A Trajectory Tracking Robust Controller of Surface Vessels with Disturbance Uncertainties

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Abstract—This paper considers the problem of a trajectory tracking control for marine surface vessels with unknown time-variant environmental disturbances. The proposed mathematical model of the surface ship movement includes Coriolis and centripetal matrix and nonlinear damp terms. The observer is constructed to provide an estimation of unknown disturbances, which is applied to design a novel trajectory tracking robust controller through a vectorial backstepping technique. It is proved that the controller can force the ship to track the arbitrary reference trajectory and guarantees that the signals of the closed-loop trajectory tracking control system are globally uniformly ultimately bounded. The simulation results and comparisons illustrate that the proposed controller is effective and robust to external disturbances.

Index Terms—trajectory tracking control of vessels, disturbance observer, vectorial backstepping, nonlinear, robust

I. INTRODUCTION

Trajectory tracking control of surface vessels is an important control problem. It is of great significance for navigation in safety, energy-saving and emission-reduction [1]. It has attracted a great deal of attention from the control systems community both in theory and in practice. In [2], the simplified linear model was used to develop an adaptive high precision track controller for ships through a combination of feedforward and linear-quadratic-Gaussian (LQG) feedback control. In fact, the tracking control for a ship has an inherently nonlinear character. Taking advantage of the model free intelligent control techniques, the fuzzy logic control and proportional-integral-derivative (PID) track autopilot of ships was presented in [3] and a ship neural network trajectory tracking controller was developed in [4]. In recent years, several significant results have been presented using nonlinear control techniques through ship maneuvering mathematical models. Jiang [5] proposed two global tracking control laws for under actuated vessels using Lyapunov’s direct method. Petterson and Nijmeijer [6] illustrated a semi-global exponential stabilization of the tracking error for any desired trajectory using an integrator backstepping approach. Furthermore, they developed an exponential tracking control law of the reference trajectory for the ship using tracking control of chained form systems through a coordinate transformation, which was validated by the experimental results on a scale 1:70 model of an offshore supply vessel in the laboratory [7]. Yu et al. [8] introduced the second-level sliding mode surface approach to design a trajectory tracking control law for an under actuated ship with parameter uncertainties. Wondergem et al. [9] considered the problem of trajectory tracking for fully-actuated surface ships in the presence of the Coriolis and centripetal induced forces and moments and a nonlinear damping. They presented a controller-observer output feedback control scheme with semi-global exponential stability by its ship motion mathematical model.

On the other hand, the ships in the sea are always exposed to the environmental disturbances induced by wind, waves, and ocean currents. It is necessary to develop robust controllers for external disturbances. Under constant disturbances, a nonlinear trajectory tracking control law was designed for fully-actuated ship simultaneously considering Coriolis and centripetal matrix and nonlinear damp in [10]. Aschemann and Rauh [11] presented two alternative nonlinear control approaches to track the trajectories through the extended linearisation technique. The tracking accuracy was improved significantly by introducing a compensating control action provided by a disturbance observer for constant disturbances. Using a backstepping technique, a discontinuous feedback control law [12] and a new family of smooth time-varying dynamic feedback laws [13] have been derived respectively.

Mostly the mathematical model of ships doesn’t simultaneously consider Coriolis and centripetal matrix and nonlinear damp terms, or the uncertain time-variant environmental disturbances aren’t dealt with during the control design procedures. However, the sea state is constantly changing during the navigation of ships. For under-actuated ships, Do [14] provided a solution for the practical stabilization through several nonlinear coordinate changes, the transverse function approach, the backstepping technique, the Lyapunov’s direct method, and utilization of the ship dynamics.

For fully-actuated surface vessels, this paper presents a novel approach to solve the trajectory tracking control problem. The mathematical model of the ship movement
simultaneously contains Coriolis and centripetal matrix and nonlinear damp terms. The disturbances induced by wind, waves and currents are considered. Our proposed approach is featured such that a disturbance observer is introduced to estimate time-variant uncertain environmental disturbances.

II. PROBLEM FORMULATION

Definition of the coordinate frames

The earth-fixed frame $O_XO_YO_Z$ and the body-fixed $AXY$ coordinate frames.

Fig. 1. Definition of the earth-fixed $O_XO_YO_Z$ and the body-fixed $AXY$ coordinate frames.

