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Subjective Probability in Behavioral Economics and Finance: A Radical Reformulation

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Behavioral finance depends intimately on the notion of subjective probability, which has been universally treated as one of the two forms of probability. A substantial body of work and recent experimental results show conclusively that this approach is invalid: subjective and objective probabilities cannot be treated as two sides of the same coin. This raises serious questions about calculations based on that assumption, decisions based on those calculations, and what to do if assigning numerical values and calculating expected values based on subjective probabilities is invalid. This paper presents a radical re-formulation of subjective probability, showing that what have been called "subjective probabilities" are properly formulated as uncertainty appraisals, re-descriptions of states of affairs carrying tautological implications for action. A novel formulation of the decision maker's field-of-view, based on the concept of Actor, Observer, and Critic roles, combined with the uncertainty appraisal formulation, is used to develop new methods for evaluating data, finding patterns in data, and integrating probabilities and uncertainty appraisals, that is, those aspects that have, until now, been called "subjective probabilities."

Keywords: Subjective probability, Decision theory, *Homo commmunitatis*, Probability theory

Almost every decision in finance, whether by individual or institutional investors, involves the use of what is called "subjective probability," either explicitly or implicitly. Virtually every formulation of subjective probability treats it as though it and objective probability were two sides of the same coin, and accordingly essentially the same methods for incorporating subjective and objective probabilities into decisions have been employed: assign a numerical value to a probability (whether objective or subjective), estimate or calculate the value of the associated event, and calculate the expected value (perhaps with adjustments to the probability values based on some form of prospect theory [Kahneman and Tversky (1979)]).

Unfortunately, a substantial body of work and recent experimental results show conclusively that this approach is invalid: objective and subjective probabilities address entirely different things, and further numerical values for subjective probabilities do not, in general, satisfy the axioms of probability. "Subjective probabilities" are not probabilities at all. As a result, any calculation based on treating those numerical values as though they satisfied the axioms of probability, such the expected value of a security or performance of a market sector, are at best highly suspect. The impact on quality of decisions involving those expected values is obvious.

A new formulation of the factors that have been known as "subjective probabilities" is clearly needed. Equally necessary are new methods for incorporating those uncertainties in decisions. The straightforward expected value calculation outlined above is invalid, but the uncertainties in the picture are inescapable; we need ways to incorporate "subjective probabilities" and actual probabilities. This paper addresses the need for a new formulation (the second section) and presents several new methods based on it (the fourth section). We believe the formulation and methods have significant

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implications for behavioral economics and finance, both academically and professionally.

APPROACH

Since the time of Daniel Bernoulli, economics and finance have rested on two foundations: that decisions are based on expected value of outcomes and probability. Previous work (Jeffrey and Putman [2013], Jeffrey [2010]) extensively analyzed the flaws in the first of these and presented an alternative, *homo communitatis* paradigm, a rigorous articulation of the full range of factors affecting human action, thereby providing a basis for full analysis of economic behavior. *Homo communitatis* is, to our knowledge, the first new formulation of choice since the inception of the study of decision under uncertainty.

In this paper we use *homo communitatis* to re-formulate "subjective probability." We show that, contrary to the 17th century formulation of objective and subjective probability as two versions of the same thing, they are actually two entirely distinct concepts applicable to two distinct phenomena: Objective probability is a relationship between two numerical states of affairs, whereas "subjective probability" is a relationship between an individual and an action. Confusing the two phenomena is a serious error and has had serious consequences. It has resulted, for example, in the unwarranted conclusion that an experimental subject who states, "Based on Sam's personality, he is more likely to be a fighter pilot than an insurance salesman" is doing Bayesian probability calculations, but doing them badly. Perhaps more seriously, it also results in the assumption that statements such as "It is unlikely that subprime mortgage default will result in serious financial disruption" are made more precise by restating them numerically, rather than by identifying possible actions and assigning them relative priorities.

Homo communitatis may be summarized by the following 7 principles:

1. Choice is choice of behavior, of which outcome is only one aspect.
2. "Behavior" means *intentional action*, formulated parametrically as <I, W, K, Kh, P, A, PC, S>.
3. The paradigm case of human behavior is *deliberate action*, that is, intentional action in which the individual knows what he is doing and choosing to do it, represented formally by including the behavior (by name) in the specification of the values of the K and W parameters, respectively.
4. Behavior choices are made in light of the individual's reasons to engage in one behavior or another.
5. People choose what matters to them; that is, choices reflect values, which are shown in the pattern of a person's choices over time.
6. Every behavior is an instance of engaging in a social practice of a community.
7. For any person, a particular state of affairs may be real, actually possible, or merely possible.

Though stated in ordinary English, these principles constitute a formal articulation of the concepts of choice, behavior, reasons for choice, value, and the relationships between them, much as the axioms of Euclidean geometry are stated in ordinary language but are an articulation of geometric concepts and their relationships.[11,22] The term *homo communitatis* reflects the centrality of the relationship between behavior and community: every action is a case of engaging in a structured pattern of actions, a social practice of a community. The new paradigm yields novel insights into several phenomena such as the Friedman-Savage "paradox," loss aversion, the endowment effect, and framing effects, by articulating the full range of factors involved in each decision or behavior. The apparent "irrationality" often attributed to economic actor is an illusion due to basing analyses on defective descriptions of the choices, much as a fun-house mirror produces the illusion of peculiar body shapes.

This paper is organized as follows. The first section discusses the relationship between objective and "subjective" probabilities, showing that they cannot be two sides of the same coin. The second section uses the principles of *homo communitatis* to give a new conceptualization of "subjective" probability (and its equivalent "degree of belief"). The novel conceptualization yields novel methods for handling uncertainty, methods quite unlike traditional ones, but in order to develop those methods we must first address the issue of an action as seen by the person *when they are acting*, as contrasted with how the actor+action situation look to an observer, and we do that in the third section. The fourth section develops the pragmatic implications of the new conceptualization in research and professional practice in economics and finance.

As is customary (see, e.g., Tversky and Kahneman [1974], Kahneman and Tversky [1979]), while our interest here is primarily behavioral economics and finance, we develop the concepts and methods in the general context of decision under uncertainty.

"SUBJECTIVE PROBABILITY" VS. ACTUAL PROBABILITY

Two important themes run through the history of probability since it emerged in Western scientific thought around 1660. One is that from the beginning it has been "Janus-faced" (Hacking [1975]), dealing on one hand with states of affairs involving relative frequency of outcomes of repeated trials and on the other hand with a person's "degree of belief" in propositions. The second is that both

concepts have been considered fundamentally related to decisions. In the language of philosophy, one aspect is ontological, the other epistemological, but both are aspects of making decisions. In this section we address the question of whether it makes sense to consider relative frequency and degree of belief as two sides of the same coin.

Frequency and degree of belief have come to be considered essentially interchangeable, with concepts and methods suitable for the objective used to address the "subjective" and vice versa, without regard to applicability of concepts, particularly in decision theory, psychology, and economics. (Interestingly, this is much less the case in engineering disciplines.) For example, it is currently common practice in behavioral economics to ask experimental subjects degree of belief questions and then analyze their answers by asserting that subjects arrive at their answers by using sample sets and (defective) calculations based on them (Tversky and Kahneman [1974], Barberis and Thaler [2003], Camerer [2000]).

Similar language (e.g., "probable," "likely") is often used to talk about the two concepts, increasing the confusion. The work that first introduced the concept and term "representativeness bias" (Tversky and Kahneman [1974]) is a good illustration of the problem. The authors introduce their discussion as follows:

> Many of the probabilistic questions with which people are concerned belong to one of the following types: What is the probability that object A belongs to class B? What is the probability that event A originates from process B? What is the probability that process B will generate event A?

A few lines later, they continue:

> Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail. How do people assess the probability that Steve is engaged in a particular occupation from a list of occupations?. . .In the representativeness heuristic, the probability that Steve is a librarian. . .is assessed by the degree to which he is representative of, or similar to, the stereotype of a librarian. . ..This approach to the judgment of probability leads to serious errors. . . . (p. 44)

Thus, the initial discussion is in terms of membership in a class of events resulting from processes, such as picking a ball from an urn of blue and red balls or a spinning roulette wheel. Actual processes are the basis of a sample space: a set of elements with a probability distribution, from which repeated samples are drawn. The same language is then used to describe a task in which there *is no* sample space—no repeated trials and no relative frequencies—and therefore one in which the objective probability is entirely inapplicable. Subjects' answers are *interpreted as though* 1) it were known that the subject understood the question as an objective probability question, despite there being a number of commonly recognized nonprobabilistic meanings to the question; and 2) answers had been produced by selection from a sample set. Subjects' answers are then described in terms of errors they *would* have been making, had they been selecting an object from a sample set. The presentation, though, does not say they *would* have been making those errors; it says they *are*. Thus, an initial discussion of objective probability moves seamlessly to a case of "subjective" probability in which the concepts of probability do not apply. Describing experiments as though subjects were literally selecting from a sample space, without regard for whether there is such a space or samples from it, is standard practice in behavioral economics.

