MHD Natural Convection Inside an Inclined Trapezoidal Porous Enclosure with Internal Heat Generation or Absorption Subjected to Isoflux Heating

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A steady laminar two-dimensional magneto-hydrodynamic natural convection flow in an inclined trapezoidal enclosure filled with a fluid-saturated porous medium is investigated numerically using a finite difference method. The left and right vertical sidewalls of the trapezoidal enclosure are maintained at a cold temperature. The horizontal top wall is considered adiabatic while the bottom wall is subjected to isoflux heating. A volumetric internal heat generation or absorption is embedded inside the trapezoidal enclosure while an external magnetic field is applied on the left sidewall of the enclosure. In the current work, the following parametric ranges of the non-dimensional groups are used: Hartmann number is varied as $0 \leq Ha \leq 50$, Darcy number is taken as $Da = 10^{-3}, 10^{-4},$ and $8 \times 10^{-5}$, Rayleigh number is varied as $10^3 \leq Ra \leq 10^5$, Prandtl number is considered constant at $Pr = 0.7$, the dimensionless internal heat generation or absorption parameter is varied as $\Delta = -0.2, 0, 1,$ and $2.0$, while the trapezoidal enclosure inclination angle is varied as $0^\circ \leq \psi \leq 90^\circ$. The results indicated a strong flow circulation occurs when the Darcy and the Rayleigh numbers are high. In addition, it is found that the Hartmann number, internal heat generation or absorption parameter and inclination angle have an important role on the flow and thermal characteristics. It is also found that when the enclosure inclination angle and Hartmann number increase the average Nusselt number along the hot bottom wall decreases.


Key words: natural convection, magneto-hydrodynamics, heat generation or absorption, trapezoidal enclosure, porous medium, Darcy model, isoflux heating

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1. Introduction

Natural convection heat transfer in fluid-saturated porous media has received much attention during the past decades. This attention has been supported by many engineering applications, for example, heat exchange between soil and atmosphere, high performance insulation for buildings, chemical catalytic reactors, food processing, electrochemistry, metallurgy, grain storage, cooling of radioactive waste containers, and compacted beds for the chemical industry. Porous media are also of interest in relation to the underground spread of pollutants, solar power collectors, and geothermal energy systems [1]. Previous studies related to natural convection heat transfer in various enclosures filled with a porous media are reported in the literature by Nithiarasu et al. [2], Baytas and Pop [3], Kim et al. [4], Pakdee and Rattanadecho [5], Basak et al. [6], Pourshaghaghy et al. [7], and Sathiyamoorthy et al. [8]. One of the basic problems considering heat transfer in porous media is the natural convection in a differentially heated square or rectangular enclosure, whose vertical sidewalls are maintained at different constant temperatures while the top and bottom walls are well insulated. In fact, actual enclosures occurring in practice often have a geometry different from a rectangular shape.

For example, various panels of electronic equipment and solar energy collectors are of non-rectangular form. For this reason, many studies begin to appear in the literature which deal with natural convection heat transfer with and without porous media for non-rectangular enclosures like triangular, trapezoidal, and parallelogram-shaped enclosures. Special attention has to be focused on trapezoidal enclosures because the selection of a trapezoidal shape enclosure is very useful for enhancing the heat transfer rate due to its extended top surface. Peric [9] investigated numerically natural convection in trapezoidal cavities with a series of systematically refined grids from \((10 \times 10)\) to \((160 \times 160)\) control volume and observed the convergence of results for grid-independent solutions. Kuyper and Hoogendoorn [10] studied laminar natural convection flow in trapezoidal enclosures to study the influence of the inclination angle on the flow and also the dependence of the average Nusselt number on the Rayleigh number. Baytas and Pop [11] performed a numerical study of the steady natural convection heat transfer within an inclined trapezoidal enclosure filled with a fluid-saturated porous medium. The top cylindrical surface of the enclosure was cooled while the bottom cylindrical surface was heated. The remaining two non-parallel plane sidewalls of the enclosure were considered adiabatic. The Darcy and energy equations (in non-dimensional stream function and temperature formulation) were solved numerically using the alternative direction implicit (ADI) finite-difference method. Flow and heat transfer characteristics (stream lines, isotherms, and average Nusselt numbers) were investigated for a wide range of values of the Rayleigh number, inclined angle, and cavity aspect ratio. Mahmud and Fraser [12] examined the flow, temperature, and entropy generation fields inside a square porous cavity under the influence of a magnetic field using the Darcy model. The momentum equation including the Navier–Stokes inertia term and Brinkman viscous diffusion term were derived for the porous media in the presence of a magnetic field. Mohamed [13] studied numerically the laminar natural heat and mass transfer in a symmetrical trapezoidal enclosure. The base and ceiling were isothermal and iso-concentration surfaces, while the lateral walls were considered adiabatic and impermeable. Both aiding and opposing buoyancy forces had been studied. The investigation was made for a wide range of buoyancy ratio \((N)\), \(-1 \leq N \leq 10\), inclination angle \((\varphi)\), \(0^\circ \leq \varphi \leq 18.44^\circ\), Lewis number \((Le)\), \(1 \leq Le \leq 5\) and thermal Grashof number \((Gr_T)\), \(2 \times 10^3 < Gr_T < 5 \times 10^3\) with a fixed aspect ratio \((A)\), at \(A = 3\), and Prandtl number \((Pr)\), at \(Pr = 0.7\). The effects of the Lewis number, buoyancy ratio, and thermal Grashof number on both the average Nusselt and average Sherwood
numbers were studied. In addition, the average Nusselt and Sherwood numbers were correlated in terms of buoyancy ratio and Lewis number.

