Some dependent aggregation operators with 2-tuple linguistic information and their application to multiple attribute group decision making

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A B S T R A C T

We investigate the multiple attribute group decision making (MAGDM) problems in which the attribute values take the form of 2-tuple linguistic information. Motivated by the ideal of dependent aggregation [Xu, Z. S. (2006). Dependent OWA operators. Lecture Notes in Artificial Intelligence, 3885, 172–178], in this paper, we develop some dependent 2-tuple linguistic aggregation operators: the dependent 2-tuple ordered weighted averaging (D2TOWA) operator and the dependent 2-tuple ordered weighted geometric (D2TOWG) operator, in which the associated weights only depend on the aggregated 2-tuple linguistic arguments and can relieve the influence of unfair 2-tuple linguistic arguments on the aggregated results by assigning low weights to those “false” and “biased” ones and then apply them to develop some approaches for multiple attribute group decision making with 2-tuples linguistic information. Finally, some illustrative examples are given to verify the developed approach and to demonstrate its practicality and effectiveness.

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1. Introduction

The ordered weighted aggregation (OWA) operator (Yager, 1988) as an aggregation technique has received more and more attention since its appearance (Merigó & Casanovas, 2009; Merigó & Gil-Lafuente, 2009; Merigó, 2010; Wei, 2008a, 2008b, 2008c, 2009a, 2009b, 2010a, 2010b, 2010c, 2010d; Wei, Zhao, & Lin, 2010; Wei, 2011a, 2011b, 2011c; Wei, Wang, Lin, & Zhao, 2011a, 2011b, Wei, Wang, & Lin, 2011c; Wei, 2011d, 2011e, 2011f, 2011g; Wei & Zhao, 2012; Zhang & Chu, 2009; Xu & Da, 2003). One important step of the OWA operator is to determine its associated weights. Many authors have focused on this issue, and developed some useful approaches to obtaining the OWA weights. We classify all these approaches into the following two categories: argument-independent approaches and argument-dependent approaches (Filev & Yager, 1998; Fullér & Majlender, 2001; Xu, 2005; Yager, 1993; Yager & Filev, 1994) and argument-dependent approaches (Filev & Yager, 1998; Xu & Da, 2002; Xu, 2005, 2006, 2008; Yager, 1993). The weights derived by the argument-independent approaches are associated with particular ordered positions of the aggregated arguments, and have no connection with the aggregated arguments, while the argument-dependent approaches determine the weights based on the input arguments. With respect to the argument-dependent approaches, Yager (1993) introduced some families of the OWA weights, including the ideal of aggregate dependent weights. Filev and Yager (1998) developed two procedures to obtain the OWA weights, the first one learns the weights from a collection of samples with their aggregated value, and the second one calculates the weights for a given level of orness. Xu and Da (2002) established a linear objective-programming model for obtaining the weights of the OWA operator by utilizing the given arguments under partial weight information. Xu (2005) developed a new dependent OWA (DOWA) operator and developed a new argument-dependent approach to determining the OWA weights, which can relieve the influence of unfair arguments on the aggregated results. Xu (2006) developed some dependent uncertain ordered weighted aggregation operators, including dependent uncertain ordered weighted geometric (DUOWG) operators, in which the associated weights only depend on the aggregated interval arguments and can relieve the influence of unfair interval arguments on the aggregated results by assigning low weights to those “false” and “biased” ones.

However, in many situations, the input arguments take the form of 2-tuples linguistic variables (Fan & Liu, 2010; Herrera & Martínez, 2000, 2001; Herrera, Martínez, & Sánchez, 2005; Herrera, Herrera-Viedma, & Martínez, 2008; Merigó, Casanovas, & Martínez, 2010; Tai & Chen, 2009; Wang, 2009a, 2009b; Wei, 2010c, 2010d, 2011a; Wei, Lin, Zhao, & Wang, 2010a, Wei, Lin, Zhao, & Wang, 2010b) because of time pressure, lack of knowledge, and people’s limited expertise related with problem domain. In this paper, we will pay attention on the second category, and develop some argument-dependent approach to determining the OWA weights with 2-tuples linguistic information. The remainder of this paper...
is set out as follows. In the next section, we introduce some basic concepts and operational laws of 2-tuple linguistic variables. In Section 3 we develop some dependent 2-tuple linguistic aggregation operators: the dependent 2-tuple ordered weighted averaging (D2TOWA) operator and the dependent 2-tuple ordered weighted geometric (D2TOWG) operator, in which the associated weights only depend on the aggregated 2-tuple linguistic arguments and can relieve the influence of unfair 2-tuple linguistic arguments on the aggregated results by assigning low weights to those “false” and “biased” ones. In Section 4 we develop an approach to multiple attribute group decision making based on these dependent aggregation operators with 2-tuple linguistic information, which is straightforward and has no loss of information. In Section 5, we give an illustrative example to verify the developed approach and to demonstrate its feasibility and practicality. In Section 6 we conclude the paper and give some remarks.