A similar but even more striking example is cited by Barberis and Thaler [2003]:

> Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. When asked which of "Linda is a bank teller" (statement A) and "Linda is a bank teller and is active in the feminist movement" (statement B) is more likely, subjects typically assign greater probability to B. *This is, of course, impossible* [emphasis added]. (p. 1064)

Here, even when experimental results contradict the mathematics of probability, the researchers are, in effect, insisting on the use of the concept of probability to analyze them.

Use of probability language to refer to situations in which there is no sample space is by no means unusual, as shown in Table 1. Such use often results in describing "subjective probabilities" with numerical values that do not satisfy the definition of probability. Specifically, it is not uncommon for subjective probabilities assigned to disjoint events to be subadditive (sum to less than one) or superadditive (sum to more than one) (Macchi, Osherson, and Krantz [1999], Mandel [2005]). In finance, law, political science, intelligence estimation, and a number of other fields ambiguity, a concept distinct from uncertainty, commonly results in situations in which, on considering all available facts, both X *and* not-X appear likely. In such cases the numerical representations of these "probabilities" would sum to more than one. Quantities (or equivalently a function from an event set to the positive real numbers) that in general do not satisfy the axioms of probability are not probabilities, just as 3-sided closed plane figure is not a different sort of rectangle. No sample set, no probability.

Objective and "subjective" probability have to do with situations that are crucially different, depending on whether or not a sample set is in use. They are, however, both ways of talking about uncertainty. Further, as noted at the beginning of this section, without exception they have to do with

TABLE 1
Examples of Conflation of Objective and Subjective Probabilities

Phenomenon	Brief description	Evidence subjects used sample set?	Subjective or objective	Presented as subjective or objective
Representation heuristic	Give personal description; ask occupation (Tversky and Kahneman [1974]). Variation: ask occupation plus group membership (Barberis and Thaler [2003])	N	S	O
Insensitivity to priors	Give personal description; sample set present; ask occupation (Tversky and Kahneman [1974])	N	S	O
Insensitivity to priors	No personal description; sample set present; ask occupation (Tversky and Kahneman [1974])	Y	O	O
Insensitivity sample size	Give M/F births at 2 hospitals; ask probable events at each (Tversky and Kahneman [1974])	Y	O	O
Insensitivity to priors	Select balls from urns; predict outcomes (Tversky and Kahneman [1974])	Y	O	O
Misconceptions of chance	Predict head-tail sequences (Tversky and Kahneman [1974])	Y	O	O
Insensitivity to predictability	Present company descriptions; predict performance (Kahneman and Tversky [1979])	N	N	O
Insensitivity to predictability	Present teacher descriptions; predict performance (Kahneman and Tversky [1979])	N	S	O
Anchoring	Ask subjects to predict Dow Jones price (Kahneman and Tversky [1979])	N	S	O

"decision under uncertainty," which means a person deciding what to do when uncertain about something relevant to the decision. Uncertainty is a fundamental fact about the world, while probability is not; the error has been to equate the uncertainty with probability.

As decisions are central to understanding uncertainty, and therefore to making sense of "subjective probability" in light of the fact that it is not probability at all, let us look more closely at decisions. Making a decision is depicted below in Figure 1. Known as the CRJ diagram (Ossorio [2006], p. 228, Jeffrey and Putman [2013]), it is a depiction all the factors that pertain to a decision.

Discursively, the diagram represents the fact that person P, in light of circumstances C_1, …, C_m, which constitute reasons R_1,…R_n to do B_1,…B_z, chooses to do B_k, reflecting the relative priorities P assigns to the reasons. Or, less formally: P's circumstances give her reasons to do various things, and the one she does reflects the relative priorities she accords each reason. (It is important to understand that Figure 1 does not depict a process of any sort. It is a reconstruction of everything related to the decision, not a sequence of steps, either overt or "internal.")

As Figure 1 shows, the logic of a decision may be complex, involving a number of states of affairs:

1. There are a number of behaviors, B_1,…, B_z that the person has an opportunity to do.
2. Each behavior B_i is described by 8 parameters, I_i, W_i, K_i, Kh_i, P_i, A_i, PC_i, S_i
3. The Circumstances C_1,…,C_m are those that the person takes to be the case and to be relevant to the decision. They are only those the person takes to be relevant, not the perhaps-much-larger set an observer might identify, and certainly not the enormously larger set that could in some way be related to the decision.
4. Each C_i is appraised by the person as providing one or more reasons to engage in one of the B_i. The entire set of reasons to do one B_i or another is R_1,…,R_n.. The C_i are the relevant circumstances; the R_j are how each circumstance is relevant.
5. The reasons have relative weights w_1,…,w_n for the person. These are the relative priorities the R_j have for the individual, in this case.

As discussed in Ossorio [2006], Jeffrey [2010], and Jeffrey and Putman [2013], there are four kinds of reasons: hedonic, prudential, ethical, and esthetic. As a result, the entire value of circumstance C_i to the person consists of four incommensurate constituents, namely the total strength of each of the four kinds of reasons. Mathematically, this means that value is a four-element vector, and 4-vectors cannot be ordered. As a result, current axiomatic treatments, all of which are premised on value as a scalar, are problematical.

What are circumstances? Circumstances here refer to what the individual takes to be the case, the perceived facts, with no connotations of correctness or degree of verification. The term is approximately equivalent to "beliefs," as that term is used technically, with the important proviso that to say, "P believes X" is stating *only* that P takes X to be the case and is prepared to act on it.

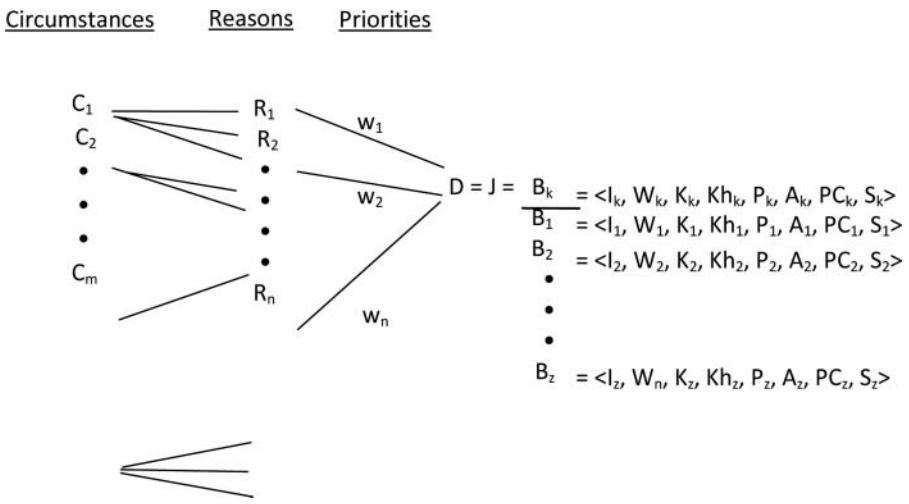

FIGURE 1 Circumstances, Reasons, and Judgment

What are reasons? Reasons are a very precisely defined concept: valuing a state of affairs gives an individual reason to act to try to achieve it. When an individual has a reason to do X and sees an opportunity to do X, he has motivation to do it. (This is the classic concept of cause, but generalized to the case of an action by a person rather than a physical objects and processes.) Circumstances, by themselves, provide opportunities to act, but they do not provide motivation. Reasons provide the immediate connection to action: reason to do B plus an opportunity to do B constitutes motivation to do B.

How are circumstances and reasons related? Fundamentally, the relationship between a circumstance and a reason is a part-whole relationship. Circumstance C gives person P reason R to do B when, and only when 1) C is a constituent of state of affairs R, and 2) bringing about C will bring about R.

Bernoulli's example of a prisoner needing money to purchase his freedom (Bernoulli [1738/1954]) is a good illustration. If a man has 2,000 ducats, that is a circumstance; that he may win 2,000 more in a lottery is also a circumstance, a possible state of affairs. If that man is "a . . . prisoner who needs two thousand ducats more to repurchase his freedom", the fact that he can win 2,000 gives him reason to play the lottery, as follows: having 2,000 and winning the additional 2,000 are constituents of the larger state of affairs of having sufficient ducats to become free.

In general, circumstance C, together with other circumstances and the relationships between them, comprise larger state of affairs R that the person values. That the C-R relationship depicted in Figure 1 is part-whole, not probabilistic, has important implications in articulating just what is meant by "subjective probability" in the section below.