Because environmental energy applied to the ship is limited. The matrix $R(\psi)$ is rotation matrix defined as

$$R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

with the property $R^{-1}(\psi) = R^T(\psi)$. Here $M$ is nonsingular, symmetric and positive definite inertia matrix, $C(\nu)$ is the matrix of Coriolis and centripetal terms, and $D(\nu)$ is the damping matrix. They are respectively

$$M = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix},$$

$$C(\nu) = \begin{bmatrix} 0 & 0 & -m_{22}v - m_{23}r \\ 0 & 0 & m_{22}v + m_{23}r \\ -m_{11}u & m_{11}u & 0 \end{bmatrix},$$

$$D(\nu) = \begin{bmatrix} d_{11}(u) & 0 & 0 \\ 0 & d_{22}(v, r) & d_{23}(v, r) \\ 0 & d_{32}(v, r) & d_{33}(v, r) \end{bmatrix}.$$  

In above equations

$$m_{11} = m - X_\dot{u},$$

$$m_{22} = m - Y_\dot{v},$$

$$m_{23} = mx_g - Y_\dot{r},$$

$$m_{32} = mx_g - N_\dot{v},$$

$$m_{33} = I_z - N_\dot{r},$$

$$d_{11}(u) = -X_\dot{u} - X_{u|u}|u|,$$

$$d_{22}(v, r) = -Y_\dot{v} - Y_{v|v}|v| - Y_{r|r}|r|,$$

$$d_{23}(v, r) = -Y_\dot{v} - Y_{v|v}|v| - Y_{r|r}|r|,$$

$$d_{32}(v, r) = -N_\dot{v} - N_{v|v}|v| - N_{r|r}|r|,$$

$$d_{33}(v, r) = -N_\dot{r} - N_{v|v}|v| - N_{r|r}|r|,$$

where $m$ is the mass of the ship, $I_z$ is the moment of inertia about the yaw rotation, and the other symbols, for example $Y_\dot{u} = \partial Y/\partial \dot{u}$, are referred to as hydrodynamic derivatives. A reader may consult [15] for more details.

The control objective in this paper is to design a feedback control law $\tau$ for the system (1)–(3) such that the position and orientation $\eta(t)$ of ships tracks arbitrary smooth reference trajectory $\eta_d(t)$, while it is guaranteed that all signals of the resulting closed-loop trajectory tracking system of a ship are globally uniformly ultimately bounded.

Assumption I: The desired smooth reference signal $\eta_d$ is bounded and has the bounded first and second time derivatives $\dot{\eta}_d$ and $\ddot{\eta}_d$.

III. CONTROLLER DESIGN

In this section, a disturbance observer is designed to estimate the unknown time-variant external environmental disturbances of the system (1)–(3). Then, we present the robust trajectory tracking controller for ships that solves the control objective as stated in Section II. The closed-loop
trajectory tracking control system of a ship mainly consists of two parts: the ship subjected to external disturbances, and the trajectory tracking controller with the disturbance observer. The schematic diagram is depicted in Figure 4.

Fig. 2. Diagram of the trajectory tracking control system of a ship.

A. Disturbance Observer Design

By means of the exponential convergent observer for a general nonlinear system from the reference [14], we constructed the disturbance observer for the disturbance vector \( b \) of the system (11)-(2) as follows:

\[
\dot{b}(t) = \beta + K_0 M \nu
\]

\[
\dot{\beta} = -K_0 \beta - K_0 [-C(\nu) \nu - D(\nu) \nu + \tau + K_0 M \nu]
\]

where \( \dot{b} \) is a disturbance estimation, \( K_0 \) is a 3-by-3 positive definite symmetric observer gain matrix, and \( \beta \) is a three dimension intermediate auxiliary vector generated by (7).

