PSF Estimation in SAR Imagery Restoration Based on Corner Reflectors

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Abstract—Point Spread Function (PSF) estimation is a key issue influencing the quality of imagery restoration. There are many PSF estimation methods for optical imagery but few for SAR imagery. This paper puts forward a new method of PSF estimation for restoration of SAR imagery. By setting corner reflectors in the field, according to the brightness and scattering features, the PSF of SAR imagery can be simulated adopting the elliptic paraboloid model; therefore, the parameters of the model can be deduced. According to the model mentioned above, SAR imagery restoration experiments were carried out. The results show that PSF based on the elliptic paraboloid model is feasible. The algorithm of the model is simple and the quality of the restored SAR imagery is enhanced.

Keywords—SAR imagery restoration; PSF; corner reflector; elliptic paraboloid

I. INTRODUCTION

With the successful launches of Synthetic Aperture Radar (SAR) satellites, the application field of SAR imagery is gradually expanding. As a result, the interpretation of target objects in SAR imagery becomes a hot issue. The disturbance of SAR signals by other signals results in blur of SAR imagery after signal processing, furthermore influencing the post interpretation and other applications. So, the image restoration technique becomes a main method to enhance the quality of images. Image restoration is a series of procedures to remove or reduce imagery degradation that is produced during image capture and processing procedures, and it reconstructs the degraded images to non-degraded or approximately non-degraded images. It is common to perform image restoration through an estimated degradation function while the PSF estimation is a common method of estimating the degradation function.

For an optical system, when the incident light source is a point light source, the light field distribution in the image is called the Point Spread Function (PSF), or the Impulse Response Function (IRF). During optimal image restoration, through giving some atmosphere parameters in advance, scholars adopted the Monte Carlo method to simulate the PSF of the sensor under the atmosphere condition of imaging [1]. But this method has high requirements for imaging parameters, experiment conditions and a large amount computation. Other scholars worked out the PSF of Gauss style in blind image restoration [9]. Forster et al. [10], Ruiz et al. [11] have discussed how to use the line type target in SPOT imagery to estimate PSF, and have used an inverse filter to restore images, which obtained better results. Z. J. Liu et al. adopt an experiential simulating method to get PSF through finding line type targets in the imagery. The disadvantage of this method is that sometimes it is hard to find ideal pixels with ideal brightness and contrast; therefore the estimated PSF is empirical [2].

The resolution of former spaceborne SAR imagery such as ERS-1/2 and JERS-1 is relatively low; therefore there are only a few studies on SAR image restoration. Recently high resolution SAR satellites such as COSMO-SkyMed, Radarsat-2, and TerraSAR-X have emerged, which enhanced the ability of SAR object recognition so that SAR image restoration has become a hot issue. Till now, there have been only a few studies on PSF estimation of spaceborne SAR images. Blacknell and Quegan have analyzed the relationship between PSF and SAR image restoration and proved the feasibility of image restoration in theory by estimating imaging PSF, but it is not proved by experiments [5,7]. Majumder and Temple carried out experiment to analyze the PSF of CSAR (Circular Synthetic Aperture Radar); simulated results show an impulse response of a radially displaced point scatterer that is asymmetric and tends to become an elliptic shape [12].

At present, research on PSF estimation of high spaceborne SAR imagery is limited. This paper uses high resolution SAR imagery as the data source to study PSF estimation for image restoration. Through taking advantage of brightness and scattering features of corner reflectors, an elliptic paraboloid model of PSF estimation is put forward and applied in SAR image restoration.

II. THE PSF ESTIMATION MODEL OF SPACEBORNE SAR IMAGERY

Under sparse prior condition, the observation model of optical and SAR images are able to uniform. When PSF of the system is known, the restored imagery can be obtained. By analyzing the PSF model of CSAR and IRF of a SAR system,
this paper puts forward the elliptic paraboloid model to simplify PSF estimation. The method of deduction and restoration method based on this model are given below.