Uncertainty

Figure 1 depicts all the factors that pertain to a decision. Referring to it, we can see that a decision maker may be uncertain about whether:

1. Circumstances-related issues:
   a. Each of the circumstances is as perceived.
   b. All the relevant circumstances have been identified.
   c. All circumstances identified as relevant actually are.
2. Reasons-related issues:
   a. Each circumstance actually gives the reason identified.
   b. All the reasons have been identified.
3. Priority issues: The relative priorities of the reasons are appropriate.
4. Behavior-related issues. Each Circumstance C_i appears in the specification of the K parameter of the behaviors – i.e., the specification of the states of affairs being acted on, and so Circumstance-related uncertainty is uncertainty about what one knows (K), but the person may also be uncertain about whether:
   a. I: they are the correct individual to carry out the action.
   b. W: they actually want the result of the action.
   c. Kh: they have the necessary competences for one or more of the behaviors.
   d. P: they can carry out the performance of one or more of the behaviors, that they are carrying out the performance of the chosen behavior correctly, or that they would recognize an incorrect performance.
   e. A: the performance of one or more of the behaviors will produce the intended state of affairs,

that they would recognize success or failure, that they would recognize the relevant other consequences of the behavior.

f. PC: they have the necessary personal characteristics. These may be personality characteristics (e.g., attitudes, character traits) or other relevant facts about them, such as wealth or other "external" attributes.

g. S: the larger state of affairs that would be brought about by accomplishing one or more of the behaviors; that they would recognize that state of affairs if it did come about.

Objective probabilities have to do with a narrow subset of these uncertainties: some of the circumstances and one of the behavior parameters. Objective probability is a numerical relationship between two numerical states of affairs: ratio of occurrence of some kind to all possible occurrences. It is therefore a circumstance—a state of affairs relevant to the decision. As discussed above, the relationship between circumstances and reasons is part-whole, not a ratio of cardinalities; the relationship between circumstances and reasons is not derivable via any probability calculation even when a sample space is present. Priorities are a (partial) ordering on reasons, not ratios of cardinalities. Thus, of the C's, R's, and w's, only circumstances could possibly be probabilistic. Of the behavior parameters, the only one that could possibly be a ratio of cardinalities is whether achievement A will result if procedure P is carried out, such as the probability that the outcome of rolling the dice will be seven.

In his 1926 work *Truth and Probability*, Ramsey notes that ". . .the general difference of opinion between statisticians who for the most part adopt the frequency theory of probability and logicians who mostly reject it renders it likely that the two schools are really discussing different things. . .." (p. 157). We can now see that Ramsey's observation is correct: "subjective probabilities" have to do with *uncertainties that are not probabilistic*.

In the following section we examine exactly *what* they have to do with those uncertainties.

"SUBJECTIVE PROBABILITIES" ARE APPRAISALS

We have seen that "subjective probability" language cannot be talking about the same concept as objective probability language. What then is it talking about?

Pragmatic Assurance of Success

The paradigm case of human behavior is that a person recognizes that behavior B is called for and engages in it, and the outcome of B is the desired state of affairs. *Pace* Aristotle, I need milk for my cereal; there is none; I go to the store and buy it; I come home; I put some milk on my cereal. There is no uncertainty, nothing I am unsure of, and no probability, whether objective or "subjective." A great deal of ordinary life and a great deal of economic behavior, including financial behavior, is of this sort—buying groceries, dining out, preparing dinner, teaching a class (including answering interesting, not previously encountered student questions), conducting a behavioral economics experiment, going to a basketball game, celebrating an anniversary, and so forth—almost ad infinitum. In these situations, the individual has what Ossorio [1981] terms *pragmatic assurance of success*, which means that engaging in B results in the intended state of affairs *and* no other behavior is called for to ensure that W = A.

It is important to realize that "assurance of success" does *not* refer to some sort of absolute guarantee, or that Probability(X) = 1.0. "Pragmatic assurance," of the success of B or that X is the case, means *only* that P is prepared to do B, or act on X, in one of the ordinary ways of doing B and with no further actions to ensure the success of B. Saying, "There is no uncertainty," should not be taken as saying, "It is guaranteed."

When There Is No Pragmatic Assurance of Success

While the paradigm case is 1) to see that behavior B is called for and 2) to do it, there is of course a large and important class of behaviors in which there is uncertainty about one or the other, or both of these. One may be uncertain whether to buy a car or take vacation, whether to enroll in college or get a job; one can engage in a series of actions to checkmate an opponent, increase the value of an investment portfolio, win a hand of poker, launch an important new product, etc., but each of these are *courses of action*: one cannot simply do them and be assured of success, as with buying milk. Games, including gambling games, are a domain in which players routinely engage in courses of action, and indeed this appears to be a *sine qua non* of games.

Appraisals

We noted above that one may be uncertain about one's circumstances, reasons, relative weights to give the reasons, and each of the aspects of the possible behaviors. Each of those items is a state of affairs. To say, "X is uncertain," is to say, "X cannot be counted on," that is, "Acting on X has no pragmatic assurance of success." It is, in other words, an *appraisal* of X: a re-description of X with tautological implications for action (Ossorio [1990]). By "tautological," we mean that to re-describe X as Y *is* to say that Y constitutes reason to act in certain ways, and not in others. To say, "Driving drunk is dangerous," is to say is to say that one has reason to do some things and not do others; in exactly the same way, to say, "Whether Greece will default

on its sovereign debt is uncertain," is to say that one has reason to engage in certain actions and not others with respect Greek default. Similarly, to say one is uncertain whether the circumstances are as perceived is to say, "Certain ways of acting on these circumstances are called for, and others are not."

Danger is of course not an all-or-nothing concept, and thus we refer to "mildly dangerous," "very dangerous," etc., situations, calling for different actions, such as immediate escape, casual re-evaluation, taking mild precautions, and taking extreme precautions. The same holds for other kinds of appraisals (e.g., enjoyable, unethical, wise), and in particular it holds for uncertainty. "Probable," "unlikely," "barely conceivable but not impossible," etc., are distinct *uncertainty appraisals*.

The answer to the question posed at the beginning of this section is therefore:

> Subjective probability language is language for expressing **uncertainty appraisals**, that is, re-descriptions of facts, in terms of nonprobabilistic uncertainty, carrying tautological implications for action.

In other words, "X is probable," "I believe X will occur," "X is unlikely," "X is barely conceivable but not impossible," etc., are ways of stating uncertainty appraisals of X, that is, re-descriptions of X that imply reasons to act in certain ways in light of the uncertainty.

Appraisal is not to be confused with assessment. An assessment is an evaluation or analysis of something in order to determine its characteristics, implications, or other related facts. One may assess characteristics of a situation or some aspect of it, qualitatively or quantitatively; one may derive other facts, empirically or logically; certain kinds of facts permit assessment of probability of occurrence. Appraisals, in contrast, are a very particular kind of evaluation: re-descriptions of circumstances as reasons to do some things and not others.

The customary linguistic forms for expressing uncertainty about the wide range of phenomena that are not probabilistic are not imprecise language for something properly expressed mathematically; they are customary forms for stating uncertainty appraisals. "I'm unsure whether do X," "He's not certain what will happen with Greek bonds," "I'm not sure whether the milk is spoiled," and "I'll move my pawn, but I'm not sure that will work," are appraisals of possible facts, each of which refers, tautologically, so some set of possible actions. In current usage, it is also common to use the *language* of probability, even though the state of affairs referred to is not probabilistic, and that is the language of "subjective probability," in all its forms: "There's a probability of 0.01 that Spain will default on its bonds," etc., "P's degree of belief in X is 0.2," "I'm 90% sure we will complete this project on time," and so forth. *"Subjective probability" language is probabilistic language for referring to appraisals*, not references to a distinct form or probability.

The particular appraisal constitutes reasons to act in certain ways and not in others, and the strength of those reasons. The kinds of differences that may be involved are:

- Performing one or more of the behaviors of the practice in a nonstandard manner. For example, a driver is unsure whether he can make a turn without hitting the curb, and makes the turn very slowly.
- Assessing one or more of the factors about which one may be uncertain, as enumerated in the previous section, before, during, or after the course of action. For example, an investor is unsure whether to buy an offering of a company's bonds, and investigates the company's financial situation with unusual thoroughness.
- Pausing to assess whether to do a particular version of the course of action. The frequent advice to persons of a certain age to assess the status of their retirement plans, in recognition of the fact that planning for retirement has no pragmatic assurance of success, is of this sort.
- Pausing to assess whether to change to a different course of action entirely, as in, "I took a look at my job prospects with a philosophy degree, and changed my major to computer science."
- Checking progress and avoiding dangers while engaging in the course of action, including other practices necessary to do so. The practices involved in testing software during its development are a paradigm case. More generally, there is rarely if ever a project in business or industry for which there is pragmatic assurance of success, and so in every firm one finds the management practices of project management, including monitoring and directing subordinates.

"X is likely," "X is probable," "X is barely possible," etc., are each uncertainty appraisals of X, that is, re-descriptions of X that identify 1) that X is a *possible* state of affairs; 2) P cannot count on X, that is, has no pragmatic assurance of success of acting on X; 3) other behaviors, which address the specifics of the uncertainty about acting on X, are called for; and 4) those other behaviors have particular priorities for P.

Whereas objective probabilities are made precise by carrying out practices to determine more specific numerical values, uncertainty appraisals are made precise by identifying the behaviors called for in light of the appraisal and the relative priorities those behaviors have. "Subjective probability" language, and uncertainty language more generally, convey that acting on state of affairs S has no pragmatic assurance of success. The central question then becomes, "What behaviors *are* called for, and what priorities should those behaviors have?"

