IMPROVING PHYSICAL BEHAVIOR IN IMAGE REGISTRATION

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ABSTRACT
During the past years, many different registration methods have emerged. These usually lead to different results and one faces the problem of choosing the most appropriate. To mitigate this issue, several algorithms have been developed to include prior knowledge in order to force the registration to have a physical behavior. The use of most of these methods in daily practice is cumbersome because of the need to create data-specific meshes. This paper introduces a local convolutive filter in order to iteratively make the result of any registration method converge towards a linear elastic solution. This method was tested and validated on virtual data and on a real silicone rubber cube.

Index Terms—Registration, regularization, linear elasticity, iterative filter.

1. INTRODUCTION

Estimating deformation in multiple time sequence images is a well known problem that has been addressed in many ways. The idea is to make a correspondence between images of the same object at different time points. Some applications, such as dose accumulation in radiotherapy [1], even require an exact point to point correspondence, also called voxel tracking.

This is usually done by using deformable image registration. This method computes a non-rigid transform in order to optimize a metric which is a measure of the image matching. This appears to be an ill posed problem which can lead to multiple solutions. Indeed, image-based metrics do not always bring sufficient constraint to make the registration process converge to the true physical deformation. To tackle this problem, smoothness criteria are often used to reduce the number of possible solutions by regularizing the deformation field. For instance, gaussian kernels are used in the demons algorithm [2] to regularize the field, while b-spline interpolation naturally leads to smooth solutions in b-spline registration [3]. Recent studies have shown that, even with such kind of regularizations, different methods lead to quite different results in homogeneous regions when applied to the same images [4]. Even worse: the same method with different parameters may lead to different results. Finding the closest transform to the ‘real’ physical deformation between the two time points is not trivial.

Several methods have been developed to include physical behavior in specific registration applications (for example in [5]), but they are often highly complex to extend to other applications. Many of them use finite element methods on meshes. However, good meshes are not easy to create.

In many cases, the fields resulting from simple registration methods can be improved without increasing the metric. As, in homogeneous regions, images do not have information to estimate the accuracy of the registration, additional a-priori criteria have to be introduced. Many people chose linear elasticity, as it is a good estimation of the behavior of many materials. But most methods do not make straightforward correspondence between method parameters and physical parameters [6]-[7]. Besides, a compromise usually has to be done between regularization and metric [8]. Furthermore, these methods are included in specific, usually home-made registration processes and hence cannot be compared easily to other general methods. In this paper, we introduce a simple method in order to improve the elastic behavior of a deformation field as post-processing of different kinds of registration.

2. METHODS

Image registration usually optimizes either an intensity-based metric or tries to fit contours one onto another. In all cases, one can assume that a good registration is able to globally fit the contours and shapes represented in the images. The difference between methods usually lies in the homogeneous regions of the image, as they contain no contrast information. Indeed, the deformation field in homogeneous regions is usually obtained using interpolation of the field from the closest heterogeneous regions. This interpolation is based on an arbitrary smoothing function usually implicitly used by the method. Furthermore, even on shape contours, the field is relevant mainly in the direction normal to the edge and an uncertainty remains about the tangential component of the displacement vector. It is then reasonable to consider that certainty about the displacement vector found by the registration in the direction of the gradient increases where this gradient is important. The proposed method is based on this assumption and is a post-processing method to regularize de-
formation fields in homogeneous regions according to linear elasticity.

2.1. Convolutive filter

Linear elasticity equations are derived from the continuum mechanics.

\[
\sigma = \frac{E}{1 + \nu} \varepsilon + \frac{E \nu}{(1 + \nu)(1 - 2\nu)} \text{tr}(\varepsilon) I
\]

where \( \varepsilon = \frac{1}{2} (\nabla u + (\nabla u)^T) \)

\[
\nabla \cdot \sigma + F = 0
\]

where \( \sigma \) is the stress tensor, \( \varepsilon \) is the strain tensor, \( u \) is the displacement vector, \( F \) is the external forces vector, \( I \) is the unity matrix, \( E \) is Young’s modulus and \( \nu \) is Poisson’s ratio.

Equation 3 is elliptic and hence if conditions are set on the boundaries, there is only one field that will comply to linear elasticity. This solution may be reached by solving these equations on a mesh according to the finite element method [9]. However, creating a data-specific mesh is cumbersome (especially for 3D images) and placing nodes at good position (typically on high contrast voxels) is not an easy task.

The proposed method avoids using a mesh by computing the displacement locally instead of globally. For each voxel of the image, the value of the deformation field \( u(t) \) is updated by adding to its initial value an increment corresponding to the elastic regularization. This increment \( v(t) \) is computed as the difference between the field previously computed \( u(t) \) and the field which minimizes the internal forces taking the direct neighbors as border conditions \( u_{el} \). In order to compute \( u_{el} \) for each voxel, equation 3 was discretized in order to compute the displacement vector of a voxel taking the surrounding voxels as boundary conditions. As the geometry is the same for every voxel of the image, the coefficients of the equation will remain the same for every voxel and will only depend on Poisson’s ratio of the material at that point. This allows us to implement it with a set of simple convolutions. Starting with the field resulting from the registration as initial condition, these convolutions are applied iteratively, making the deformation field converge to the linear elastic solution [9].