Natarajan et al. [14] used a penalty finite element analysis with bi-quadratic elements to investigate the influence of uniform and non-uniform heating of a bottom wall on natural convection flows in a trapezoidal cavity. The bottom wall was uniformly and non-uniformly heated while the two vertical walls were maintained at a constant cold temperature and the top wall was well insulated. Parametric study for a wide range of Rayleigh numbers \((Ra)\), \(10^3 \leq Ra \leq 10^5\) and Prandtl numbers \((Pr)\), \(0.07 \leq Pr \leq 100\) showed consistent performance of their numerical approach to obtain the solutions in terms of stream functions and temperature profiles. They concluded that non-uniform heating of the bottom wall produced a greater heat transfer rate at the center of the bottom wall than the uniform heating case for all Rayleigh numbers but the average Nusselt number showed overall lower heat transfer rate for the non-uniform heating case.

Basak et al. [15] studied numerically the phenomena of natural convection in a trapezoidal enclosure filled with a porous matrix. A penalty finite element analysis with bi-quadratic elements was performed to investigate the influence of uniform and non-uniform heating of the bottom wall while two vertical walls were maintained at a constant cold temperature and the top wall was well insulated. The parametric study for a wide range of Rayleigh numbers, \(10^3 \leq Ra \leq 10^5\), Prandtl numbers, \(0.015 \leq Pr \leq 1000\) and Darcy numbers, \(10^{-3} \leq Da \leq 10^{-5}\) explained the consistent performance of their numerical approach to obtain the solutions in terms of stream function and isotherm contours. A symmetry was observed for temperature and flow simulations. They concluded that non-uniform heating of the bottom wall produced a greater heat transfer rate at the center of the bottom wall than the uniform heating case for all Rayleigh and Darcy numbers but the average Nusselt number showed an overall lower heat transfer rate for the non-uniform heating case. It was observed that the conduction was dominant irrespective of Rayleigh number for \(Da = 10^{-5}\). As the Rayleigh number increased, there was a change from a conduction-dominant region to a convection-dominant region for \(Da = 10^{-3}\). The correlations between average Nusselt number and three parameters [Rayleigh number \((Ra)\), Prandtl number \((Pr)\), and Darcy number \((Da)\)] were also obtained. Varol et al. [16] conducted a numerical study to investigate the steady free convection flow in a two-dimensional right angle trapezoidal enclosure filled with a fluid-saturated porous medium. The left vertical wall of the cavity was heated; the inclined wall was partially cooled; and the remaining walls were considered insulated. Three different cases were considered. While in Case I, the cooler wall was located adjacent to the top wall, in Case II, it was located in the middle inclined wall. In Case III, it was located adjacent to the bottom wall. Flow and heat transfer characteristics were studied for a range of parameters: the Rayleigh number, \(100 \leq Ra \leq 1000\); and the aspect ratio, \(AR = 0.25, 0.50\), and \(0.75\). Numerical results indicated that there existed significant changes in the flow and temperature fields as compared with those of a differentially heated square porous cavity. They concluded the position of minimum heat transfer across the cavity. This is of great interest in the thermal insulation of buildings and other areas of technology. From the other side, the natural convection problem of a fluid-saturated porous media in an enclosure in the presence of a magnetic field was studied numerically by several researchers. Khanafer and Chamkha [17] investigated numerically hydro-magnetic natural convection heat transfer in an inclined square enclosure filled with a fluid-saturated porous medium with heat generation. They concluded that the magnetic field and the porous medium reduced the heat transfer and fluid circulation within the cavity.
Wang et al. [18] numerically simulated natural convection of a fluid in an inclined enclosure filled with porous medium in a high magnetic field. The physical model was heated from the left-hand side vertical wall and cooled from the opposing wall. Above their enclosure an electric coil was set to generate a magnetic field. The Brinkman–Forchheimer extended Darcy model was used to solve the momentum equations, and the energy equations for fluid while solid were solved with the local thermal non-equilibrium (LTNE) model. Calculations were performed for a range of the Darcy number from $10^{-5}$ to $10^{-1}$, inclination angles from 0 to 90°, and a magnetic force parameter from 0 to 100. They concluded that both the magnetic force and the inclination angle had a significant effect on the flow field and heat transfer in porous medium. Akbala and Bayta [19] investigated the effects of non-uniform porosity on double diffusive natural convection in a porous cavity with a partially permeable wall, while Grosan et al. [20] studied the effects of a magnetic field and internal heat generation on the free convection in a rectangular cavity filled with a porous medium. Zeng et al. [21] numerically investigated natural convection in an enclosure filled with a diamagnetic fluid-saturated porous medium under a strong magnetic field. The Brinkman–Forchheimer extended Darcy model was used to solve the momentum equations, and the energy equations for fluid while the solids were solved with the local thermal non-equilibrium (LTNE) model. Computations were performed for a range of Darcy number from $10^{-6}$ to $10^{-3}$, Rayleigh number from $10^{3}$ to $10^{6}$, and magnetic force parameter from –200 to 0. The results showed that the magnetic force had a significant effect on the flow field and heat transfer in a diamagnetic fluid-saturated porous medium. Mansour et al. [22] performed a numerical investigation of unsteady magneto-hydrodynamic free convection in an inclined square cavity filled with a fluid-saturated porous medium and with internal heat generation. A uniform magnetic field inclined with the same angle of the inclination of the cavity was applied. The governing equations were formulated and solved by a direct explicit finite-difference method subject to appropriate initial and boundary conditions. Two cases were considered: the first case when all the cavity walls were cooled and the second case when the cavity vertical walls were kept adiabatic. The results explained the influence of the Hartmann number, the Rayleigh number, the inclination angle of the cavity, and the dimensionless time parameter on the flow and heat transfer characteristics such as the streamlines, isotherms, and the average Nusselt number. The velocity components at the mid-section of the cavity as well as the temperature profiles were reported graphically.