For example, Mandel [2005] reported that in a group of 135 students at the University of Victoria the median estimated probability of a terrorist attack within two months was 0.10 while the median value the probability of no attack was 0.50. The statements, "P(attack) = 0.10" and "P(no attack) = 0.50" are cases of using probability language to express uncertainty appraisals of the two states of affairs, "attack" and "no attack." Stipulating the customary correspondence between numerical values and verbal characterizations, this means the participants, on average, appraised a terrorist attack as "unlikely," and appraised having no attack as "maybe-can't tell." (Participants were instructed to express their "estimates" as decimal values, with no verbal interpretations, so it cannot be determined whether participants used the customary correspondence. Thus, we are stipulating these verbal characterizations, for illustrative purposes.) These appraisals tautologically indicate and contra-indicate various actions, with various relative priorities: changing one's travel plans, investigating situations where terror attacks might occur, deciding whether a location of interest is a terrorist target, and so forth. (On Sept. 12, 2001, some residents of suburban Chicago cancelled dentist appointments, having appraised a terrorist attack in their area as highly likely.)

Consider a classic actual gamble, a sports bet. P decides to bet $20 on the Cubs at 100:1. He does that, instead of some other behavior with the $20—take the children to McDonald's, buy an inexpensive watch, buy his wife a small present, and so forth. The circumstances include having the $20, having other wealth in some amount W, how his spouse will react to knowing about the bet, his personal values, etc. Each of these circumstances gives P reasons to bet or do something else, such as the prudential value of the $2,000 that may be won, the hedonic value of betting on the Cubs, or the esthetic (i.e., appropriateness) value of supporting the home team. P's appraisal of the odds constitutes reasons to act in various ways, with various relative priorities that reflect P's personal characteristics. For many persons, particularly those in "tight" financial circumstances, the 100:1 odds will result in the person assigning a low priority to betting the $20. If P said, "In our situation every dollar counts, and my wife hates me gambling, but even though the odds were 100:1 I took the bet," it would call for explanation. The kinds of explanation available are either that the man had some other previously unrecognized reason, or a that he was "just the kind of guy" to take such a bet in such a circumstance (an attribution of a personality characteristic, one which most in this culture would consider pathological).

To summarize:

- Objective probability refers to a numerical relationship between cardinalities of event sets; "subjective probability" refers to uncertainty appraisal.
- The particular appraisal gives the person reasons to engage in that behavior and others.
- The person's choice reflects the relative weights, or priorities, of the reasons for that person.

RELATED WORK

Vagueness vs. Incompleteness

Verbal characterizations such as "likely," "probable," etc., have traditionally been considered vague or ill-defined language, to be replaced with numerical values. When talking about objective probabilities, that approach is correct, but it is not correct when talking about uncertainty appraisals. The issue is one of complexity, not vagueness. Verbal identifications of appraisals, which are almost always single words or short phrases, necessarily "encode" a great deal of information, including all the practices either indicated or contra-indicated and the relative priority of each practice in the current context. In addition, practices are in general complex and may be done in a number of ways, some of which are consistent with the appraisal and others of which are not, to varying degrees. All of that information is "carried" in the brief terms such as "likely," "probable," etc. The language is not vague; it is merely incomplete, eliding a great deal of information about the connection between the appraisal and a number of practices.

Degree of Belief

That "subjective probability" or degree of belief refers to appraisals, and therefore indirectly to indicated or contra-indicated behaviors, is consistent with Ramsey's [1926] characterization of degree of belief as "the extent to which we are prepared to act on [the belief]" (p. 65). Ramsey rather neatly disposes of the other interpretation, namely that degree of belief is some kind of internal strength or intensity:

> We can . . . suppose that the degree of a belief is something perceptible by its owner; for instance that beliefs differ in the intensity of a feeling by which they are accompanied, which might be called a belief-feeling or feeling of conviction, and that by the degree of belief we mean the intensity of this feeling. This view would be very inconvenient, for it is not easy to ascribe numbers to the intensities of feelings; but apart from this it seems to me observably false, for the beliefs which we hold most strongly are often accompanied by practically no feeling at all; no one feels strongly about things he takes for granted. (Ramsey [1926], p. 65)

Mathematical Treatments

Based on the observation that degree of belief can only be defined in terms of action, Ramsey used the idea of betting

to define it: P's degree of belief in X is the lowest odds P would accept. de Finetti [1937] used the same concept to axiomatize degree of belief, which, as Ramsey notes, means the extent to which one is prepared to act on it:

> Let us suppose that an individual is obliged to evaluate the rate p at which he would be ready to exchange the possession of an arbitrary sum S (positive or negative) dependent on the occurrence of a given event E, for the possession of the sum pS; we will say by definition that this number p is the measure of the degree of probability attributed by the individual considered to the event E, or, more simply, that p is the probability of E (according to the individual considered; this specification can be implicit if there is no ambiguity)... Your degree of belief in E is p iff p units of utility is the price at which you would buy or sell a bet that pays 1 unit of utility if E, 0 if not E. (de Finetti [1937], p. 62)

The method derives a quantitative measure of how sure a person is by directly asking, "To what extent are you willing to act on what you take to be the case (the crude form being, "Put your money where your mouth is"). The classic sports bet, for example, "I'll bet you $10 the Cubs will finally win the World Series this year," is an everyday example. Professional bookies famously determine odds in exactly this way. However, it is important to keep in mind what these "odds" are: they are not a calculation based on samples of a sample space; they are straightforwardly a ratio of monies received.

Unfortunately, de Finetti's axioms do not axiomatize the phenomenon at hand because as noted in the first section, the numerical values violate finite, that is, "probabilities" for disjoint events may sum to more or less than one.

ACTORS, OBSERVERS AND CRITICS

In order to develop the practical implications of the uncertainty appraisal formulation, we need to articulate a particular aspect of the relationship between a person and what they are doing.

It is a universal fact about persons that what a person sees, and how they value what they see, depends on the position from which they are seeing, that is, the role they occupy. In the role of chess player, a person sees pieces, configuration, tactics and possible tactics, strategies and possible strategies, opponent chess characteristics, possible states of affairs on the board, and so forth, and we do not wonder that a chess player, playing chess, rarely notices anything outside what is happening or could happen on the board. One simply does not see anything outside the domain corresponding to the role. It is similar to but substantially more pronounced and stronger than, the widely known phenomenon of figure-ground. What hold for chess players holds equally for baseball players, financial analysts, economists, computer programmers, chief executive officers, and so forth.

It also holds for three other roles, ones that an individual always has: Actor, Observer, and Critic. Carrying out actions in the world, including making decisions in the face of uncertainty, requires three different kinds of tasks: one must act, one must observe the act and how it is going, and one must assess or critique the situation so as to be able to change or correct how things are going as necessary. Acting, observing, and critiquing each involve several tasks, and require different knowledge and skills, so it is useful to articulate them in terms of jobs and job descriptions, as follows:

- Actor: In the role of Actor, the individual's "job" is to act, to carry out the behavior called for. Doing will be seen by someone in the observer role as constituting a choice from among possible behaviors, as depicted in Figure 1, but that reconstruction is not part of the Actor's job description.
- Observer-Describer: The individual observes the behavior and describes it via various concepts.
- Critic: The individual assesses the description of the behavior and gives "feedback" to the actor. (The term "feedback" in this context originated with Norbert Weiner, who observed that behavior has the characteristic of the "feedback loop" in electronics; the term has since become ubiquitous. Although the feedback loop is a useful analogy to Actor-Observer-Critic functioning, it differs in one very important way. Unlike the operation of a feedback loop, ordinarily an individual occupies all three roles, and does all three jobs, simultaneously. An individual acts, observes their action, and appraises it for corrective action all at once.)

This distinction is well-known in economics. Moscati and Tubaro [2009] point out:

> ... it is the economist who rationalizes the decision maker's choices as if they were generated by utility maximization. Therefore, the utility function and its maximization are in the economist's mind rather than in the decision maker's, so that the psychology of the latter is not at issue. (p. 1)

In other words, decision makers act; economists make sense of those actions and decisions.

Homo economicus is a set of observer concepts (self interest, monetary quantification of it, objective probability, and expected value) and a principle for making sense of choices. *Homo communitatis* is a different set of systematically related Observer concepts. An actor's behavior *conforms to* the CRJ diagram, but in the role of Actor an individual does not identify circumstances and their implications for action, identify what social practice they would

be engaging in by carrying out some behavior, assess the significance of a behavior, consider their personal characteristics and the priorities each reason should have, and so forth. Figure 1 is a reconstruction, from the Observer perspective, of the factors involved in the individual's action, not a depiction of the world *as seen from the perspective of Actor*.