A. Image Degradation/Image Restoration Model

Under sparse prior condition, the observation model of optical and SAR systems both can be expressed as function (1).

\[ g(x, y) = h(x, y) \ast f(x, y) + n(x, y) \]

(1)

\[ f(x, y) \]

Degradation function \[ h(x, y) \]

Restoration filter

Optimized estimation for \[ f(x, y) \]

Restoration Procedure

Figure 1. Image degradation/Image restoration model

g(x,y) is the observation image; h(x,y) is the PSF of the imaging system; f(x,y) is the target imagery; n(x,y) is noise [8]. Image degradation and image restoration are inter-reversible procedures. If only taking the degradation caused by h(x,y) into consideration, by estimating the PSF of imaging procedure, the approximate image of the original image can be calculated.

B. PSF Model of SAR

In SAR imagery, though the size of the corner reflectors is far smaller than the size of a resolution unit, the echo of point targets will generate a disturbance, which is larger than a pixel on the image. This disturbance is caused by combined factors and is represented by h(x, y), which is the PSF of the imaging system. As for spaceborne SAR images, the blur of the image depends on not only on the PSF of sensors but also atmosphere conditions [6]. Therefore, even under the same imaging conditions, the quality of the imager is not the same. Through analyzing the IRF of SAR and the PSF of airborne CSAR, an improved PSF model is introduced.

1) The PSF model of SAR

The PSF Model of SAR is represented by h(x,r) [8]:

\[ h(x, r) = \Re \left[ e^{-j2\pi 1} \frac{\sin \left( \frac{r}{\rho_s} \right)}{\pi \frac{r}{\rho_s}} L_0 e^{j2\pi \frac{x}{\rho_s}} \frac{\sin \left( \frac{x}{\rho_s} \right)}{\pi \frac{x}{\rho_s}} \right] \]

(2)

\[ \rho_s = \frac{l}{2}, \rho_r = \frac{c}{2B}, L_s = \frac{\lambda R}{l} \]

(3)

where B is the bandwidth; C is the light velocity; \( l \) is the antenna aperture size; \( \bar{F} \) is the signal Pulse Width; \( (x, r) \) is azimuth-slant range binary variable; \( \rho_r \) is resolution in the range direction; \( \rho_s \) is resolution in the azimuth direction; \( \bar{K} \) is Linear Frequency Modulated slope; \( \bar{K} \) is the minimum distance between target and satellite; \( I \) is range timing; \( L_s \) is the length of the synthetic aperture. In formula 2, the impulse response of the SAR system has the form of \( \sin c \) function in both the range and azimuth directions, not the ideal \( \delta \) unit impulse function. So, it makes the simulating model and the system theoretical model closer by simulating PSF from the shape of IRF.

2) The PSF of airborne CSAR

As for the PSF of airborne CSAR, Soumekh [3] provides an analytical expression for the 2D PSF of a point target. For a target located at the origin, i.e., \( (x_0, y_0) = (0, 0) \), the 2D PSF is given by:

\[ f_s(x, y) = \rho_{\text{max}} \frac{J_1(r \cdot \rho_{\text{max}})}{r} - \rho_{\text{min}} \frac{J_1(r \cdot \rho_{\text{min}})}{r} \]

(4)

\[ r = \sqrt{x^2 + y^2} \]

(5)

\[ \rho_{\text{max}} = 2k_{\text{max}} \cos(\theta) \]

(6)

\[ \rho_{\text{min}} = 2k_{\text{min}} \cos(\theta) \]

(7)

where \( J_1 \) is a first-order Bessel function of the first kind, \( k_{\text{max}} \) is the maximum wave number of radar signal, \( k_{\text{min}} \) is the minimum wave number of radar signal, and \( \theta \) is the average depression angle of target area. Simulated results show that the impulse response of a radially displaced point scatterer is asymmetric and tends to become an elliptic in shape.