The values of average Nusselt number for various parametric conditions were also presented in tabular form. Mamun et al. [23] investigated numerically by using the finite element method the effects of magnetic force, acting vertically downward on natural convection within a porous trapezoidal enclosure saturated with an electrically conducting fluid. The bottom wall of the enclosure was subjected to a constant hot temperature and the top wall experienced a constant cold temperature whereas the remaining sidewalls were kept adiabatic. The physical problems were represented mathematically by different sets of governing equations along with the corresponding boundary conditions. For natural convection in a porous medium, the influential parameters are the modified Rayleigh number ($R_{a_m}$), the fluid Rayleigh number ($R_{a_f}$), the inclination angle of the sidewalls of the cavity, the rotational angle of the enclosure ($\Phi$), and the Hartmann number ($Ha$), through which different thermo-fluid characteristics inside the enclosure were obtained. The results showed that with an increasing Hartmann number, the diffusive heat transfer became prominent even though the modified Rayleigh number increased. Optimum heat transfer rate was obtained at higher values of modified Rayleigh number in the absence of magnetic force. Very recently, Ashorynejad et al. [24] presented a numerical study of the magneto-hydrodynamic (MHD) flow in a square cavity filled with porous medium by using the lattice Boltzmann method (LBM). The left and right vertical walls of
the cavity were kept at constant but different temperatures while both the top and bottom horizontal walls were insulated. The results showed that heat and mass transfer mechanisms and the flow characteristics inside the enclosure depended strongly on the strength of the magnetic field and Darcy number. They concluded that the average Nusselt number decreased with rising values of the Hartmann number while it increased with increasing values of the Darcy number. However, to our best knowledge, magneto-hydrodynamics natural convection flow in an inclined trapezoidal enclosure filled with a fluid-saturated porous medium with both effects of magnetic field and internal heat generation or absorption has not been considered in any paper previously. Thus, the main objective of this paper is to examine this problem in detail. The studied problem is not easy to study because of the sloping walls of the trapezoidal enclosure. In general, the mesh nodes do not lie along the sloping walls and consequently from a program-using and computational point of view, the effort required to simulate the flow and heat characteristics increases significantly.