The World from Actor and Observer Perspectives

The "job" of the actor is to moment by moment carry out or continue a behavior (unless the actor recognizes it is no longer called for). What is relevant to fulfilling that "job description" is the current behavior, possible behaviors, and anything relevant to one or more of them: reasons for or against one or more, presence or absence of anything needed to do one or more, and whether or not anything that must be so in order to carry out one or more is the case. We may summarize this by saying that to the Actor, the world appears as a field of action (Ossorio [2006], p. 254).

The Observer's "job" is to observe and describe. To fulfill that job description with respect to behaving persons, an Observer needs a set of concepts articulating the concepts of person, behavior, and the real world. The principles of *homo communitatis* are such an articulation, allowing an Observer to re-describe what an Actor sees in terms of circumstances, reasons, relative priorities of possible behaviors, behavior itself being a multifaceted concept. Thus, an observer watching a person pick up and drink a glass of water describes it as, "He wanted a drink; he saw the water; he picked it up and drank it." The person, *when drinking*, recognizes something to drink and drinks it. He does *not* see the glass of water, somehow deduce the possibility of drinking it, and enact that action. Nor does he recognize each of the parameters of Intentional Action (Principle 2) and then in some fashion deduce that the thing to do is drink.

Actions take place in the real world. Looking at that whole state of affairs, Actor-acting-in-world, we can observe that there are two kinds of facts, facts about the actor and the rest. Facts about the actor are aspects of the actor in their position in their world; it is "how they are" at that moment. The rest, the externals, are what the Actor, to act, must find out about by observation. Observers have no access to actor facts, and so must, if they need to make sense of an action, construct formulations using concepts such as states of affairs, intentional and deliberate action, states of affairs, and priorities.

Appraisals, as states of affairs, are parts of the world, and like everything else look different from Actor and Observer perspectives. Consider a familiar example: person P asks friend Q for a favor; Q does the favor. An Observer describes this interaction as 1) P and Q have the relationship "friend"; 2) having that relationship is grounds for Q to do the favor requested; 3) doing the favor has sufficiently high priority for Q, in this case, that he did it. From the perspective of Actor, though, things look very different. As an Actor, Q *knows* his reasons and how strongly they count for him, *sees* actions and possible actions, and *does* the one called for. Where Observers see circumstances, appraisals of them, and priorities, Actors see what action is called for.

Now let us apply these principles to our topic of interest, "subjective probability." P observes that X (e.g., Greek bond default) is "unlikely." "Unlikely" is an appraisal of X by P. That appraisal of X constitutes grounds for engaging in various actions (e.g., sell euros, buy Greek bonds, monitor the stock much more closely than they would otherwise) and the relative priorities of those actions for him. In the role of Actor, though, P sees possible actions, knows the reasons and how much they count for him, and acts. If after his first observation P continues to act as an Observer, he may construct the elaboration of the state of affairs of the unlikely-but-possible event of Greek bond default just outlined. However, he may choose not continue as Observer, and simply act. Asked "Why did you buy Greek bonds?" P may reconstruct his action in term of circumstances, reasons, and priorities, but he is then giving an account, as an Observer, of what he did as an Actor, not reporting what happened *at the time of acting*.

If P responds with, "Seemed like the thing to do" or "I felt like it," he is not voicing ignorance; he is refusing to engage in the reconstruction. "I knew what to do, but I didn't know why," "I couldn't tell you why I did that," "I don't know why, but I'm sure," and the famous, "I had a feeling" are all language for expressing knowing what to do but not being able to articulate the reasons. They are *Actor language*, that is, ways of talking about the world *as seen by the Actor*. Probably the most famous of these is "intuition": the ability to discern, without being able to articulate reasons.

The fourth section discusses new methods for decision analysis, including financial decisions, based on the reformulation of "subjective probability" as uncertainty appraisal. Several of these technique rely on the concept of Actor perspective for dealing with decision making under uncertainty. Since the Actor's world looks very unlike the Observer/Critic's, the new methods in many cases bear little resemblance to traditional decision-theoretic methods. It is therefore significant that the effectiveness of these methods has been repeatedly demonstrated via use in actual business decisions.

IMPLICATIONS AND APPLICATIONS

The uncertainty appraisal reformulation has two kinds of implications for research and professional practice: addressing confusion and misunderstandings caused by treating the objective and subjective as two sides of the probability coin, and new methods for precise specification

and use of uncertainty appraisals. Since actual probabilities are refined by improving data and calculating a numerical value, while uncertainty appraisals are refined by improving the specification of actions potentially called for, reasons for or against each, and relative priorities, integrating probabilities and uncertainty appraisals requires integrating two very different kinds of information, and therefore very different methods than simply doing an expected value calculation. (It is our view that adopting the term "uncertainty appraisal" and avoiding the traditional term "subjective probability" would be a valuable initial move on the part of researchers and professionals who must communicate about such phenomena, both to other professional and to lay persons.)

Correcting Mistakes

Talking about and treating probability and uncertainty appraisals as though they were versions of the same thing has caused difficulties, sometimes serious ones, virtually everywhere that the two concepts are involved, which is to say virtually everywhere that persons make decisions under uncertainty. The difficulties range from relatively simple misunderstandings that are easily cleared up with a short discussion to deeply embedded concepts and research practices yielding spurious conclusions and flawed practices.

One kind of problem of this sort is failure to carefully distinguish between estimating an (objective) probability and clarifying an appraisal. An excellent example of this error is the following:

> There are situations in which people assess . . . the probability of an event by the ease with which instances . . . can be brought to mind. For example, one may assess the risk of heart attack among middle-aged people by recalling such occurrences among one's acquaintances. Similarly, one may evaluate the probability that a given business venture may fail by imagining various difficulties it could encounter. (Tversky and Kahneman [1974], p. 1127)

Risk of heart attack among middle-aged people is a probabilistic fact; the sample set is middle-aged people, and the subset is those who will have a heart attack during some specified period. Actual data can be gathered, and the estimate compared to the result. By contrast, the probability that *this specific individual* will have a heart attack, or the probability that a given business venture may fail, are not a probabilities at all, because there is no event set: the question involves a specific individual, not a collection to be sampled. These are cases of using probability language to give an appraisal of the possible success of the business venture. While there is a sample space of business ventures, or business ventures having various characteristics, the question—the central question in all decisions to buy or sell a corporate bond—is whether the *particular* business venture will fail, not whether *some* venture will. The authors' point, that persons often estimate poorly, is valid and important, but they have illustrated it by citing entirely different kinds of assessment, the first a case of (actual) probability and the second an uncertainty appraisal. What the two assessments have in common is that they both involve asking persons to identify facts or possible facts (heart attacks or business difficulties), and the authors correctly note that that task is often done poorly. Unfortunately, the authors' well-taken point obscures the fact that the estimates are of entirely different thing, numerical facts and appraisals.

The first section discussed a second problem, one exhibited by a good deal of the foundational research in probability fallacies and biases. This is the assumption that if an experimenter asks a question using probability language, such as "probable," "most likely," etc., subjects will interpret the question as a probability estimation task and do some kind of probability calculation on some sample space. For example, that persons use representativeness is beyond question; that it is a probability bias, as Tversky and Kahneman [1974] characterize it, is based on the assumption that subjects take "What is the probability that Steve is a librarian?" to be a probability question rather than, for example, "Which occupation does Steve most resemble?" There are similar problems with several other purported errors in probability computations, such as the conjunction fallacy (Tversky and Kahneman [1983]), availability bias, and base rate fallacy.

Recommendations

Conflating measurement of actual probability, estimation of actual probability, and giving uncertainty appraisals (*née* "subjective probability") is improper scientific procedure. It has resulted in a great deal of confusion among researchers and practitioners, and significantly impacted lay persons in economics and in all fields dealing with risk and decision under uncertainty, ranging from medical decision making (Gigerenzer [2012]) to national intelligence estimates (Central Intelligence Agency [1999]) to civil engineering (U.S. Department of the Interior [2011]). In our view corrective action is called for, at a minimum including:

- Insistence by journal editors and reviewers on clear identification of which task—counting, estimation of a count, or uncertainty appraisal—is the focus of an experiment.
- Insistence by journal editors and reviewers on specification of measures taken to verify that questions couched in the language of objective probability are actually interpreted by subjects as probability questions. In the "Linda" experiment, a favorite example of many authors writing about heuristics and biases, there are at least five interpretations of the question

"Which of the statements [about Linda] is most likely?" in addition to the probabilistic one assumed (Jeffrey [2010]). Failure to take such measures is failure to control a crucial variable, an elementary experimental error.

- Insistence by journal editors and reviewers that an objective probability description of a "subjective probability" question be accompanied by a specification of the sample space and the repeated trials, and the justification for assuming that the space is actually being used by the subjects, that is, is a distinction on which they are acting. In our view such specifications are as fundamental as any other aspect of experimental procedure description, such as identification of the source of experimental subjects or statistical methods used.
- The above two items should be adopted as standards to be met by all Ph.D. dissertation work.

New Methods

The elaboration of the principles of *homo communitatis* above, particularly the Actor-Observer-Critic distinction, yields a number of practical methods for addressing decisions in the face of uncertainty, several of which have been extensively tested in practice over the past three decades. Three of these are presented below.