Define the estimation error vector \( \tilde{b}(t) = [\tilde{b}_1(t), \tilde{b}_2(t), \tilde{b}_3(t)]^T \) of disturbance vector \( b \) as:

\[
\tilde{b}(t) = b(t) - \hat{b}(t)
\]

From (7), (8) and (9), we have

\[
\dot{\tilde{b}} = \dot{\beta} + K_0 M \nu
\]

\[
= K_0 [b - (\beta + K_0 M \nu)]
\]

\[
= K_0 (b - \hat{b})
\]

Then the derivative of (10) along (11) is

\[
\dot{\tilde{b}} = \dot{b} - K_0 (b - \hat{b})
\]

\[
= \tilde{b} - K_0 \tilde{b}.
\]

Consider the following Lyapunov function candidate:

\[
V_c = \frac{1}{2} \tilde{b}^T \tilde{b}.
\]

The time derivative of \( V_c \) along the solution of (11) is

\[
\dot{V}_c = \tilde{b}^T (-K_0 \tilde{b} + \hat{b})
\]

\[
= -\tilde{b}^T K_0 \tilde{b} + \hat{b}^T \hat{b}.
\]

According to the following complete square inequality

\[
\tilde{b}^T \tilde{b} \leq \varepsilon \tilde{b}^T \tilde{b} + \frac{1}{4 \varepsilon} \hat{b}^T \hat{b},
\]

where \( \varepsilon \) is a small positive constant, (13) can be rewritten as

\[
\dot{V}_c \leq -\lambda_{\min}(K_0) \tilde{b}^T \tilde{b} + \varepsilon \tilde{b}^T \tilde{b} + \frac{1}{4 \varepsilon} \hat{b}^T \hat{b}
\]

\[
\leq -2[\lambda_{\min}(K_0) - \varepsilon] \tilde{b}^T \tilde{b} + \frac{C_d^2}{4 \varepsilon}
\]

\[
\leq -\alpha V_c + c
\]

where

\[
c = \frac{C_d^2}{4 \varepsilon}, (16)
\]

\[
\alpha = 2[\lambda_{\min}(K_0) - \varepsilon], (17)
\]

\[
\lambda_{\min}(K_0) - \varepsilon > 0,
\]

and \( \lambda_{\min}(\cdot) \) represents the smallest eigenvalue of a matrix. Therefore, we have the following theorem.

Theorem 1: The disturbance observer (7)-(10) guarantees that the disturbance estimation error \( \hat{b} \) of disturbances exponentially converges to a ball \( \Omega_b \) centered at the origin with the radius \( R_d = C_d/\sqrt{8 \sqrt{\varepsilon} (\lambda_{\min}(K_0) - \varepsilon)} \). The estimation error \( \hat{b} \) of disturbances can be made arbitrarily small by appropriately adjusting the design matrix \( K_0 \) and parameter \( \varepsilon \) satisfying the condition (18).

Proof: Solving (15), we have

\[
0 \leq V_c(t) \leq \frac{\varepsilon}{\alpha} \left[ V_c(0) - \frac{c}{\alpha} \right] e^{-\alpha t}
\]

It is known from (12) that \( V_c \) is ultimately bounded and exponentially converges to a ball centered at the origin with the radius \( R_V = C_d^2/\sqrt{8 \sqrt{\varepsilon} (\lambda_{\min}(K_0) - \varepsilon)} \). Furthermore, it is known from the definition of \( V_c \) that the disturbance estimation error \( \hat{b} \) exponentially converges to a ball \( \Omega_b \) centered at the origin with the radius \( R_d = C_d/\sqrt{8 \sqrt{\varepsilon} (\lambda_{\min}(K_0) - \varepsilon)} \). Therefore, the theorem is proved.

Remark 1: In the case \( C_d = 0 \), i.e., the disturbance vector is unknown constant vector, the disturbance observer is exponentially stable. The disturbance estimation error \( \hat{b} \) exponentially converges to zero.

B. Control Law Design

Let the desired position and orientation of ships be \( \eta_d = [x_d, y_d, \psi_d]^T \). First define the error vectors as follows:

\[
\eta_e = \eta - \eta_d
\]

\[
\lambda_e = \nu - \lambda_1
\]

where \( \lambda_1 \) is the stabilization function vector of subsystem (1), \( \nu \) is taken as the virtual control input vector. The control law design consists of two steps.

Step 1: Consider the following Lyapunov function candidate:

\[
V_1 = \frac{1}{2} \eta_e^T \eta_e.
\]

The derivative of \( \eta_e \) is given by

\[
\dot{\eta}_e = \dot{\eta} - \dot{\eta}_d
\]

\[
= R(\psi) \lambda_e + R(\psi) \lambda_1 - \dot{\eta}_d.
\]

Then the time derivative of \( V_1 \) along the solution of (23) is

\[
\dot{V}_1 = \eta_e^T \dot{\eta}_e
\]

\[
= \eta_e^T [R(\psi) \lambda_1 - \dot{\eta}_d] + \eta_e^T R(\psi) \lambda_e.
\]

We choose the stabilization function vector

\[
\lambda_1 = R^{-1}(\psi)(-C_1 \eta_e + \dot{\eta}_d)
\]
where $C_1$ is a positive definite symmetric design parameter matrix.