2. Preliminaries

Let \( S = \{s_1, s_2, \ldots, s_l\} \) be a linguistic term set with odd cardinality. Any label, \( s_i \), represents a possible value for a linguistic variable, and it should satisfy the following characteristics (Herrera & Martínez, 2000, 2001):

1. The set is ordered: \( s_i > s_j \) if \( i > j \);
2. Max operator: max \((s_i, s_j) = s_j\) if \( s_i \geq s_j \);
3. Min operator: min \((s_i, s_j) = s_i\) if \( s_i \leq s_j \). For example, \( S \) can be defined as

\[
S = \{s_1 = \text{extremely poor}, s_2 = \text{very poor}, s_3 = \text{poor}, s_4 = \text{medium}, s_5 = \text{good}, s_6 = \text{very good}, s_7 = \text{extremely good}\}
\]

Herrera and Martínez (2000, 2001) developed the 2-tuple fuzzy linguistic representation model based on the concept of symbolic translation. It is used for representing the linguistic assessment information by means of a 2-tuple \((s_i, d_j)\), where \( s_i \) is a linguistic label from predefined linguistic term set \( S \) and \( d_j \) is the value of symbolic translation (Herrera & Martínez, 2000, 2001).

Definition 1. Let \( \beta \) be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set \( S \), i.e., the result of a symbolic aggregation operator, \( \beta \in [1, l] \), being the cardinality of \( S \). Let \( i = \text{round}(\beta) \) and \( \alpha = \beta - i \), where \( i \in [1, l] \) and \( \alpha \in [-0.5, 0.5] \) then \( \alpha \) is called a Symbolic Translation (Herrera & Martínez, 2000, 2001).

Definition 2. Let \( S = \{s_1, s_2, \ldots, s_l\} \) be a linguistic term set and \( \beta \in [1, l] \) is a number value representing the aggregation result of linguistic symbolic. Then the function \( \Delta \) used to obtain the 2-tuple linguistic information equivalent to \( \beta \) is defined as:

\[
\Delta : [1, l] \rightarrow S \times [-0.5, 0.5),
\]

\[
\Delta(\beta) = \left\{ \begin{array}{ll}
  s_i & \text{if } i = \text{round}(\beta), \\
  \alpha = \beta - i, \quad \alpha \in [-0.5, 0.5] & \text{if } \alpha = \text{round}(\beta)
\end{array} \right.
\]

(1)

where \( \text{round}(.) \) is the usual round operation, \( s_i \) has the closest index label to \( \beta \) and \( \alpha \) is the value of the symbolic translation (Herrera & Martínez, 2000, 2001).

Definition 3. Let \( S = \{s_1, s_2, \ldots, s_L\} \) be a linguistic term set and \( \{s_i, \alpha_i\} \) be a 2-tuple. There is always a function \( \Delta^{-1} \) can be defined, such that, from a 2-tuple \( \{s_i, \alpha_i\} \) it return its equivalent numerical value \( \beta \) in \([1, L] \in \mathbb{R} \), which is (Herrera & Martínez, 2000, 2001).

\[
\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [1, L],
\]

\[
\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta.
\]

(3)

3. Some dependent aggregation operators with 2-tuple linguistic information

Yager (1988) provides a definition of the ordered weighted averaging (OWA) operator as follows:

Definition 5. An OWA operator of dimension \( n \) is a mapping, OWA: \( \mathbb{R}^n \rightarrow \mathbb{R} \), that has an associated weighting vector \( w = (w_1, w_2, \ldots, w_n)^T \) with the properties \( w_j \in [0, 1] \), and \( \sum_{j=1}^{n} w_j = 1 \), such that

\[
\text{OWA}_w(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j a_j,
\]