Pragmatic Evaluation (Putman [1980])

The traditional approach to decision making under uncertainty, on which virtually all traditional methods are based, is what might appropriately be called the "truth-seeking" approach: evaluate the situation to produce a clear picture, and when we have enough data, the choice will be clear. The problem with this approach is that, as any number of sociologists, philosophers, anthropologists, and psychologists have pointed out, data virtually never "speaks for itself." To make matters worse, as Gigerenzer [1994] points out, in actual practice a person is looking at a particular representation of the data, and the representation greatly affects whether the person can understand and act on the data. Any evaluation, even the most systematic and comprehensive, produces data that is a picture of the situation, and the picture cannot direct the decision.

Truth-seeking approaches to decision making proceed by finding the data, assessing its implications, and then seeking the correct relative weighting of the implications. Pragmatic evaluation, by contrast, proceeds in the reverse manner. It corresponds to going right-to-left on the CRJ diagram (Figure 1), rather than the customary left-to-right:

1. An evaluation is always done for one purpose: to enable an individual (or individuals) to make a (behavioral) choice. Therefore begin by identifying 1) Who is making the choice, and 2) What choice are they making? That choice will always be whether to do Y (perhaps in lieu of doing Z, W, etc.): Do I invest in security X? Do I increase my holdings in large-cap mutual funds? Do I sell my Greek bonds? Do I recommend that a client re-balance his/her portfolio?
2. Individuals vary enormously in terms of what kind of information they want, and in what form, for the particular choice they need to make. In deciding whether to release a new version of a major software project, for example, one manager may want extensive reliability reports and analyses of the mathematical characteristics of the history of the reliability data, while another may value only the professional judgments of three trusted subordinates. Having determined that the evaluation of the situation is being done to enable person P to decide whether to do Y, the second step is to identify what kind of information "counts" for P for this choice, that is, constitutes a reason, with a particular P to act in one way or another. "Information" here means both the state of affairs that implies action *and* the representation of that information. It is important to recognize that the information that counts for P may be much broader than the particular thing being evaluated. It may include relationships between P and others in his/her organization, P's standing in his/her organization, relationships between P's organization and other organizations, such as their customers, and so forth. For example, a legal firm requests that a litigation support provider do a pilot study evaluation of new technology, but in deciding whether to hire the provider bases their decision in part on their legal clients' reaction to the use of new technology in their case.

   This is perhaps the single greatest problem faced by practitioners of behavioral finance. It is crucial to know what kind of information counts for the decision maker, for this decision, and that may be quite difficult to find out. It is also one of the common sources of error in decision analysis, resulting in inappropriate attributions of overoptimism bias and sunk cost fallacy when an executive persists in a course of action that does not succeed.
3. Having identified what P needs to know, and the form in which it needs to be presented to him or her, the evaluator (or decision maker) is now in the position to ask, "What data—what facts and circumstances—need to be gathered, and how does that data need to be processed, to determine the information P needs?" A number of practical considerations that must be addressed to prepare for and carry out the data-gathering and analysis, including cost, requisite permissions, any necessary collaboration or cooperation, etc., are addressed in this final step, to ensure the

practicality of gathering and presenting to P the information he/she needs for this purpose, that is, deciding whether to do Y.

Complete Situation Analysis

The traditional approach of assigning "subjective" probabilities numerical values and then calculating expected values has the advantage that it neatly uses the same framework for both actual and "subjective" probabilities and appears to offer a straightforward way to approach complex decisions. The major disadvantage is that as we have seen "subjective" probabilities are not probabilities at all, and as a result it is entirely unclear what the "expected values" mean, or indeed whether they mean anything at all. This is not "mere semantics." It means that the numbers used to represent "subjective probabilities" violate the definition of a probability measure, and therefore the calculation does not in fact produce expected values. Specifically, numbers attached to distinct events of a set of possible outcomes may add to more than 1.0 (Macchi [1999]) or less than 1.0 (Tversky and Koehler [1994]). Mandel [2005], for example, found that in a group of 135 students at the University of Victoria the median estimated probability of a terrorist attack within two months was 0.10, while the median value the probability of no attack was 0.50.

Tversky and Koehler [1994] state,

> The major conclusion of the present research is that subjective probability, or degree of belief, is nonextensional and hence nonmeasurable in the sense that alternative partitions of the space [of possibilities] can yield different judgment. . . .The evidence reported here and elsewhere indicates that both qualitative and quantitative assessments of uncertainty are not carried out in a logically coherent fashion, and one might be tempted to conclude that they should not be carried out at all. However, . . .in general, there are no alternative procedures for assessing uncertainty.

They conclude,

> . . .judgments of uncertainty . . . play an essential role in people's deliberations and decisions. The question of how to improve their quality . . . poses a major challenge to theorists and practitioners alike. (p. 565)

A relatively recent approach to the problem is the one advocated by Gigerenzer [2005, 1994]. Health care providers frequently face a different form of the uncertainty-relationship question: "What are my chances?" Similar questions arise in all fields in which there is uncertainty: "What are the chances this dam will fail?" or "What are the chances this stock will go up?" and so forth. They are uncertainty appraisal, not probability, questions because they are single events; there is no sample space and therefore no probability.

Gigerenzer's recommended approach is to respond to a "single-event probability" question with a frequency answer, for example, "Of 100 patients like you, 10 will recover." Frequency answers can always be given and they are well-understood by professionals and lay-persons (Gigerenzer [1994]). Unfortunately, the frequency-answer approach does not address the challenge posed by Tversky and Koehler, because a frequency response is not an answer to the question asked. "What are my chances" is a question about an individual; "What are the chances of someone like me" is a question about a group. Giving frequency answers is a method for changing the subject, not answering the question.

Judgments of uncertainty matter because of their role in decisions. The call for improving the quality of judgments of uncertainty is a call for a better method for making decisions involving actual and "subjective" probabilities, that is, decisions involving both probabilistic facts and uncertainty appraisals. Complete Situation Analysis addresses that need.

The "presenting question" in Pragmatic Evaluation is, "What are the facts," and the first step is to change that question to, "Who is making what behavioral choice?" With "subjective" or single-event probabilities the "who" is the questioner, and the initial question is, "What should *I* do," or, more completely stated, "What should *I/we* do, in light of *these* circumstances and *these* uncertainty appraisals?"

Accordingly, we proceed in two phases, essentially separating uncertainties from probabilities. In Phase 1, after specifying the behavioral choice, the decision maker identifies the significance of each alternative action, incorporating all the factors unique to them and their specific circumstances, including the uncertainty appraisals (i.e., the factors the individual has "a degree of belief" in). The result is a set of pragmatically complete "Complete Situation" descriptions, each of which identifies the impact of each behavior in all the ways that matter to the individual. (The point of the word "complete" here is to emphasize the breadth of the outcome-significance analysis involved.) In Phase 2, the actual probabilities are incorporated, thereby associating the probabilities with the Complete Situations, e.g., "20% chance of CS_1 and an 80% chance of CS_2." Recalling that the core of the difficulty here is the distinction between "What are the chances of someone just like me?" and "What are *my* chances?" Phase 1 builds an event set that takes into account all the relevant factors unique to the individual and all the factors about which they are uncertain (factors that have customarily, and as we have seen, misleadingly been described as having associated "subjective probabilities"). Phase 2 applies actual probabilities to the events in that set.

The method is:

1. Specify the behavioral choice to be made, including specification of the actor or actors involved.
2. Identify the actually possible outcomes of each choice.
3. Expand the descriptions of each action.
4. Associate the (actual) probabilities with each Complete Situation.
5. Decide.

To elaborate the method a bit:

1. Specify the behavioral choice to be made, including the actor. (Including specification of the actor is a pragmatic, not logical, necessity. The actor is known. But the method is to be carried out *by the person(s) making the decision*. An actor will frequently give different answers to "Should I do this?" than they will to "Should this be done?")
2. Identify the actually possible outcomes of each choice. The possible outcomes are the ones the decision maker considers the relevant possibilities.
3. Expand the descriptions of each action. The result of these first two steps is a list of names of actions and possible outcomes. The crucial next step is to expand the descriptions of each action to include:
    a. The process involved in the action and any facts specifically related to the process, such as cost, time, other events that may occur during the process, etc.
    b. The significance of each outcome, including 1) the larger practices the actor is doing by doing each alternative action and 2) the impact of each possible outcome on each state of affairs of importance to the actor. Those states of affairs will include effects on other persons with which the individual has a relationship that matters to them, choice principles of any community the person is part of, and, very importantly, the individual's place in those communities, e.g., their family, their company, their church, etc. This specifically includes the individual's concept of "what kind of person he/she is," that is, self-concept.
4. Associate the (actual) probabilities with each Complete Situation: for each expanded action+outcomes, ask: "What is the probability of this happening?"
5. Decide.