Substituting (23) into (24) yields
\[
\dot{V}_1 = \eta_T^T \left[ R(\psi) R^{-1}(\psi) \right] (-C_1 \eta_e + \dot{\eta}_d) - \eta_T R(\psi) \dot{X}_c
- \eta_T^T C_1 \eta_e + \eta_T^T R(\psi) \dot{X}_c. \tag{26}
\]

The coupling term $\eta_T^T R(\psi) \dot{X}_c$ will be cancelled in the next step.

Step 2: From (24) and (27), we have
\[
\dot{X}_c = \dot{\nu} - \dot{\hat{X}}_1
= M^{-1} [-C(\nu) \nu - D(\nu) \nu + \tau + b - M \dot{X}_1]. \tag{27}
\]

Consider the augmented Lyapunov function candidate:
\[
V_2 = V_1 + \frac{1}{2} \dot{X}_c^T M X_c + \frac{1}{2} \dot{\nu}^T \dot{\nu}. \tag{28}
\]

In terms of (11), (40) and (27), the time derivative of $V_2$ is
\[
\dot{V}_2 = \dot{V}_1 + \dot{X}_c^T M \dot{X}_c + \dot{\nu}^T \dot{\nu}
= -\eta_T^T C_1 \eta_e + \eta_T^T \left[ R(\psi) \eta_e - C(\nu) \nu - D(\nu) \nu + \tau + b - M \dot{X}_1 \right] - \dot{\nu}^T \dot{\nu}.
\]

We choose the control input vector as
\[
\tau = C(\nu) \nu + D(\nu) \nu + M \dot{X}_1 - R(\psi) \eta_e - C_2 \dot{X}_c - \dot{\nu}, \tag{30}
\]
where $C_2$ is a positive definite symmetric design parameter matrix.

According to (24) and the property $R^{-1}(\psi) = R^T(\psi)$, we calculate the derivative of $X_1$ as follows:
\[
\dot{X}_1 = R^T(\psi) \left[ -C_1 (\eta - \eta_d) + \dot{\eta}_d \right]
+ R^{-1}(\psi) \left[ -C_1 (\dot{\eta} - \dot{\eta}_d) + \dot{\eta}_d \right]. \tag{31}
\]

In addition, we have from (33)
\[
\hat{R}(\psi) =
\begin{bmatrix}
-\bar{r} \sin \psi & -\bar{r} \cos \psi & 0 \\
\bar{r} \cos \psi & \bar{r} \sin \psi & 0 \\
0 & 0 & 0
\end{bmatrix}
= \begin{bmatrix}
\sin \psi & -\sin \psi & 0 \\
\cos \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
= R(\psi) S(r) \tag{32}
\]
where $S(r) =
\begin{bmatrix}
0 & -r & 0 \\
r & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}$. Then, we obtain
\[
\dot{X}_1 = \left[ R(\psi) S(r) \right]^T \left[ -C_1 (\eta - \eta_d) + \dot{\eta}_d \right]
+ R^T(\psi) \left[ -C_1 (\dot{\eta} - \dot{\eta}_d) + \dot{\eta}_d \right]. \tag{33}
\]

By substituting (22), (24), (27) and (33) into (31), (30) can be written as
\[
\tau = - (MS^T R^T C_1 + R^T + C_2 R^T C_1) (\eta - \eta_d) + M \dot{R}^T \dot{\eta}_d
+ (MS^T R^T + M \dot{R}^T C_1 + C_2 R^T) \bar{\eta}_d + [C(\nu) + D(\nu)] \\
+ M \dot{R}^T C_1 R - C_2 - K_0 M \nu - \beta. \tag{34}
\]