(5)

where \( \{a_1, a_2, \ldots, a_n\} \) be a set of 2-tuples and \( a = (a_1, a_2, \ldots, a_n) \) be the weighting vector of 2-tuples \((r_j, a_j)\) \((j = 1, 2, \ldots, n)\) and \( a_j \in [0, 1] \). Since its appearance in 1988, the OWA operator has been used in a wide range of applications, such as engineering, neural networks, data mining, decision making, image process, expert systems, etc. Consider that the OWA operator aggregates only the exact inputs having been reordered, Herrera and Martínez (2000, 2001) extend WA operator and OWA operator to accommodate the situations where the input arguments are linguistic variables.

Definition 6. Let \( x = [(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)] \) be a set of 2-tuples and \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) be the weighting vector of 2-tuples \((r_j, a_j)\) \((j = 1, 2, \ldots, n)\) and \( \omega_j \in [0, 1] \). The 2-tuple weighted average is

\[
\text{2TWA}_\omega((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) = \Delta \left( \sum_{j=1}^{n} \omega_j \Delta^{-1}(r_j, a_j) \right).
\]

(6)

Definition 7. Let \( x = [(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)] \) be a set of 2-tuples and \( w = (w_1, w_2, \ldots, w_n)^T \) be an associated weighting vector, \( w_j \in [0, 1] \). The 2-tuple OWA for linguistic 2-tuples is computed as

\[
\text{2TOWA}_\omega((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) = \Delta \left( \sum_{j=1}^{n} w_j \Delta^{-1}(r_j, a_j) \right).
\]

(7)

where \( \{r_1, r_2, \ldots, r_n\} \) be a permutation of \((1, 2, \ldots, n)\), such that \( (r_{\pi(j)}, \omega_{\pi(j)}) \geq (r_{\pi(k)}, \omega_{\pi(k)}) \) for all \( j = 2, \ldots, n \).

Recently, Jiang and Fan (2003) extended the WG operator and OWA operator to accommodate the situations where the input arguments are linguistic variables.

Definition 8. Let \( x = [(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)] \) be a set of 2-tuple and \( \omega = (\omega_1, \omega_2, \ldots, \omega_n)^T \) be the weighting vector of 2-tuple \((r_j, a_j)\) \((j = 1, 2, \ldots, n)\) and \( \omega_j \in [0, 1] \), \( \sum_{j=1}^{n} \omega_j = 1 \). The 2-tuple weighted geometric operator is (Jiang & Fan, 2003)

\[
\text{2TWG}_\omega((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)) = \Delta \left( \prod_{j=1}^{n} \Delta^{-1}(r_j, a_j) \right)^{\omega_j}.
\]

(8)
**Definition 9.** Let \( x = \{ (r_1,a_1), (r_2,a_2), \ldots, (r_n,a_n) \} \) be a set of 2-tuple, A 2-tuple ordered weighted geometric operator of dimension \( n \) is a mapping \( \text{TOWG}: \mathbb{R}^n \rightarrow \mathbb{R} \) that has an associated vector \( w = (w_1, w_2, \ldots, w_n) \) such that \( w_j > 0 \) and \( \sum_{j=1}^{n} w_j = 1 \). Furthermore,

\[
\text{TOWG}_w((r_1,a_1), (r_2,a_2), \ldots, (r_n,a_n)) = \Delta \left( \prod_{j=1}^{n} \Delta^{-1}(r_{x_j}, a_{x_j}) \right)^{w_j},
\]

where \( (\pi(1), \pi(2), \ldots, \pi(n)) \) is a permutation of \( (1,2,\ldots,n) \), such that \( (s_{\pi(j-1)}, a_{\pi(j-1)}) > (s_{\pi(j)}, a_{\pi(j)}) \) for all \( j = 2, \ldots, n \).

**Definition 10.** Let \( x = \{ (r_1,a_1), (r_2,a_2), \ldots, (r_n,a_n) \} \) be a set of 2-tuples, the 2-tuple arithmetic mean is computed as

\[
(f, \bar{a}) = \Delta \left( \frac{1}{n} \sum_{j=1}^{n} \Delta^{-1}(r_j, a_j) \right), \quad f \in S, \quad \bar{a} \in [-0.5, 0.5].
\]

**Definition 11.** Let \( (r_a, a) \) and \( (r_j, a_j) \) be two 2-tuples, then we call

\[
d((r_a, a), (r_j, a_j)) = \frac{\Delta^{-1}(r_a, a) - \Delta^{-1}(r_j, a_j)}{f},
\]

the distance between \( (r_a, a) \) and \( (r_j, a_j) \).