C. The simplified elliptic paraboloid model

From the analysis above, this paper puts forward a simplified elliptic paraboloid model to simulate the imaging of corner reflectors, as shown in formula (8) and (9), which can satisfy the shape and feature of point target imaging. In (8), \( p, q \) represent transverse and lengthwise openings of an ellipse, equal to the diffuse region in azimuth and range directions; \( x, y \) is the location of the pixel; \( x_0, y_0 \) is the coordinate of the point with maximum height; \( z \) is the actual pixel value; \( z_0 \) is the value of pixel moved upward, it is equal to the maximum brightness value of the corner reflector. If the opening is downward, then \( p, q \) and \( z_0 \) are negative. Fig. 2 shows the model of the Elliptic paraboloid. They are similar to IRF in shape.

\[ \frac{(x-x_0)^2}{2p} + \frac{(y-y_0)^2}{2q} = z + z_0 \]

(8)

That is:

\[ F = q(x-x_0)^2 + p(y-y_0)^2 - 2pq(z + z_0) \]

(9)

(a) 3D view   (b) Transverse view   (c) Lengthwise view
The model of elliptic paraboloid.

Five parameters, \( p, q, x_0, y_0, z_0 \) need to be calculated. The original formula is nonlinear and it needs linearization and (10) is the result after differentiation, where \( I \) is the constant item, worked out by using the least squares method.

\[
\frac{\partial F}{\partial x_0} dx_0 + \frac{\partial F}{\partial y_0} dy_0 + \frac{\partial F}{\partial z_0} dz_0 + \frac{\partial F}{\partial p} dp + \frac{\partial F}{\partial q} dq + I = 0
\]

(10)

Image restoration is a converse question and it is ill-conditioned. Thus, it needs more attention paid to the solution method and the initial value of the parameters and the noise should be reduced to the minimum. As initial values, the values of \( x_0, y_0, z_0 \) can be set as the location of the point with maximum brightness of corner reflectors with its gray value, which can also be given by an interactive equation. The initial values of \( p, q \) are given by an interactive equation or the mean value of the simulated results of each corner reflector. Taking the imaging quality of corner reflectors into account, the weighted method is adopted. When the variation of each parameters is less than the given threshold value, the interactive procedure ends.

D. The Restoration Method for a Single SAR Image Based on PSF

The restoration procedure for a single SAR image based on the Elliptic Paraboloid Model:

a) Filtering the noise from the original image to reduce the influence of noise on the following procedures.

b) Working out the parameters- \( p, q, x_0, y_0, z_0 \) of the elliptic paraboloid, then using the parameters to construct the PSF.

c) Wiener filtering restoration based on the PSF.

d) Evaluation of the restored image by comparison of the original and restored images; the result can be evaluated by mean, standard deviation, radiation resolution and equivalent number looks (ENL).

III. EXPERIMENTAL VERIFICATION

A. The Design of the Experiment

The research object of this paper is the gray intensity of SAR images, so it refers to some PSF estimate methods for optical images. The PSF can be determined by adopting a empirically fitted equation through picking up point or line objects. Since lines can be considered as the assembly of many points, here we adopt the point PSF estimation method. In the optical image PSF estimation, sometimes it is hard to find ideal pixels with ideal brightness and contrast. Since SAR is sensitive to the metal target, the corner reflectors are bright white points in images. This paper calculates the PSF based on the elliptic paraboloid model by analyzing the imaging feature of corner reflectors, performs the restoration experiments and evaluates the results.

1) Data acquisition

Trihedral corner reflectors have good directivity when the incident direction of the radar signals are parallel to the normal of the corner reflector; the scattering cross section is then at the maximum. The size of trihedral corner reflector is 1.2m×1.2m×1.2m in this experiment. Before the satellite transits the experiment region, the azimuth and vertical directions of the corner reflectors are adjusted according to the parameters of satellite orbit, which makes the echo signal the strongest. The corner reflectors are laid in a single background region, such as pools, or open areas without metal objects and coordinate information recorded. In this experiment, five or six corner reflectors of the same size were laid in each image. Fig. 3 (a) is an image of a corner reflector, Fig. 3 (b) is a bright point that represents the corner reflector in a SAR image. The imaging parameters are shown in TABLE I.