Substituting (34) into (25) results in
\[
\dot{V}_2 = - \eta_T^T C_1 \eta_e + X_e^T R(\psi) \eta_e - C(\nu) \nu - D(\nu) \nu + C(\nu) \nu + D(\nu) \nu + M \dot{X}_1 - R(\psi) \eta_e - C_2 X_e
+ \bar{\eta} + b - M \dot{X}_1 - \dot{\nu} \dot{\nu} + \ddot{b} \dot{b} + \ddot{b} \dot{b}
= - \eta_T^T C_1 \eta_e - X_e^T C_2 X_e - \dot{X}_c^T \dot{b} + \dot{\nu} \dot{\nu} + \ddot{b} \dot{b}. \tag{35}
\]
Considering (13) and the following complete square inequalities
\[
- X_e^T \dot{b} \leq \varepsilon_1 X_e^T X_e + \frac{1}{4 \varepsilon_1} \dot{b} \dot{b}, \tag{36}
- X_e^T C_2 X_e \leq - \lambda_{\min}(C_2 M^{-1}) X_e^T M X_e \tag{37}
\]
where $\varepsilon_1$ is a small positive constant, then, (35) can be rewritten as
\[
\dot{V}_2 \leq - \lambda_{\min}(C_1) \eta_e^T \eta_e - \lambda_{\min}(C_2 M^{-1}) X_e^T M X_e + \varepsilon_1 X_e^T X_e + \frac{1}{4 \varepsilon_1} \dot{b} \dot{b} - \lambda_{\min}(K_0) \dot{b} \dot{b} + \dot{\nu} \dot{\nu}
+ \frac{1}{4 \varepsilon_1} \ddot{b} \ddot{b}
\leq - 2 \min \left[ \lambda_{\min}(C_1), \lambda_{\min}(C_2 M^{-1}) - \varepsilon_1 \lambda_{\max}(M^{-1}) \right]
\lambda_{\min}(K_0) - \frac{1}{4 \varepsilon_1} - \varepsilon > 0, \tag{38}
\]
where
\[
\lambda_{\min}(C_2 M^{-1}) - \varepsilon_1 \lambda_{\max}(M^{-1}) > 0, \tag{39}
\lambda_{\min}(K_0) - \frac{1}{4 \varepsilon_1} - \varepsilon > 0, \tag{40}
\]
and $\lambda_{\max}(\cdot)$ represents the largest eigenvalue of a matrix. Therefore, there is the following theorem.

**Theorem 2:** Under Assumption 1, for the 3-DOF nonlinear motion mathematical model of ships with unknown time-variant disturbances given by (10) and (22), the control input vector $\tau$ described by (24) together with (8) guarantees that the actual trajectory of ships tracks the arbitrary reference trajectory with the desired accuracy and all the signals of the closed-loop trajectory tracking system of ships are globally uniformly ultimately bounded by appropriately choosing the design parameter matrix $C_1$, $C_2$ and $K_0$ satisfying the conditions (39) - (40).

**Proof:** Notate
\[
\mu = \min \left[ \lambda_{\min}(C_1), \lambda_{\min}(C_2 M^{-1}) - \varepsilon_1 \lambda_{\max}(M^{-1}) \right], \tag{41}
\lambda_{\min}(K_0) - \frac{1}{4 \varepsilon_1} - \varepsilon \tag{42}
\]
and then (38) can be rewritten as
\[
\dot{V}_2(t) \leq -2 \mu V_2(t) + \sigma. \tag{43}
\]
Solving the above inequality, we have
\[
0 \leq V_2(t) \leq \frac{\sigma}{2 \mu} + \left[ V_2(0) - \frac{\sigma}{2 \mu} \right] e^{-2 \mu t}. \tag{44}
\]
It is seen from (43) that \( V_2(t) \) is globally uniformly ultimately bounded. Hence, \( \eta_e, \mathcal{X}_e \) and \( \hat{b} \) are globally uniformly ultimately bounded according to (48), then \( \mathcal{X}_1 \) and \( \nu \) are globally uniformly ultimately bounded. From the boundedness of \( \eta_e \) and \( \hat{b} \), we know that \( \eta \) and \( \hat{b} \) are bounded.

From (48) and (44), we can obtain
\[
\|z_1\| \leq \sqrt{\frac{\sigma}{\mu}} + 2 \left[ V_2(0) - \frac{\sigma}{2\mu} \right] e^{-2\mu t}.
\] (45)

It follows that, for any \( \mu_{z_1} > \sqrt{\sigma/\mu} \), there exists a constant \( T_{z_1} > 0 \), such that \( \|z_1\| \leq \mu_{z_1} \) for all \( t > T_{z_1} \). Therefore, the trajectory tracking error \( z_1 \) of the ship can converge to the compact set \( \Omega_{z_1} := \{ z_1 \in \mathbb{R}^3 \| z_1 \| \leq \sqrt{\sigma/\mu} \} \). Since \( \sqrt{\sigma/\mu} \) can be made arbitrarily small if the design parameters \( C_1, C_2 \), and \( K_0 \) are appropriately chosen, the actual trajectory of the ship can track the arbitrary reference trajectory. Theorem 2 is thus proved.

