i) r_p(i) \] is calculated recursively according to the following equations:
\[ y_p(i) = -\frac{\beta}{\xi_p(i)} y_p(i-1) + y_p(i) \] (82)
\[ \Gamma_p(i) = \frac{\beta}{\xi_p(i)} y_p(i-1) - \frac{\xi_p(i)^2}{1 + \xi_p(i)^2 y_p(i)} y_p(i) y_p(i)^H. \] (83)

B. Comparison with Single-User-Based Blind Space-Time Processing

Several single-user-based blind adaptive space-time processing methods have been proposed in the recent literature, e.g., [13], [22], [26], which are essentially adaptive versions of the single-user space-time algorithms discussed in Section IV-B. In [13] and [26], it is assumed that the users employ random spreading sequences, and this assumption is essential in deriving the corresponding single-user blind algorithms. In [22], a blind adaptive array method is proposed for interference rejection in CDMA systems employing short spreading codes. We next extend this method to the multipath channel and show further that if the system employs random spreading code, then this method is equivalent to the single-user space-time MMSE receiver.

Adapting to our notation, the method proposed in [22] can be extended to multipath channels as follows. Suppose we are to demodulate the \( l \)th symbol of the \( k \)th user. Assume that the receiver knows the \( k \)th user’s spreading waveform and multipath delays. Given the continuous-time received signal \( x(t) \) in (4), the receiver synchronizes with the desired signal from each path and samples the received signal at chip rate to obtain \( y(t) \) in \( \mathbb{C}^{P \times N} \).
\[ x(t) = [x(t_1) \cdots x(t_{N-1})], \quad l = 1, \cdots, L \] (84)
where
\[ y(t) = \int_{t-\tau_m}^{t+\tau_m+(n+1)\tau_c} z(t) \delta(t - \tau - \tau_m - n\tau_c) \, dt \] (85)

Denote \( X(i) = [x(i)^T \cdots x(i)^T]^T \in \mathbb{C}^{L \times P \times N} \). For a given combining weight \( w_k \in \mathbb{C}^{L \times P} \), the signal \( X(i) \) is spatially combined and then temporally correlated with the desired user’s spreading sequence to yield the decision statistic \( z(i) = w_k^H X(i) s_k \), where \( s_k \) is the normalized signature sequence of the \( k \)th user \( s_k = (1/\sqrt{N}) [s_k(0) \cdots s_k(N-1)]^T \) [cf. (2)]. In order to blindly adapt the combining vector \( w_k \), an error signal is defined as \( e(i)^T = w^H X(i) - z(i)s_k^T \).

The weight vector \( w_k \) is then chosen as the solution to the following optimization problem:
\[ \min_{w \in \mathbb{C}^{L \times P}} E\{|e(i)|^2\}, \quad \text{s.t.} \ E\{|z(i)|^2\} = 1. \] (86)

The solution to the above optimization problem is given by the generalized eigenvector corresponding to the minimum eigenvalue of the matrix pencil \( (R_x, R_{xz}) \)
\[ R_x = E\{X(i)X(i)^H\}, \quad \text{and} \quad R_{xz} = E\{X(i)s_k X(i)^H\}. \] In Section V-C, the performance of this single-user-based blind space-time method is compared with the multiuser space-time blind algorithm proposed in Section V-A.

Finally, we point out that if the users employ random spreading sequences, then the above algorithm is equivalent to the single-user MMSE method in (41). To see this, note that under the condition of random spreading sequence, we have [3], [13]
\[ R_x = A_2^2\hat{p}_k\hat{p}_k^H + R_f \] (88)
\[ R_{xz} = A_2^2\hat{p}_k\hat{p}_k^H + \frac{1}{N} R_f \] (89)
where \( \hat{p}_k = [g_{kl} a_{kl}^T \cdots g_{kl} a_{kl}^T]^T \) is the desired user’s channel response, and where \( R_f \) denotes the autocorrelation matrix of the interference plus noise. [In fact, the methods in [3] and [13] compute the desired user’s channel response \( \hat{p}_k \) based on (88) and (89), i.e., \( \hat{p}_k \) is the principal eigenvector of the matrix \( (NR_{xz} - R_x) \).] Now substituting (88) and (89) into (87), we have
\[ \left( 1 - \frac{1}{N} \right) R_f w \]
\[ = (\lambda - 1) A_2^2 \hat{p}_k^H w_k \]
\[ \lambda = N \]
\[ \text{if } w \text{ satisfies } \hat{p}_k^H w = 0 \]
\[ \Rightarrow \lambda = \frac{1 + A_2^2 R_f^{-1} \hat{p}_k}{1 + A_2^2 R_f^{-1} \hat{p}_k} < N \] (91)
Hence, the optimal weight vector is given by \( w_k = \alpha R_f^{-1} \hat{p}_k \).
Now, using (88) and the matrix inversion lemma, it follows that \( w_k = \alpha R_x^{-1} \hat{p}_k \) (with scalar constant \( \alpha' \)), which is the MMSE single-user combining vector [3], [13].