**Nomenclature**

- $B_0$: magnitude of the magnetic field, Tesla
- $C_p$: specific heat at constant pressure, $\text{j/kg } ^\circ\text{C}$
- $Da$: Darcy number
- $g$: gravitational acceleration, $\text{m/s}^2$
- $H$: height of the trapezoidal enclosure, $\text{m}$
- $Ha$: Hartmann number
- $K$: permeability of the saturated porous media, $\text{m}^2$
- $k$: fluid thermal conductivity, $\text{W/m } ^\circ\text{C}$
- $n$: the normal direction with respect to the inclined left side wall
- $L$: length of the trapezoidal enclosure, $\text{m}$
- $Nu$: average Nusselt number
- $P$: dimensionless pressure
- $p$: pressure, $\text{N/m}^2$
- $Pr$: Prandtl number
- $Q_o$: volumetric internal heat generation or absorption, $\text{W/m}^3$
- $q$: heat flux, $\text{W/m}^2$
- $Ra$: Rayleigh number
- $T$: temperature, $^\circ\text{C}$
- $t$: time, $\text{s}$
- $U$: dimensionless velocity component in $x$-direction
- $u$: dimensional velocity component in $x$-direction, $\text{m/s}$
- $V$: dimensionless velocity component in $y$-direction
- $v$: dimensional velocity component in $y$-direction, $\text{m/s}$
- $X$: dimensionless coordinate in horizontal direction
- $x$: Cartesian coordinate in horizontal direction, $\text{m}$
- $Y$: dimensionless coordinate in vertical direction
- $y$: Cartesian coordinate in vertical direction, $\text{m}$

**Greek Symbols**

- $\alpha$: thermal diffusivity, $\text{m}^2/\text{s}$
2. Mathematical Modeling

2.1 Governing equations and geometrical configuration

The magneto-hydrodynamic natural convection flow of a trapezoidal enclosure of height \( H \) and length \( L \) filled with a fluid-saturated porous medium is considered, as shown in Fig. 1 along with the important geometric parameters. The vertical left and right sidewalls of the trapezoidal enclosure are maintained at constant cold temperatures \( T_c \) while the horizontal top and bottom walls are considered adiabatic and subjected to a constant heat flux \( q \), respectively. A volumetric internal heat generation \( (Q_0 > 0) \) or absorption \( (Q_0 < 0) \) is embedded inside the trapezoidal enclosure while
an external magnetic field of magnitude \((B_o)\) is applied on the left sidewall of the enclosure parallel to the heated bottom wall and at an angle of \((\Phi)\) normal to the inclined sidewall. The Hartmann number is varied as \(0 \leq Ha \leq 50\), the Darcy number is taken as \(Da \leq 10^{-3}, 10^{-4}\), and \(8 \times 10^{-5}\), the Rayleigh number is varied as \(10^{3} \leq Ra \leq 10^{5}\) to cover both buoyancy and magnetic field dominant flow regimes, the Prandtl number is considered fixed at \(Pr = 0.7\), the dimensionless internal heat generation or absorption parameter are varied as \(\Delta = -0.2, 0, 1,\) and \(2.0\) while the trapezoidal enclosure inclination angle is varied as \(0^\circ \leq \psi \leq 90^\circ\), respectively. In the present work, the following assumptions are utilized:

- Two-dimensional laminar flow, i.e., variation in \(z\)-direction is neglected.
- Steady-state condition, i.e.,
  \[
  \frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} = 0
  \]
- Incompressible flow, i.e., \(\rho = \text{constant}\).
- Gravity acts vertically downward.
- Electrically conducting Newtonian fluid.
- Neglecting Forschheimer’s inertia term.
- Viscous dissipation effects are considered to be neglected.
- The flow properties are considered constant except for the density variation which is modeled according using Boussinesq approximation.
- Radiation and Joule heating neglected.
- Small magnetic Reynolds number, hence the induced magnetic field is negligible compared to applied magnetic field.

The flow and thermal fields inside the trapezoidal enclosure are modeled by the Navier–Stokes and the energy equations, respectively, which are given in a dimensional structure as follows [25, 26]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(2)

\[
\frac{1}{\rho} \frac{\partial \rho}{\partial x} + \frac{1}{\rho} \frac{\partial \rho}{\partial y} = - \frac{1}{\rho} \frac{\partial \rho}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + g \beta \left( T - T_{\text{ref}} \right) \sin(\psi) - \frac{v}{K}
\]

(3)

\[
\frac{1}{\rho} \frac{\partial \rho}{\partial x} + \frac{1}{\rho} \frac{\partial \rho}{\partial y} = - \frac{1}{\rho} \frac{\partial \rho}{\partial x} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + g \beta \left( T - T_{\text{ref}} \right) \cos(\psi) - \frac{v}{\rho - \sigma_e B_o^2 \nu}
\]

(4)

\[
\frac{1}{\rho C_p} \frac{\partial \rho}{\partial x} + \frac{1}{\rho C_p} \frac{\partial \rho}{\partial y} = \frac{k}{\rho C_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q_0}{\rho C_p} \left( T - T_{\text{ref}} \right)
\]

(5)

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The non-dimensional form of the governing equations are given by