![Figure 3. Images of corner reflector](image)

TABLE I. THE IMAGING PARAMETERS

<table>
<thead>
<tr>
<th>Satellite Type</th>
<th>Cosmo-SkyMed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imaging Time</td>
<td>20100121</td>
</tr>
<tr>
<td>Incident Angle*</td>
<td>22.16-26.03</td>
</tr>
<tr>
<td>Polarization</td>
<td>HH</td>
</tr>
<tr>
<td>Production Type</td>
<td>SCS</td>
</tr>
</tbody>
</table>

2) The imaging feature analysis of point target of corner reflectors

A 30×30 window was selected with the point target at the center and shown in normalization. Fig. 4 shows the features in 3D view, azimuth and range direction. The size of the point target is about 3×3 at the center and other pixels are the background information. The feature of corner reflection is a cone with similar downward opening in azimuth and range direction; this shape is similar to the SPF of SAR and an elliptic paraboloid.

![Figure 4. The normalization of point target (3D, azimuth and distance view).](image)

3) Point target analysis based on elliptic paraboloid model

Taking a Cosmo-SkyMed image gathered on January 21, 2010 as an example, the light energy distribution model is simulated in azimuth and range direction firstly by adopting the elliptic paraboloid model, shown as Fig. 5. The curves of x1-x5, x6 represent the independent simulation of five corner reflectors in azimuth direction and the combined simulation;
the curves of y1-y5, y6 represent the independent simulation in range direction and the combined simulation. It can be seen from each simulated curve that the trend of light distribution is similar in azimuth and range direction, but some points are different from others, such as point 2; all those are relative to the state of corner reflector, background and imaging environment. But this does not mean that those points are worthless. As for point 2, the trend is similar in azimuth and range direction; the reason that leads to its curve differing from others is that its gray value is lower than those of others. We can distribute a weight to each point so that all points can be used to work out parameters. The simulated result of Cosmo is shown in TABLE II and the parameters 3 images of Cosmo are shown in TABLE III.

![Figure 5. The simulated curves of cosmo image in azimuth and range direction.](image)

**TABLE II. THE SIMULATED RESULT OF COSMO-SKREMED**

<table>
<thead>
<tr>
<th>X axis</th>
<th>Y axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i^{10^4}$</td>
<td>$x_o$</td>
</tr>
<tr>
<td>1</td>
<td>-2.179</td>
</tr>
<tr>
<td>2</td>
<td>-2.268</td>
</tr>
<tr>
<td>3</td>
<td>-3.623</td>
</tr>
<tr>
<td>4</td>
<td>-2.336</td>
</tr>
<tr>
<td>5</td>
<td>-2.653</td>
</tr>
<tr>
<td>6</td>
<td>-2.512</td>
</tr>
<tr>
<td>7</td>
<td>-2.418</td>
</tr>
</tbody>
</table>

a. Notes: 1-5 for the fitting results of five corner reflector points, 6 for Direct simulation, 7 for the weighted simulation.

**TABLE III. THE PQ VALUE CONTRAST OF 3 COSMO IMAGE (DIRECT SIMULATION)**

<table>
<thead>
<tr>
<th>Image time</th>
<th>X axis</th>
<th>Y axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i^{10^4}$</td>
<td>$x_o$</td>
<td>$z_o$</td>
</tr>
<tr>
<td>20100121</td>
<td>-2.524</td>
<td>0.04266</td>
</tr>
<tr>
<td>20100131</td>
<td>-2.185</td>
<td>0.08267</td>
</tr>
<tr>
<td>20100205</td>
<td>-1.517</td>
<td>0.09282</td>
</tr>
</tbody>
</table>

Analyzing the data in TABLE II and TABLE III, the following conclusions can be drawn:

a) The values of $p,q,x_0,y_0,z_0$ are concerned with the gray level of pixels, the larger the gray level, the smaller is the $P,Q$ value.