C. Simulation Examples

In this subsection, we assess the performance of the various blind multiuser and single-user space-time processing methods discussed above by computer simulations. The simulated CDMA system is the same as described in Table I of Section IV-D.
Example 6—Performance of the Batch Algorithm for Blind Space-Time Multiuser Detection: First, we illustrate the performance of the batch algorithm in Table II for blind space-time multiuser detection. The number of symbols per frame is $M = 250$. The bit error rate performance for users 3 and 8 is shown in Fig. 7. The performance of the single-user matrix pencil blind method discussed in Section V-C is also plotted in the same figure. It is clear that the proposed blind method significantly outperforms the single-user blind array method.

Example 7—Performance of the LMS Blind Adaptive Space-Time Multiuser Detector: Next, we illustrate the performance of the LMS version of the adaptive blind space-time multiuser detector discussed in Section V-B1. The user of interest is user 8. The step size of the first 250 iterations is $\mu = 0.01$, for the next 250 iterations $\mu = 0.005$, and finally, the step size becomes $\mu = 0.002$. The convergence behavior of the multipath filter is shown in Fig. 8(a), where the total mean output energy $|y(i)|^2$ is plotted against the number of iterations. The theoretical minimum mean output energy, which is given by $\sum_{i=1}^{P}(S^T C_i^{-1} S)^{-1}$ [cf. (59) and (60)], is also plotted in the figure as the dashed line. The steady-state bit error rate performance is plotted in Fig. 8(b). Here, error is counted for consecutive 2000 iterations after the initial 500 iterations and then averaged over 200 independent runs.

Example 8—Performance of the RLS Blind Adaptive Space-Time Multiuser Detector: Finally, we illustrate the performance of the QR-RLS version of the adaptive blind space-time multiuser detector discussed in Section V-B2. The user of interest is again user 8. The forgetting factor is $\beta = 0.998$. The convergence behavior of the multipath filter is shown in Fig. 9(a), where the total mean output energy $|y(i)|^2$ is plotted against the number of iterations. The steady-state bit error rate performance is plotted in Fig. 9(b).
Fig. 9. Performance of the RLS blind adaptive space-time multiuser detector. (a) Convergence of the multipath filters (the dashed line corresponds to the theoretical minimum mean output energy). (b) Bit error rate in the steady state.

Fig. 10. Space-time multiuser receiver structure.

VI. CONCLUSIONS

In this paper, we have considered the problem of space-time multiuser detection in multipath CDMA channels with receiver array antennas. First, we have derived the multiuser space-time receiver structure based on sufficient statistic, which is illustrated in Fig. 10. It is seen that the front end of the receiver consists of a bank of matched filters, followed by a bank of array combiners, and then followed by a bank of multipath combiners, which produces the sufficient statistic. The maximum likelihood multiuser sequence detector and the linear state bit error rate performance is plotted in Fig. 9(b). Here, error is counted for consecutive 2000 iterations after the initial 200 iterations and then averaged over 200 independent runs.
multicarrier detection based on serial iterative interference cancellation, are derived. Note that since the detection algorithms in Fig. 10 operate on the sufficient statistic, their complexities are functions of only the number of users ($K$) and the length of the data block ($M$) but not of the number of antennas ($P$), or the number of paths ($L$). We have then developed blind adaptive space-time multiuser detection techniques that require the prior knowledge of only the signature waveform and the timings of the desired user. Simulation results demonstrate that the proposed nonadaptive and adaptive multiuser space-time processing methods can substantially outperform the single-
user-based space-time methods.