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]
\[
\frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + Ra Pr \frac{\partial \theta}{\partial Y} \sin(\theta) - \frac{Pr}{Da} U
\]
\[
\frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra Pr \frac{\partial \theta}{\partial X} \cos(\theta) - \frac{Pr}{Da} V - H \frac{Pr}{Da} V
\]
\[
\frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + \Delta \theta
\]

The dimensionless boundary conditions of the trapezoidal enclosure are expressed as follows:

1. The left vertical inclined sidewall is maintained at constant cold temperature \( T_c \) and subjected to an external magnetic field of magnitude \( B_o \) so that

\[
X + Y \tan \Phi = 0, \ 0 \leq Y \leq 1 \Rightarrow U = V = \Theta = 0
\]

2. The right vertical inclined sidewall is maintained at constant cold temperature \( T_c \) so that

\[
X - Y \tan \Phi = \tan \Phi, \ 0 \leq Y \leq 1 \Rightarrow U = V = \Theta = 0
\]

3. The upper horizontal wall of the trapezoidal enclosure is considered adiabatic, so that

\[
Y = 1, - \tan \Phi \leq X \leq 1 + \tan \Phi \Rightarrow U = V = \frac{\partial \Theta}{\partial Y} = 0
\]

4. The lower horizontal wall of the trapezoidal enclosure is subjected to a constant heat flux, so that

\[
Y = 0, 0 \leq X \leq 1 \Rightarrow U = V = \Theta = \sin \pi X
\]

The previous non-dimensional governing equations are produced by using the following non-dimensional variables:

\[
\left[ X, Y \right] = \left[ \frac{x}{L_{ref}}, \frac{y}{L_{ref}} \right]; \left[ U, V \right] = \left[ \frac{u}{u_{ref}}, \frac{v}{u_{ref}} \right]; P = \frac{p}{p_{ref}}; \Theta = \frac{T - T_{ref}}{\Delta T} \text{ where, } \Delta T = \frac{q L_{ref}}{k}
\]

and the above set of non-dimensional numbers are defined as follows:

\[
Pr = \frac{\nu}{a} = \frac{\rho C_p \nu}{k}; \quad Ra = \frac{g \beta \Delta T L_{ref}^3}{\nu a}; \quad H = \frac{B_o L_{ref} \sqrt{\sigma_e}}{\sqrt{\beta \nu}}; \quad Da = \frac{K L_{ref}^2}{\rho \sigma a C_p}
\]

By considering the following definition of non-dimensional stream function and vorticity

\[
U = \frac{\partial \psi}{\partial Y}; V = - \frac{\partial \psi}{\partial X}; \Omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} = - \nabla^2 \psi
\]

and differentiating Eqs. (7) and (8) with respect to \( Y \) and \( X \), respectively, and subtracting them gives the following equation:

\[
\frac{\partial \psi}{\partial Y} \frac{\partial \Omega}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \Omega}{\partial Y} = Pr \left[ \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right] + Ra Pr \left[ \cos(\psi) \frac{\partial \Theta}{\partial X} - \sin(\psi) \frac{\partial \Theta}{\partial Y} \right] - H a^2 Pr \frac{\partial V}{\partial X} - \Omega \frac{Pr}{\Theta}
\]
The local Nusselt number along the heated wall is defined by

\[ Nu = -\frac{\partial \theta}{\partial Y} \bigg|_{Y=0} \]  \hspace{1cm} (17)

The average Nusselt number now can be written as

\[ Nu_{av} = \frac{1}{L} \int_{0}^{L} Nu(X) dX \]  \hspace{1cm} (18)

3. Numerical Method and Verification

The dimensionless governing equations, Eqs. (9)–(15), and (16), together with the boundary conditions, Eqs. (10)–(13), were discretized using a finite difference method. Central difference quotients were used to approximate the derivatives in both the X- and Y-directions. The resulting system of discretized equations was solved iteratively. As convergence criteria, $10^{-6}$ is chosen for all dependent variables and the value of 0.1 is used for the under-relaxation parameter. The number of grid points is taken as $129 \times 61$ with uniform spaced mesh in both X- and Y-directions. The average Nusselt numbers at the bottom wall have been compared with the corresponding results reported by Basak et al. [15] as shown in Table 1. The comparison is made using the following dimensionless data: $Pr = 0.7$, $Da = 10^{-3}$, $Ha = \Delta = 0$, $\psi = 0^\circ$. A good agreement is obtained between the present computation and the previous values obtained by Basak et al. [15], which validates the current computations indirectly. Further verification is performed by using the present numerical algorithm to simulate the same problem considered by Basak et al. [15] using the same geometry and boundary conditions for laminar natural convection flow in a trapezoidal enclosure filled with porous medium. The comparison for both streamlines and isotherms is shown in Fig. 2 using the following dimensionless parameters: $Pr = 0.7$, $Da = 10^{-3}$, $Ra = 10^5$, $Ha = \Delta = 0$, $\psi = 0^\circ$. Again an excellent agreement is obtained. Therefore, the computational procedure is practicable and can predict magneto-hydrodynamic natural convective flow in an inclined trapezoidal enclosure; thus, the previous verification indicates the promise of the present numerical model for dealing with recent physical problems.