We illustrate with two examples. We begin with a health care example, a decision widely recognized as involving both actual probabilities and nonprobabilistic uncertainties, that is, uncertainty appraisals. Consider a patient with a diagnosed cancer whose doctor has recommended a course of chemotherapy. The patient asks, "What are my chances?"

1. The behavioral choice: do I undergo chemotherapy?
2. Actually possible outcomes:
    a. Undergo chemo and survive.
    b. Undergo chemo and live 6 months longer.
    c. Undergo chemo and die within 2 years
    d. Forgo chemo and die within 2 years.
    e. Forgo chemo and recover.

    (A different individual might distinguish different possible outcomes, e.g., living 3 months, living 6 months, living one year, and recovery.)
3. Expand outcomes to Complete Situations:
    a. Undergoing chemo and recovery means CS_1:

- Months of very unpleasant sickness, very low life quality and inability to carry out normal duties in my family and work.
- I'm affirming my self-image as a fighter.
- My spouse sees I did all I possibly could.
- My spouse will see me suffering during the treatment.
- I'll be able to finish the research project I'm working on, which means a lot to me.
- I get to attend my daughter's wedding in 10 months.
- I may see grandchildren.

    b. Undergoing chemo and live 6 months longer means CS_2:

- Months of very unpleasant sickness, very low life quality and inability to carry out normal duties in my family and work.
- I'm affirming my self-image as a fighter.
- My spouse sees I did all I possibly could.
- My spouse will see me suffering during the treatment.
- I'll be able to finish the research project I'm working on, which means a lot to me.

    c. Undergo chemo and die within 2 years means CS_3:

- Months of very unpleasant sickness, very low life quality and inability to carry out normal duties in my family and work.
- I'm affirming my self-image as a fighter.
- My spouse sees I did all I possibly could.
- My spouse will see me suffering during the treatment.

    d. Forgoing chemo and dying in 2 years means CS_4:

- A 2-year decline.
- Much better time with my family during the two years.
- I'll be able to finish a research project that is important to me.
- I'll have time to make peace with my passing.
- I'm doing something that conflicts with my image of myself as a fighter.
- My spouse will not see me suffering until the end.

- I'll be able to finish the research project I'm working on, which means a lot to me.
- I get to attend my daughter's wedding in 10 months.
  e. e.
  Forgoing chemo and recovery means CS_5
- Moderate sickness and disability for 6 months.
- I'll be able to finish a research project that is important to me.
- Affirms my self-image as a uniquely fortunate person.
- I'll be able to continue my life and all that implies: finishing my research project, attending my daughter's wedding in 10 months, perhaps having grandchildren, etc.
- Relevant facts include: 1) I am much healthier than the average person, 2) I have a history of medical treatments being successful, and 3) I see myself as a lucky person—things generally turn out well for me. (We have deliberately presented here a mixture of the kinds of factors typical of such a situation. two are "objective," empirical, facts; the third is a typical expression of self-concept.)

4. Apply known statistics for survival rates for this cancer and this treatment regime to these Complete Situations, for example: $p(CS_1) = 0.60$; $p(CS_2) = 0.30$; $p(CS_3) = 0.10$; $p(CS_4) = 0.98$; $p(CS_5) = 0.02$.
5. Decide.

Step 3 is the point at which the particulars of the situation, including the individual's values, their relationships, their "place" in their life and the impact on that place and what they care about of the possible actions is taken into account. Persons do not make decisions based simply on outcome, a fact discussed extensively by Jeffrey and Putman [2013], and the immediate-outcome-only picture of a decision situation omits most of what matters to an individual. In this step these other factors are re-incorporated. An investor is not asking, "What is the probability that a stock of this kind will go up 10% over the next 6 months?" out of intellectual or academic interest; they are asking, "What is the probability that *this* stock will go up 10% over the next 6 months," and what they are doing by asking that question is asking, "Do I invest *this* amount of money in *this* stock, given *my* life circumstances, including but not limited to the financial ones?"

This step is an articulation of the impact of each outcome on the individual, *from the Actor's point of view* (which is why we have stated the significance of each outcome in the first person. Actor statements are first-person). The circumstances and values involved in those impacts are, as illustrated in the example, highly specific to both the decision maker and the facts of the situation. For example, most decision makers in Western culture will eschew betting the family fortune on a 100:1 shot, but some, such as persons whose self-image is, "That's how it is for most, but I'm a lucky guy," or, "I'm the kind of guy who always goes for it!" So it is with the person who agrees to the 1 in 100 chance of success of a particular treatment, or a 1 in 100 chance of their investment behaving in a certain way. A particular uncertainty appraisal ("likely," "barely possible," etc.) is, to the Actor, reason, with a relative priority with respect to other reasons, to do X. That priority depends on the decision maker's personal characteristics and other circumstances, including cultural choice principles. If the decision is an organizational one, the circumstances include the organization's choice principles. "X a long shot" is an appraisal of X. To some, that state of affairs constitutes reason to not do X; to others, it is the opposite.

Complete Situation Analysis is explicitly descriptive, rather than prescriptive. Current standard practice is to attribute acting in certain ways in the face of uncertain outcomes to various cognitive errors, such as over-optimism bias (Shefrin [2000]). While that may sometimes be the case, it may equally reflect a particular personal characteristic on the part of the decision maker, such as extraordinary determination to succeed. It would not seem appropriate, for example, to ascribe Eisenhower's decision to proceed with the D-Day invasion to a cognitive error. By contrast, Complete Situation Analysis immediately clarifies Eisenhower's decision, the one shown clearly in his D-Day Orders (Eisenhower [n.d.]): "the elimination of Nazi tyranny."

Now let us apply Complete Situation Analysis to a finance decision, one involving, as in health care, probabilistic and non-probabilistic uncertainties. A bond trader asks, "What are the chances that Company X, whose bonds are rated BBB+, will go bankrupt in 6 months?" There are well-known default statistics, but the trader is asking, "I know the statistics, but I know how *this* company does business and *its* financials. What is *its* probability of default?" The first step is to recognize that the presenting question is in the service of deciding, "Do I invest amount A in *these* bonds?" Thus, we have:

1. The behavioral choice: do I invest amount A in Company X's bonds?
2. Actually possible outcomes:
   a. Invest and after 6 months bond price has increased *p* percent.
   b. Invest and after 6 months X is bankrupt.
   c. Forgo investment in X and after 6 months it has increased *p* percent.
   d. Forgo investment in X and after 6 months X is bankrupt.
3. Expanded outcomes:
   a. I have affirmed my image of myself as an a smart, decisive trader; company X has performed as I anticipated based on my knowledge of it; I have made a profit on X; my portfolio is increased in value; my standing among my peers is improved, though this is not very important to me; I am

acting in accord with my organization's culture.

In the cancer treatment example, the importance of including aspects of the situation related to the individual's character and values is obvious. It is perhaps not so obvious that one must explicitly identify that range of facts in *all* decisions, including financial ones, perhaps because of a belief that that decisions should be based purely on "objective" (i.e., not personal) factors, such as the financial aspects. The final item here illustrates that the character of the specific person is always a necessary part of identifying the expanded outcomes. For many, the last statement would be incorrect—standing among peers is quite important to them. Equally important is that people often not skilled at recognizing this kind of fact about themselves, or have strong reasons to not recognize them. For example, an individual who has a strong belief they should not be concerned what their peers think, or is working in an organization that puts a high priority on assertiveness and independence, is unlikely to recognize it when they do in fact care. As a result, consultation with colleagues or trusted friends can be very valuable in identifying expanded outcomes. "Know thyself" is the common recommendation, but it one that people often require assistance to follow.

In the case of institutional investors, the impact of organizational culture or choice principles on investor decisions (Jeffrey and Putman [2013]) must always be taken into account. That organizations vary along dimensions such as aggressiveness/caution, degree of acceptable risk, individualism/collaborativeness, etc., and that these principles affect individual choices, is universally recognized. These principles are as much a part of the complete situation as the financials of prospective investment. Crucially, "organization" refers to the immediate cohesive work group the investor is operating as a member of; this may or may not be the entire firm. Differences in principles between different divisions, even different supervisory groups, are well known.

b. Company X has performed contrary to what I expected, based on my knowledge of it; I am less sure of my image of myself as an a smart, decisive trader; the portfolio I manage has lost value; my manager has some question about my business judgment, though this is not very important because I trust him to see the larger picture; though it did not work out, this kind of risk is expected of someone in my organization.
c. I have acted contrary to my image of myself as an a smart, decisive trader; I see that I could have made a profit and I regret it; I have amount A to invest in other companies; I have forgone an investment I believe would have paid off, and I really hate that; I am somewhat more determined to follow my own judgment next time.
d. I have acted contrary to my image of myself as an a smart, decisive trader; I see that I would have lost money and know I "dodged a bullet"; I have amount A to invest in other companies; I have foregone an investment I believe would have paid off; I am somewhat less confident of my ability to assess the financial health of a company; my manager sees me as having wisely refrained from investing in X, and I value that, though it does not carry huge weight for me.