**APPENDIX**

**DERIVATION OF (80)–(83)**

Suppose an application of the rotation matrix $Q(\hat{\tau})$ yields the following partitioned matrix form:

$$Q(\hat{\tau})[A_1A_2] = [B_1B_2].$$  \hfill (92)

Then, because of the orthonormal property of $Q(\hat{\tau})$, i.e., $Q(\hat{\tau})^HQ(\hat{\tau}) = I$, taking the outer products of each side of (92) with their respective transposes, we get the following identities:

$$A_1^HA_1 = B_1^HB_1$$ \hfill (93)
$$A_2^HA_2 = B_2^HB_2$$ \hfill (94)
$$A_2^HA_1 = B_2^HB_1$$ \hfill (95)

Associating $A_1$ with the first $N$ columns of the partitioned matrix on the left-hand side of (80) and $B_1$ with the first $N$ columns of the partitioned matrix on the right-hand side of (80), (93)–(95) then yield the following:

$$B_2(\hat{\tau})^HB_2(\hat{\tau}) = \beta B_2(\hat{\tau})^H(\hat{\tau} - 1) + r_p(\hat{\tau})^HR_p(\hat{\tau})$$  \hfill (96)
$$B_2(\hat{\tau})^HY_p(\hat{\tau}) = B_2(\hat{\tau})^H(\hat{\tau} - 1)Y_p(\hat{\tau} - 1)$$  \hfill (97)
$$\beta Y_p(\hat{\tau})H Y_p(\hat{\tau}) = r_p(\hat{\tau})$$  \hfill (98)
$$\beta Y_p(\hat{\tau})H Y_p(\hat{\tau}) + \eta_p(\hat{\tau}) = Y_p(\hat{\tau} - 1)^H Y_p(\hat{\tau} - 1)$$  \hfill (99)
$$\eta_p(\hat{\tau})^HY_p(\hat{\tau}) + \eta_p(\hat{\tau})\xi_p(\hat{\tau}) = O_L$$  \hfill (100)
$$y_p(\hat{\tau})^HY_p(\hat{\tau}) + |\xi_p(\hat{\tau})|^2 = 1.$$  \hfill (101)

A comparison of (96)–(98) with (77)–(79) shows that $B_2(\hat{\tau})$, $Y_p(\hat{\tau})$, and $Y_p(\hat{\tau})$ in (80) are the correct updated quantities at time $\eta$. Furthermore, by direct calculation, we have $\xi_p(\hat{\tau}) = \prod_n c_n$, where $c_n$’s are the rotation factors defined in (81). Therefore, $\xi_p(\hat{\tau})$ is real.

The output of the multipath filter at time $\hat{\tau}$ is computed as follows:

$$y_p(\hat{\tau}) = W_p(\hat{\tau} - 1)^H r_p(\hat{\tau})$$
$$= \Gamma_p(\hat{\tau} - 1)^{-1}S^H C_p(\hat{\tau} - 1)^{-1} r_p(\hat{\tau})$$
$$= \Gamma_p(\hat{\tau} - 1)^{-1}S^H \left[ C_p(\hat{\tau}) - \frac{1}{\beta} r_p(\hat{\tau})r_p(\hat{\tau})^H \right]^{-1} r_p(\hat{\tau})$$
$$= \beta \Gamma_p(\hat{\tau} - 1)^{-1}S^H \left[ C_p(\hat{\tau})^{-1} \right] r_p(\hat{\tau})$$
$$+ \frac{1}{1 - r_p(\hat{\tau})H C_p(\hat{\tau})^{-1} r_p(\hat{\tau})} \cdot C_p(\hat{\tau})^{-1} r_p(\hat{\tau})r_p(\hat{\tau})^H C_p(\hat{\tau})^{-1} r_p(\hat{\tau})$$
$$= \frac{\beta}{\xi_p(\hat{\tau})^2} \Gamma_p(\hat{\tau} - 1)^{-1}S^H C_p(\hat{\tau})^{-1} r_p(\hat{\tau}) \hfill (102)$$
$$= - \frac{\beta}{\xi_p(\hat{\tau})} \Gamma_p(\hat{\tau} - 1)^{-1} \eta_p(\hat{\tau}) \hfill (103)$$

where (102) follows from the fact that $r_p(\hat{\tau})H C_p(\hat{\tau})^{-1} r_p(\hat{\tau}) = Y_p(\hat{\tau})Y_p(\hat{\tau})$ and (101), and (103) follows from the fact that $S^H C_p(\hat{\tau})^{-1} r_p(\hat{\tau}) = Y_p(\hat{\tau})^H Y_p(\hat{\tau})$ and (100). By definition, $\Gamma_p(\hat{\tau}) = S^H C_p(\hat{\tau})^{-1} S = Y_p(\hat{\tau})^H Y_p(\hat{\tau})$. From (99), we have $\Gamma_p(\hat{\tau}) = \Gamma_p(\hat{\tau} - 1)/\beta - \eta_p(\hat{\tau})^HY_p(\hat{\tau})^H$, which together with (103) leads to (83).

**REFERENCES**


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