4. Results and Discussion

Steady-state two-dimensional laminar magneto-hydrodynamics natural convection flow in a trapezoidal enclosure filled with a fluid-saturated porous medium has been studied numerically. Parametric studies of the influence of various parameters such as the Hartmann number, Rayleigh number, Darcy number, enclosure inclination angle, and dimensionless heat generation or absorption parameter on the fluid flow and heat transfer have been performed. Flow and heat transfer characteristics, streamlines, isotherms, and local and average Nusselt number are presented and discussed.

<table>
<thead>
<tr>
<th>$Ra$</th>
<th>Present</th>
<th>Basak et al. [15]</th>
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<tbody>
<tr>
<td>$8 \times 10^3$</td>
<td>1.983972</td>
<td>1.933142</td>
</tr>
<tr>
<td>$10^4$</td>
<td>2.002537</td>
<td>1.954831</td>
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Table 1. Comparison of Average Nusselt Number at the Bottom Wall at $Pr = 0.7$, $Da = 10^{-3}$, $Ha = \Delta = 0$, $\psi = 0^\circ$
in this section. In all the obtained results, the trapezoidal enclosure with $\Phi = 30^\circ$ (Fig. 1) has been considered.

### 4.1 Effect of enclosure inclination angle

Figure 3 shows the streamlines (left) and isotherms (right) for different Hartmann numbers ($Ha = 0, 25, \text{ and } 50$) and inclination angles ($\psi = 0^\circ, 30^\circ, 60^\circ, \text{ and } 90^\circ$) at $Ra = 10^5$, $Da = 10^{-3}$, $Pr = 0.7$, and $\Delta = 1.0$. These figures demonstrate that the inclination angle of the trapezoidal enclosure has an important role in the flow and thermal fields. When the inclination angle of the trapezoidal enclosure is zero ($\psi = 0^\circ$), the flow and thermal fields are governed by the effects of the Hartmann number, Darcy number, and Rayleigh number. The effects of these parameters will be discussed later.

Fig. 2. Comparison of streamlines and isotherms at $Pr = 0.7$, $Da = 10^{-3}$, $Ra = 10^5$, $Ha = \Delta = 0$, $\psi = 0^\circ$.

Fig. 3. Streamlines and isotherms for various $Ha$ at $\psi = 0^\circ$, $Ra = 10^5$, $Da = 10^{-3}$, $Pr = 0.7$, and $\Delta = 1.0$. 

<table>
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<tr>
<th>Present</th>
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<tr>
<td><img src="image1.png" alt="Image" /></td>
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in detail. From these figures, it can be observed that for the same Hartmann number \((Ha = 25)\), the strength of flow circulation and isotherm contours decrease as the enclosure inclination angle increases from \((\psi = 30^\circ)\) in Fig. 4 to \((\psi = 90^\circ)\) in Fig. 6. Moreover, the size of the re-circulating vortices diminish as the inclination angle increases until it becomes one vortex when \(\psi = 90^\circ\). From the other side, the isotherms begin to move towards the enclosure center as the inclination angle increases.

4.2 Effect of Hartmann number

Figures 3, 4, 5, and 6 illustrate the streamlines (left) and isotherms (right) for various Hartmann numbers \((Ha = 0, 25, \text{ and } 50)\) and inclination angles \((\psi = 0^\circ, 30^\circ, 60^\circ, \text{ and } 90^\circ)\) at \(Ra = 10^5, Da = 10^{-3}, Pr = 0.7, \text{ and } \Delta = 1.0\). In general, the flow field can be represented by two re-circulating clockwise and counterclockwise vortices of higher strength adjacent to the cold left and right sidewalls of the trapezoidal enclosure. The flow field inside the enclosure is created adjacent to the hot bottom wall of the enclosure due to the constant heat flux effect and then moves adjacent the left cold sidewall and then impacts the adiabatic top wall which leads to a change in its direction to flow near the cold right sidewall. This repeated motion produces the primary two re-circulating vortices inside the trapezoidal enclosure. When the Hartmann number is zero \((Ha = 0)\) or when the effect of magnetic field is absent, the strength of circulation is very strong, since the buoyancy force due to natural convection effect is the only dominant force in the enclosure.