**Definition 12.** Let \( x = \{ (r_1,a_1), (r_2,a_2), \ldots, (r_n,a_n) \} \) be a set of 2-tuples, and let \( (f, \bar{a}) \) the arithmetic mean of these 2-tuples, then we define the standard deviation of these 2-tuples as

\[
\sigma = \sqrt{\frac{1}{n} \sum_{j=1}^{n} d((r_j, a_j), (f, \bar{a}))^2}.
\]

**Definition 13.** Let \( x = \{ (r_1,a_1), (r_2,a_2), \ldots, (r_n,a_n) \} \) be a set of 2-tuples, and let \( (f, \bar{a}) \) the arithmetic mean of these 2-tuples, then we call

\[
sim((r_{x_j}, a_{x_j}),(f,\bar{a})) = 1 - \frac{d((r_{x_j}, a_{x_j}),(f,\bar{a}))}{\sum_{j=1}^{n} d((r_j, a_j), (f, \bar{a}))},
\]

where \( j = 1, 2, \ldots, n \).

**Theorem 1.** Let \( x = \{ (r_1,a_1), (r_2,a_2), \ldots, (r_n,a_n) \} \) be a set of 2-tuples, and let \( (f, \bar{a}) \) the arithmetic mean of these 2-tuples, \( \sigma \) the standard deviation of these 2-tuples, \( \phi \) the greatest value of \( (\pi(1), \pi(2), \ldots, \pi(n)) \) that \( (r_{\pi(j-1)}, a_{\pi(j-1)}) \geq (r_{\pi(j)}, a_{\pi(j)}) \) for all \( j = 2, \ldots, n \).

Then we replace (15) by

\[
\text{TOWA}((r_1,a_1), (r_2,a_2), \ldots, (r_n,a_n)) = \Delta \left( \frac{\sum_{j=1}^{n} \Delta^{-1}(r_{x_j}, a_{x_j})}{\sum_{j=1}^{n} \text{sim}(r_{x_j}, a_{x_j}, (f, \bar{a}))} \right).
\]

Similar to Xu (2006, 2008), we have the following result:
2TOWA\((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\) 
\[= \Delta \left( \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij}) \right)\] 
\[= \Delta \left( \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij}) \right)\] 
\[= \Delta \left( \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij}) \right)\] 
\[= \Delta \left( \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij}) \right)\]

Since 
\[\sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij}) = \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij})\] 

and 
\[\sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} = \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}}\] 

then, (20) can be rewritten as 
\[2TOWA\((r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)\) \[= \Delta \left( \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij}) \right)\] 
\[= \Delta \left( \sum_{j=1}^{n} e^{\frac{\left| r_j - a_j \right|}{2}} \Delta^{-1}(r_{ij}, a_{ij}) \right)\]

Obviously, (21) is also a neat and dependent 2TOWA (D2TOWA) operator.

Step 2. Utilize the decision information given in matrix \(R_k\), and the 2TWA operator 
\[z_i^{(k)} = \left( r_i^{(k)}, a_i^{(k)} \right) = \Delta \left( \sum_{j=1}^{n} c_{ij} \Delta^{-1}\left( r_i^{(k)}, 0 \right) \right)\]
or the 2TWG operator 
\[z_i^{(k)} = \left( r_i^{(k)}, a_i^{(k)} \right) = \Delta \left( \left( \sum_{j=1}^{n} \left( \Delta^{-1}\left( r_i^{(k)}, 0 \right) \right)^{c_{ij}} \right) \right)\]
to derive the individual overall preference value \(z_i^{(k)}\) of the alternative \(A_i\).