b) The values of $p,q$ are very close, commonly they have a difference at the sixth position after the decimal point.

c) The simulated values of $z_0$ are close in azimuth and range directions (that is X and Y axes). That is to say, all corner reflectors in an image have uniform maximum value in azimuth and range direction, so we can simulate in azimuth and range direction together.

d) Even the incident angle and the type of image are the same; different imaging time and different imaging conditions will make the value of $p,q$ different, that is, the PSF is concerned with the quality of the image and has nothing to do with the incident angle.

e) Therefore, it is feasible to simulate the PSF by using the elliptic paraboloid model.

B. The Experiment of PSF Restoration

The restoration experiment was carried out for point targets firstly to determine the size of template window, then the restoration of a regional image can be carried out. The parameters of Cosmo 20100121 image are calculated by using equation 9. The result is shown in TABLE IV.

**TABLE IV. THE VALUE OF PARAMETERS**

<table>
<thead>
<tr>
<th>$P$</th>
<th>$Q$</th>
<th>$x_0$</th>
<th>$y_0$</th>
<th>$z_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.00252398</td>
<td>-0.0025614</td>
<td>0.042655224</td>
<td>-0.016393442</td>
<td>-394</td>
</tr>
</tbody>
</table>

The matrix $Z$ can be worked out by the elliptic paraboloid model. Then the PSF matrix template can be obtained by normalizing the matrix $Z$. The experiment takes a template size of 3×3 and 5×5 as examples; the results of point target restoration are displayed as Fig. 6 and Fig. 7. From Fig. 6 we can see that the maximum brightness value of a pixel and first-order pixel are improved by adopting a 3×3 restoration template, but the brightness of a second-order pixel decreases. A 5×5 template restores the image, the brightness of a pixel reduces in the original 3×3 window, the brightness of pixels with high values are meaned and extends to a 10×10 window, but this makes obvious differences from the original image. Therefore, the 3×3 PSF template is adopted. Moreover, the degraded noise of the image is unknown and the signal-to-noise ratio (SNR) in restoration is an estimated value.

![Figure 6. Restoration result of point target.](image)

![Figure 7. 3D effect viewer of restoration of point target.](image)
Four indices were selected to evaluate of restored image. They are the mean, standard deviation (SD), radiation resolution, and equivalent number looks (ENL). The result is shown in TABLE V. For visual quality, the noise of the restored image is less than that of the original image while the edges become enhanced at the same time. The mean and SD value are decreased and radiation resolution is improved. The quality of the restored image is better than that of the filtered image and the information is improved.

<table>
<thead>
<tr>
<th>Standard</th>
<th>Original image</th>
<th>Restored image</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>9.354040520260130</td>
<td>8.631832416208104</td>
</tr>
<tr>
<td>SD</td>
<td>14.72681775184346</td>
<td>12.152279900368375</td>
</tr>
<tr>
<td>Radiation Resolution</td>
<td>4.106727149048091</td>
<td>3.816284805545863</td>
</tr>
<tr>
<td>ENL</td>
<td>0.403441391793071</td>
<td>0.504534039570753</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

Aiming at the PSF estimation for SAR image restoration, this paper carries out research and arrives at the following innovative conclusions.

a) Addressing the problem of lack of ideal point target in PSF estimation by laying corner reflectors on the ground.

b) Analyzing the PSF of corner reflectors, the conclusion is that the PSF is concerned with the quality of the image and has nothing to do with incident angle.

c) Putting forward the simplified elliptic paraboloid model of the PSF, giving the solution procedure and method to calculate the parameters and carrying out restoration experiment by using Wiener filtering, the experiment proves that the method is feasible.

d) In further work, there are some questions that need to be analyzed: the relationship between PSF and imaging parameters; the selection of parameters of the elliptic paraboloid model and the investigation of a calculation procedure that is stable and converges fast.

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REFERENCES