It can be observed that the streamlines are very crowded near the cold vertical left and right sidewalls since the convection heat transfer plays an important role in the absence of magnetic field effect. When the Hartmann number increases (i.e., \(Ha = 25\) and \(50\)), the Lorentz force which is created...
due to the magnetic field effect becomes greater than the buoyancy force which causes the flow circulation intensity to reduce, and as a result the convection effect begins to diminish. In fact, the existence of a magnetic field within the enclosure causes a force called the Lorentz force, which works opposite to the flow direction, and it resists the flow. Also, the streamlines begin to move towards the trapezoidal enclosure center and this movement increases as the Hartmann number increases. For this reason, it can be noticed from the results that the stream function values begin to decrease as the Hartmann number increases. With respect to isotherms, when the Hartmann number is zero ($H_a = 0$) or when the effect of magnetic field is negligible, the isotherms are highly concentrated adjacent to the bottom/base wall where the heat source (i.e., constant heat flux) exists in order to develop a thermal boundary layer. In this case the heat is transferred due to convection. As the Hartmann number increases (i.e., $H_a = 25$ and 50), the concentrated region of isotherms adjacent to the hot bottom wall becomes less compressed and the isothermal lines become smooth, merely symmetrical, and clearly parallel near the cold right and left sidewalls due to the increased effect of the magnetic field and in this case the heat is transferred inside the enclosure by conduction.

4.3 Effect of internal heat generation or absorption

Figure 7 explains the streamlines (left) and isotherms (right) for various dimensionless internal heat generation or absorption parameter ($\Delta$) at $\psi = 30^\circ$, $Ra = 10^5$, $Da = 10^{-3}$, $Pr = 0.7$, and $H_a = 25$. When there is no effect of internal heat generation or absorption (i.e., $\Delta = 0$), the flow and thermal fields are in general similar to the corresponding fields when the effect of buoyancy force is slight. When the dimensionless internal heat generation or absorption parameter ($\Delta$) increases (i.e., $\Delta = 2.0$), internal heat generation occurs inside the trapezoidal enclosure. In this case, the strength of circulation increases and both values of the stream function and isotherms contour increase. Furthermore, a high circulation and temperature can be found in the center of the trapezoidal enclosure due to an internal heat generation effect. The high temperature and flow circulation can also be observed adjacent to the cold left and right vertical sidewalls, because the hot fluid inside the enclosure cannot reject the extra heat adjacent to the left and right vertical sidewalls. But, when the dimensionless internal heat generation or absorption parameter ($\Delta$) decreases (i.e., $\Delta = -2.0$), the stream function and isotherms contour decrease. In this case, the absorption occurs inside the trapezoidal enclosure.

4.4 Effect of Rayleigh number

The streamlines (left) and isotherms (right) at various Rayleigh number ranging from $10^3$ to $10^5$ and at $\psi = 45^\circ$, $H_a = 25$, $Da = 10^{-3}$, $Pr = 0.7$, and $\Delta = 1.0$, are shown in Fig. 8, when the left and right sidewalls are maintained at cold temperatures while the bottom wall is subjected to a constant
heat flux and the top wall is considered adiabatic. When the value of the Rayleigh number is slight (i.e., $Ra = 10^3$), the magnitudes of stream function are very low and the fluid circulation is small due to the slight effect of convection when the Rayleigh number is low. But, as the values of the Rayleigh number increase (i.e., $Ra = 10^4$ and $10^5$), the fluid circulation inside the enclosure strongly increases and the magnitudes of stream function begin to increase dramatically. Furthermore, a region of high-

Fig. 7. Streamlines and isotherms for various $\Delta$ at $\psi = 30^\circ$, $Ra = 10^5$, $Da = 10^{-3}$, $Pr = 0.7$, and $Ha = 25$.

Fig. 8. Streamlines and isotherms for various $Ra$ at $\psi = 45^\circ$, $Ha = 25$, $Da = 10^{-3}$, $Pr = 0.7$, and $\Delta = 1.0$. 510
circulation flow intensity can be noticed near the bottom wall and it tries to push the fluid towards the core of the trapezoidal enclosure. For isotherms, when the Rayleigh number is low (i.e., $Ra = 10^3$), they are in general smooth and approximately parallel to the cold left and right sidewalls and the heat transfer is purely due to conduction. When the Rayleigh number increases (i.e., $Ra = 10^4$ and $10^5$), the values of temperature contours increase from low values at the cold left and right vertical sidewalls to high values at the hot bottom/base wall which satisfy the boundary conditions of the considered geometry indirectly. For this case, the isotherms are non-linear which indicates high temperature gradients and the heat is transferred due to convection. Moreover, a thermal boundary layer can be observed when the Rayleigh number is high.