Step 3. Utilize the D2TOWA operator:
\[z_i = \left( r_i, a_i \right) \[= \Delta \left( \sum_{j=1}^{n} \text{sim}\left( \left( r_i^{(k)}, a_i^{(k)} \right), \left( r_j^{(k)}, a_j^{(k)} \right) \right) \right) \]
\[= \Delta \left( \sum_{j=1}^{n} \text{sim}\left( \left( r_i^{(k)}, a_i^{(k)} \right), \left( r_j^{(k)}, a_j^{(k)} \right) \right) \right) \]

or the D2TOWG operator:
\[z_i = \left( r_i, a_i \right) \[= \Delta \left( \sum_{j=1}^{n} \text{sim}\left( \left( r_i^{(k)}, a_i^{(k)} \right), \left( r_j^{(k)}, a_j^{(k)} \right) \right) \right) \]
\[= \Delta \left( \sum_{j=1}^{n} \text{sim}\left( \left( r_i^{(k)}, a_i^{(k)} \right), \left( r_j^{(k)}, a_j^{(k)} \right) \right) \right) \]
to derive the collective overall preference values \(z_i\) \((i = 1, 2, \ldots, m)\) of the alternative \(A_i\) where \((r_i, a_i) = \Delta\left( \sum_{j=1}^{n} \Delta^{-1}\left( r_i^{(k)}, 0 \right) \right)\).

Step 4. Rank all the alternatives \(A_i\) \((i = 1, 2, \ldots, m)\) and select the best one(s) in accordance with the collective overall preference values \(z_i\) \((i = 1, 2, \ldots, m)\).

Step 5. End.

5. Illustrative example

Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from Herrera & Herrara-Viedma (2000)). There is a panel with five possible alternatives to invest the money: 1\(A_1\) is a car company; 2\(A_2\) is a food company; 3\(A_3\) is a computer company; 4\(A_4\) is a chemical company; 5\(A_5\) is a TV company. The investment company must take a decision according to the following four attributes: 1\(G_1\) is the risk analysis; 2\(G_2\) is the growth analysis; 3\(G_3\) is the social-political impact analysis; 4\(G_4\) is the environmental impact analysis. The five possible alternatives \(A_i\) \((i = 1, 2, \ldots, 5)\) are to be evaluated using the linguistic term set 
\[S = \{ s_1 = \text{extremely poor}, s_2 = \text{very poor}, s_3 = \text{poor}, s_4 = \text{medium}, s_5 = \text{good}, s_6 = \text{very good}, s_7 = \text{extremely good} \} \]
by the three decision makers \(D_k(k = 1, 2, 3)\) under the above four attributes (whose weighting vector \(\omega = (0.3, 0.1, 0.2, 0.4)\), and construct, respectively, the linguistic decision matrices are shown in Tables 1–3:

In the following, we shall utilize the proposed approach in this paper getting the most desirable alternatives:

Step 1. Transforming linguistic decision information given in Tables 1–3 into 2-tuple linguistic decision matrices which are given in Tables 4–6.

Step 2. Utilize the decision information given in matrix \(R_k\), and the 2TWA operator and 2TWG operator (Let \(\omega = (0.3, 0.1, 0.2, 0.4)\)) to derive the individual overall preference value \(z_i^{(k)}\) of the alternative \(A_i\). The results are shown in Tables 7 and 8.
Step 3. Utilize the D2TOWA operator and D2TOWG operator to derive the overall preference values $z_i$ ($i = 1, 2, 3, 4, 5$) of the alternative $A_i$. The results are shown in Table 9.

Step 4. According to the aggregating results shown in Table 9, the ordering of the alternatives are shown in Table 10. Note that "preferred to". As we can see, depending on the aggregation operators used, the ordering of the alternatives is slightly different. Therefore, depending on the aggregation operators used, the results may lead to different decisions. However, the best alternative is $A_4$.

6. Conclusion

In this paper, we have investigated the multiple attribute group decision making (MAGDM) problems in which the attribute values take the form of 2-tuple linguistic information. Motivated by the ideal of dependent aggregation (Xu, 2006, 2008), we develop some dependent 2-tuple linguistic aggregation operators: the dependent 2-tuple ordered weighted averaging (D2TOWA) operator and the dependent 2-tuple ordered weighted geometric (D2TOWG) operator, in which the associated weights only depend on the aggregated 2-tuple linguistic arguments and can relieve the influence of unfair 2-tuple linguistic arguments on the aggregated results by assigning low weights to those "false" and "biased" ones and then apply them to develop some approaches for multiple attribute group decision making with 2-tuples linguistic information. Finally, some illustrative examples are given to verify the developed approach. In the future, we shall continue working in the extension and application of the developed operators to other domains.

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References