IV. SIMULATIONS AND COMPARISONS

In this section, the simulation studies are carried out on CyberShip II, which is a 1:70 scale replica of a supply ship of the Marine Cybernetics Laboratory in Norwegian University of Science and Technology. The ship has the length of 1.255m, mass of 23.8kg and other parameters of the ship are given in detail in [17].

We carry out the simulations with two different disturbances. In the simulations, the reference trajectory is chosen as bellow
\[
\begin{align*}
x_d &= 4 \sin(0.02t) \\
y_d &= 2.5(1 - \cos(0.02t)) \\
\psi_d &= 0.02t
\end{align*}
\] (46)
which is an ellipse.

A. Trajectory tracking under constant disturbances

In this section, the disturbance vector is set as \( b = [2N, 2N, 2N \cdot m]^T \) which corresponds to the environmental disturbances due to slowly varying wind, waves and currents. Assume the initial conditions of the system are \( [x(0), y(0), \psi(0), u(0), v(0), r(0)]^T = [1m, 1m, \pi/4\text{rad}, 0\text{m/s}, 0\text{m/s}, 0\text{rad/s}]^T \) and the initial state of the disturbance observer is \( \hat{b}(0) = [0, 0, 0]^T \). The design parameter matrices are taken as \( C_1 = diag([0.05, 0.05, 0.05]) \), \( C_2 = diag([120, 120, 120]) \), \( K_0 = diag([2, 2, 2]) \) such that the conditions (59) and (61) are satisfied for \( 0.125 < \varepsilon_1 < 9.6509 \) and \( 0 < \varepsilon < 1.9741 \). The results are depicted in Figures 3-5. The external disturbances \( b \) and its estimate value \( \hat{b} \) are depicted in Figure 3 from which it is clearly seen that the disturbance observer provides the rapidly exponentially convergent estimation of unknown disturbances within about 1.5s as proved in Theorem 1. From Figure 4 it is observed that the proposed controller is able to force the ship to track the reference trajectory. Furthermore, the curves of the desired and actual positions and yaw angles are presented in Figure 5, which shows

![Fig. 3. Constant external disturbances b1, b2, b3 and their estimations b1, b2, b3.](image)

![Fig. 4. Actual trajectory and reference trajectory in xy-plane under constant disturbances.](image)
that the actual ship position \((x, y)\) and yaw angle \(\psi\) can track the desired trajectory \(\eta_d = [x_d, y_d, \psi_d]^T\) at a good precision in around 40s. The curves of the surge velocity \(u\), sway velocity \(v\) and yaw rate \(r\) versus time are shown in Figure 6. The corresponding control inputs are presented in Figure 7 which shows that the control force and torque are smooth and reasonable. These results reveal that all the signals of the closed-loop trajectory tracking system of ships are globally uniformly ultimately bounded as proved in Theorem 3. Therefore, the proposed trajectory tracking controller is effective for the ship with uncertain constant disturbances.

**B. Trajectory tracking under time-variant disturbances**

In this section, the disturbance vector is set as

\[
b(t) = [b_1(t), b_2(t), b_3(t)]^T = \begin{bmatrix}
1.3 + 2.0 \sin(0.02t) + 1.5 \sin(0.1t) N \\
-0.9 + 2.0 \sin(0.02t - \pi/6) + 1.5 \sin(0.3t) N \\
- \sin(0.09t + \pi/3) - 4 \sin(0.01t) N \cdot m
\end{bmatrix}.
\]

The initial conditions of the system and the design parameters of controller are same as the counterparts in the first case of subsection III-A. The results are depicted in Figures 8-12 which exhibit almost the same control performance as under constant disturbances despite the time-variant disturbances. It is obvious that the designed controller is effective when the ship is exposed to both unknown constant and time-variant disturbances, which demonstrates that the proposed controller is robust against unknown environmental disturbances.
C. Performance comparisons