### 4.5 Effect of Darcy number

Figure 9 displays the streamlines (left) and isotherms (right) for various Darcy numbers at $\psi = 45^\circ$, $Ha = 25$, $Ra = 10^5$, $Pr = 0.7$ and $\Delta = 1.0$. When the Darcy number is high ($Da = 10^{-4}$), a strong intensity of circulation can be observed inside the trapezoidal enclosure and the flow field can be represented by two unsymmetrical re-circulating clockwise and counterclockwise vortices. The flow circulations are higher adjacent to the enclosure core and weak at the cold left and right sidewalls due to a no-slip boundary condition. As the Darcy number decreases from $Da = 10^{-4}$ to $Da = 8 \times 10^{-5}$, the steam function values decrease and the flow circulation becomes weak. With respect to isotherms, at a high Darcy number it clustered strongly near the heat source at the bottom wall where a thermal boundary layer can be observed and the high circulation causes the convection heat transfer to be dominant. On the other hand, as the Darcy number decreases, the reduction in flow circulation makes the isotherms parallel and symmetrical and pure heat conduction dominates.

### 4.6 Local and average Nusselt numbers

Figure 10 illustrates the effects of the enclosure inclination angle on the local Nusselt number at the bottom wall for different values of the Hartmann number. It is found that the local Nusselt number decreases when the Hartmann number and enclosure inclination angle increase. But when $X = 0.5$, it begins to increase gradually with a decrease of the Hartmann number. Figure 11 explains the

![Fig. 9. Streamlines and isotherms for various $Da$ at $\psi = 45^\circ$, $Ha = 25$, $Ra = 10^5$, $Pr = 0.7$, and $\Delta = 1.0$.](image-url)
effects of enclosure inclination angle on the average Nusselt number at the bottom wall for various values of the Hartmann number when \( Da = 10^{-3} \), \( Pr = 0.7 \), \( \Delta = 1.0 \), and \( Ra = 10^5 \). It can be seen that the average Nusselt number along the hot bottom wall decreases as the enclosure inclination angle increases. Therefore, the high average Nusselt number at this wall corresponds to the low inclination angle and vice versa. The reason for this behavior is due to a reduction in the temperature gradient when the enclosure inclination angle increases. This reduction leads to a decrease in the average Nusselt number values. Also, it can be seen from this figure that the average Nusselt number decreases when the Hartmann number increases for all values of inclination angles. This is due to the effect of the magnetic field which becomes very significant with an increasing Hartmann number to the decreased strength of flow circulation and reduces the temperature gradient near the hot bottom wall and for this reason the average Nusselt number decreases. Figure 12 illustrates effects of the heat

![Figure 10](image1.png)

**Fig. 10.** Effects of enclosure inclination angle on the local Nusselt number at the bottom wall for different values of the Hartmann number.

![Figure 11](image2.png)

**Fig. 11.** Effects of enclosure inclination angle on the average Nusselt number at the bottom wall for different values of the Hartmann number.
generation/absorption parameter on local Nusselt number at the bottom wall for different values of the Rayleigh number. It can be observed that the local Nusselt number increases with the Rayleigh number. But when $X = 0.5$, it begins to decrease gradually with an increasing Rayleigh number. Also, it can be noticed that the local Nusselt number decreases when the heat generation/absorption parameter increases.

5. Conclusions

The following conclusions are found from the results of this present work:

1. When the Hartmann number increases, the applied magnetic field intensity increases, which causes a reduction in the flow field circulation. The diffusive currents begin to dominate the convective currents. The layered isothermal lines are in general symmetrical and parallel to the cold right and left sidewalls which indicates the strong influence of conduction.

2. The flow circulation inside the trapezoidal enclosure is strongly dependent on the Rayleigh number and it increases as the Rayleigh number increases and vice versa. The isotherms distribute symmetrically and parallel to the cold left and right sidewalls when the Rayleigh number is low and the conduction is dominant. As the Rayleigh number increases a clear deformation in isotherms can be observed and convection is dominant.

3. A strong flow circulation occurs when the Darcy number is high. This high circulation causes the isotherms to accumulate near the hot bottom wall and the heat transfer occurs by convection. The opposite phenomena can be noticed when the Darcy number decreases and conduction is dominant in this case.

4. When internal heat generation occurs inside the trapezoidal enclosure ($\Delta > 0$), the flow circulation intensity increases and both the values of stream function and isotherm contours increase. When there is no influence by internal heat generation or absorption (i.e., $\Delta = 0$), the flow and thermal

Fig. 12. Effects of heat generation/absorption parameter on the local Nusselt number at the bottom wall for different values of the Rayleigh number.
fields are in general similar to the corresponding fields when the effect of buoyancy force is weak. But, when absorption occurs inside the trapezoidal enclosure ($\Delta < 0$), the stream function and isotherm contours decrease.

5. When the enclosure inclination angle and Hartmann number increase the local and average Nusselt numbers along the hot bottom wall decrease.

**Literature Cited**