4. Apply known statistics for financial performance of this kind of company to these Complete Situations: $p(CS_1) = 0.30$; $p(CS_2) = 0.70$; $p(CS_3) = 0.30$; $p(CS_4) = 0.70$.
5. Decide.

As in the previous example, the particulars elaborated in Step 3 depend strongly on the individual, manager, and organization. "Psychological" factors such as effect on self-confidence, regret or relief, and manager's assessment are included in this example because they are all universally recognized as factors that affect financial decisions. As discussed in Jeffrey and Putman [2013], any description of the decision that omits them is seriously deficient. "Degree of belief" is an uncertainty appraisal; how one acts on those appraisals in the presence of multiple reasons to act one way or another is intimately related to the personality characteristic of "decisiveness"; having the appropriate balance of caution and decisiveness is an important part of any adult's self-concept and of how others in their organization see them; and finally, "appropriate" means, "in accordance with the individual's values" and "in accordance with our company's choice principles," in the cases of how an individual sees themselves and how others see them, respectively.

Experience with the Complete Situation Analysis method has shown that its success requires inclusion of the actor in the specification of the behavioral choice in Step 1. Omitting the pronoun in the sentence, converting "Do I invest?" into "Whether to invest," is particularly easy in a domain in which decisions are traditionally viewed as "impersonal" and personal factors as "irrational" influences to be avoided. The method is, by design, specific to the person making the decision and all of their reasons to act one way or another.

We invite the reader to complete Complete Situation Analyses for the following cases, with Step 1 as indicated:

- What are the chances of a recession during the next year?
  - Decision: "Do we (the Federal Reserve) raise interest rates 0.25%?"
- What are the chances of war with Iran?
  - Decision: "Do I, the President, agree to deploy Aircraft Carrier *Eisenhower* Strike Group to the Persian Gulf?"
- What are the chances that a Category 4 hurricane will strike New Orleans in the next 10 years?
  - Decision: "Do we, the U.S. Army Corps of Engineers, request $30 billion to construct a levee system for New Orleans capable of withstanding a direct strike by a Category 4 hurricane?"

The final question in this list illustrates the complexity arising from the inescapable fact that different behavioral decisions are likely to result in very different analyses of "the same" question. Consider the following alternatives to the question here:

- Do we, the United States Congress, allocate $30 billion to construct a levee system for New Orleans capable of withstanding a direct strike by a Category 4 hurricane?
- Do I, a United States Senator from Louisiana, vote to allocate $30 billion to construct a levee system for New Orleans capable of withstanding a direct strike by a Category 4 hurricane?
- Do I, a United States Representative from Idaho, vote to allocate $30 billion to construct a levee system for New Orleans capable of withstanding a direct strike by a Category 4 hurricane?
- Do I, a candidate for President of the United States, advocate the allocation of $30 billion to construct a levee system for New Orleans capable of withstanding a direct strike by a Category 4 hurricane?"

This kind of issue is present in many, perhaps most, decisions of substantial significance in organizations.

Finding the Pattern in the Data

People are often faced with a large number of observations, facts, and possible facts with no recognizable pattern, that is, no single description of what the many states of affairs "add up to," and finance is a prime example. There is an immense amount of data in the fields of economics and finance; the existence of a distinct field, History of Economics, testifies to the difficulty of finding patterns in that data. Finding the pattern in the data is of central importance in any finance policy decision and in many specific investment decisions.

We do not look for a pattern, a summary description, as an academic or intellectual exercise; rather, we do it as part of a decision analysis. One common case in which this arises is the brainstorm, which by design generates a large number of observations that may have little coherence and many of which are of little value when later evaluated critically. The standard method of brainstorming is to generate ideas and then "distill" them via a voting procedure, in which each participant is given a number of votes to distribute among the top n ideas. If the brainstorm was done properly, this distillation procedure is typically very difficult for participants, because they are forced to allocate votes among different concepts, multiple aspects of the same concept, sometimes including between small but crucial details of a concept and the overall concept itself. Consider, for example, the dilemma facing a participant voting on features of a software system for controlling elevators (Gause and Weinberg [1989]), faced with brainstormed possible functions including:

- Display selected floors
- Show passengers' floor selections
- "Scream" when passengers are assaulted
- Trap assaulters in the elevator
- Display directory information
- Give directions for delivery people

Giving delivery directions is closely related to displaying directory information; displaying floor selections is close to displaying passengers' selections; both assault measures are exotic and impractical, but "Provide a panic button" is not, though it is not on the list. Voting among alternatives provides no way to identify larger functions that would encompass these specifics, and forces participants to allocate their votes among multiple aspects of one larger one. The usual distillation is unsuitable for finding patterns and making decisions.

It would seem natural, if the goal is to find the pattern in the data, to show participants the data and ask, "What is the pattern?" Experience has shown that that procedure works poorly. Participants find it very difficult to identify and describe patterns in data, and it is quite rare for it to result in a pattern that the participants agree on, other than grudgingly. As Ryan and Bernard [2003] put it, "theme identification is one of the most fundamental tasks in qualitative research. It also is one of the most mysterious" (p. 85).

Actor-based Distillation is a different approach, based on the concept of Actor functioning. It has been has been used in a wide variety of settings, from individuals formulating psychological case studies to large complex software organizational planning tasks, and has been found to be a reliable and effective method for eliciting "the pattern" from participants.

The procedure is to present the participants with a list of the observations, facts, data, possibilities, etc., and ask them to do a simple task: look at the list of what is known and make simple observations about it and the items on it,

in the form of simple declarative sentences. (Note the contrast with the customary question, "What is the pattern here?") Any observation is acceptable, as long as it is in the form of a simple declarative sentence. (The sentence form is essential, not a detail. Simple declarative sentences are the customary English form used to identify overall states of affairs, rather than re-statements of details. Long, complex sentences identify perhaps-important details of the overall state of affairs, not the state of affairs itself, and it is that identification that is needed.)

For example, consider once more the case of deciding whether to purchase the BBB+ rated bonds of Company X. One way to approach that question is to first ask, "What is the pattern of X's performance in the industry conditions expected over the next six months?" An answer to that question is developed by first developing a list of facts, observations, data, and possibilities involving X, its industry, other companies in the industry, related industries, etc., that is, anything considered possibly relevant by a group of participants with expertise relevant in some way to this company or industry. (One would not expect, for example, specialists in the paper products industry to have relevant observations about the construction equipment rental industry, but an individual might have accounting experience with companies like X.) One then asks the participants to look at the list of observations—financial, accounting, company-specific, related-company, industry, related-industry, macro-economic—and make observations in the form of simple declarative sentences.

In practice, perhaps the most striking fact about this procedure is its speed: it typically takes only from 3 to 5 observations before some participant recognizes the overall pattern and voices it. Less experienced moderators routinely report surprise at how quickly a pattern acceptable to all the participants is articulated.

We have termed this method "Actor-based Distillation" because it is based on the concept of Actor functioning, while traditional methods are cases of Observer/Critic functioning. Actor functioning is looking at a situation and acting, without any kind of analysis or re-description, just as one picks up a glass and drinks. The actor's world is of a field of action, consisting in this case of the opportunities to act defined by the moderator's initial request.

SUMMARY

The facts and concepts that have since the 17th century been known as "subjective probability" are central to behavioral economics and finance. In the last several years, treating subjective probability as a form of probability has been shown decisively to fail, both conceptually and mathematically. Conceptually, objective probability is a ratio of cardinalities of event sets, while what have been called subjective probabilities are appraisals of the uncertainty of possible facts. Mathematically, assigning numerical values to subjective probabilities results in values that violate the axioms of probability: they are sub- or super-additive. It is no longer viable to think of objective and "subjective" probabilities as two sides of the same coin, or treat them that way. In particular, assigning numerical values to subjective probabilities and then calculating expected values with them is no longer a viable analytic method. The failure of the concept of subjective probability means a new concept and methods for analyzing and dealing with the uncertainties that cannot be dealt with by assigning them a number are needed. To address those needs, we have presented a complete re-formulation of subjective probability as *uncertainty appraisals*, re-descriptions of states of affairs carrying tautological implications for action. Whereas probabilities are refined by better specification and methods for collecting and using event set numerical data, uncertainty appraisals are refined by better specification of the actions tautologically implied by the appraisal and their relative priorities. Combining a concept new to economics and finance, the difference between the Actor and Observer perspectives, with the uncertainty appraisal formulation, new methods for decision analysis have been developed, including pragmatic (action-oriented) evaluation of data, actor-based distillation of data into patterns, and Complete Situation Analysis, which integrates probabilities and uncertainty appraisals, which replaces the old approach of calculating expected value on the basis numerical values that do not represent probabilities.

NOTES

1. A formulation of the concepts of intentional action, deliberate action, and community in mathematical formalism may be found in Jeffrey [2010].
2. Poincaré [1905/1970] noted that "axioms are definitions in disguise"; that is, they articulate the basic relationships in a mathematical domain. The principles of *homo economicus* articulate the domain of behavior. As Putman discusses in his introduction to Ossorio [2012], this does not mean that the principles function as postulates or assumed truths.

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