In this section, we compare the tracking performance of the designed controller (34) in this paper with the controller without disturbance observer:

$$
\tau_{cm} = -(MS^T R C_{cm1} + R^T + C_{cm2} R^T C_{cm1})(\eta - \eta_d) + [M(S^T R^T + R^T C_{cm1}) + C_{cm2} R^T] \dot{\eta}_d + [C(\nu) + D(\nu) - M R^T C_{cm1} R - C_{cm2}] \nu \\
- K_{cm} \int_0^t [\nu + R^T C_{cm1}(\eta - \eta_d) - R^T \dot{\eta}_d] \, d\tau
$$

(47)

which is designed using the backstepping approach for the ship with constant disturbances in [31] with gains $C_{cm1} = diag([0.05, 0.05, 0.05])$, $C_{cm2} = diag([120, 120, 120])$ and $K_{cm} = diag([2, 2, 2])$. Figures 10 and 11 illustrate the comparison of tracking performance between the two different controllers under constant disturbances and time-variant disturbances respectively. It can be seen from Figure 10 that both the controller exhibit similarly good transient and steady-state performances under the constant disturbances. However, under time-variant disturbances, it is seen from Figure 11 that the controller $\tau$ with disturbance observer in this paper performs better than the backstepping controller $\tau_{cm}$ with faster decay of tracking error and lower steady-state error value, since our observer provides an estimation of unknown disturbances. In contrast, $\tau_{cm}$ does not have disturbance compensation, it results in a larger tracking error norm.

To quantitatively compare the two controller performance, the performance under both constant and time-variant disturbances is summarized in Table I where $x_e = x_d - x$ and $y_e = y_d - y$ representing the error between the desired and actual positions, $\psi_e = \psi_d - \psi$ representing the error between the desired and actual yaw angles, and $t_{final} = 300s$. Table I clearly shows that the controller $\tau$ has better steady-state and transient performances than the backstepping controller $\tau_{cm}$. 
REFERENCES


Future research work would extend the proposed method to address the robust adaptive output feedback tracking of ships subjected to external disturbances and model uncertainties only depending on the position information $\eta = [x, y, \psi]^T$. From a practical viewpoint, it is convenient since it doesn’t have to measure directly the velocities $\nu = [u, v, r]^T$.

V. CONCLUSIONS

In this paper, a trajectory tracking robust control law has been designed for fully-actuated surface vessels in the presence of uncertain time-variant disturbances due to wind waves, and ocean currents. Both Coriolis and centripetal matrix and nonlinear damper term have been considered in the nonlinear ship surface movement mathematical model. The control strategy is introduced by the vectorial backstepping technique with our disturbance observer. The disturbance observer is employed to compensate disturbance uncertainties. It has been proved that the signals of the resulting closed-loop trajectory tracking system of the ship are globally uniformly ultimately bounded. Furthermore, the simulation results on an offshore supply ship model has illustrated that our controller is effective and robust to external disturbances. Our proposed trajectory tracking control scheme can provide good transient and steady-state performance for the considered ship system.

Future research work would extend the proposed method to address the robust adaptive output feedback tracking of ships subjected to external disturbances and model uncertainties only depending on the position information $\eta = [x, y, \psi]^T$. From a practical viewpoint, it is convenient since it doesn’t have to measure directly the velocities $\nu = [u, v, r]^T$.

TABLE I

<table>
<thead>
<tr>
<th>Disturbances</th>
<th>Constant $\tau$</th>
<th>Time-variant $\tau_{cm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{final}^{1} t_{e}(s)$</td>
<td>39</td>
<td>64</td>
</tr>
<tr>
<td>$f_{final}^{2} t_{e}(m \cdot s)$</td>
<td>19.4765 25.6207 19.8225 52.0868</td>
<td></td>
</tr>
<tr>
<td>$f_{final}^{3} t_{e}(m \cdot s)$</td>
<td>17.8455 40.7659 18.2010 55.9768</td>
<td></td>
</tr>
<tr>
<td>$f_{final}^{4} t_{e}(rad \cdot s)$</td>
<td>13.6354 35.1542 13.5816 44.8413</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 13. Comparison of tracking performance under constant disturbances.

Fig. 14. Comparison of tracking performance under time-variant disturbances.

TABLE I: Performance Index Comparison of Controllers $\tau$ and $\tau_{cm}$ Under Different Disturbances.