2.2. Anisotropism

Without anisotropy, the deformation field would converge towards a unique value, as elliptic equations act as “smoothers”. Yet, we do not want the contours matched by the registration method to move. Thus, anisotropism was added where the registration was supposed to behave correctly. The field was updated as followed:

\[
u(t + 1) = u(t) + v^T(t)D^T + v^N(t)D^N
\]

\( v^N(t) \) and \( v^T(t) \) are the projections of \( v(t) \) respectively on the normalized gradient in the moving image and its perpendicular (see figure 1).

The anisotropism coefficients \( (D^N \) and \( D^T) \) are decreasing as a function of the gradient magnitude in the fixed image. \( D^T \) is larger than \( D^N \) in order to give greater importance to displacements in the direction tangential to the surface in the moving image. Besides, both coefficients are limited to ensure numerical stability.

3. VALIDATION OF THE METHOD

In order to illustrate the utility of this elastic regularization, two experiments were made on a rubber cube. The first experiment was based on a theoretical modelization of the cube which allowed the comparison of registration results with a simulated deformation. A real rubber cube was then scanned and registered in a second experiment in order to validate the method.

3.1. Virtual rubber cube

Geometry of the problem

Let us consider a rubber cube put in a box. If one forces a displacement on the top surface using a brick (as shown in the left part of figure 2), the deformations will propagate inside the volume according to the elastic behavior of the rubber. In order to modelize the cube and to simulate its behavior, a tetrahedral mesh of 4590 elements was created. The theoretical deformation field was then computed in every node of the mesh using a finite element system based on the linear elasticity equations and constrained on the borders of the cube (dirichlet conditions on the displacement in the direction normal to the surface). Poisson’s ratio was fixed to 0.45 inside the cube, 0.49 inside the brick, and 0.2 outside. 3D noiseless images (81x81x81) representing the cube before deformation (moving image) and after deformation (fixed image) were created.
Then 3 multi-resolution registrations were applied on
these images:

- B-Spline registration with 3 levels of resolution (3x3x3
to 8x8x8 grid-size). Sum of square differences (SSD)
metric. LBFGS optimizer. Implemented with the In-
sight Toolkit (ITK [10]).
- Symmetric demon’s registration with 4 levels of reso-
lution (downsampling of 2). Implemented with ITK.
- Morphons [11] with 8 levels of resolution (downsam-
pling of $\sqrt{2}$). Implemented with Matlab.

These registrations were then repeated with an additional
elastic regularization of the field. This was done at the end of
each resolution step in order to speed up the elastic smoothing
process. Poisson’s ratio was the same as in the finite element
simulation. The number of iterations was chosen in order to
achieve a reasonable convergence of the regularization pro-
cess.

The results of these 6 registrations were compared. First,
the SSD between fixed and deformed images was computed
for each registration, in order to evaluate the possible dete-
rioration brought by the regularization on the matching. In
average, the increment in SSD was equal to 6%. Then the
norm of the difference between fields was computed for each
couple of methods with and without regularization. The aver-
age reduction between differences was of 63%.

Eventually, deformation fields were compared to the the-
eoretical solution computed by finite element simulation. The
mean norm of the difference in every nodes of the mesh was
reduced by 57% by using the regularization filter.

These results are illustrated in the left part of figure 3.

3.2. Phantom cube

A phantom cube in silicone rubber (Eurosil 10, Schouten Syn-
tec, The Netherlands) was created (see right part of figure 2).
A grid of 3x3x3 punctual markers were added to the structure.
The cube was placed on a table and deformed by pushing on
its top surface. CT images were taken with and without de-
formation. Inhomogeneous information (markers and noise) was
then removed from both images in order to get homogeneous
volumes as in the first experiment.

The same registrations as in point 3.1 were applied on
these homogeneous images. In this case, a small improve-
ment of the metric (SSD reduction of 13%) was observed,
and the average reduction between differences with and with-
out regularization was of 52%.

The deformation fields resulting from the registrations
were used to deform the original moving image containing
the markers information. The distance between center of
mass of markers in the original fixed image and in the de-
formed image was computed. In average, the mean distance
was reduced by 51%.

These results can be seen in the right part of figure 3.

![Fig. 2. Left and center: virtual cube. Right: phantom cube.](image)

![Fig. 3. Results: For each graph, black bars correspond to the
results without elastic regularization. Gray bars correspond
to the results with regularization. Black and gray lines rep-
resent their mean value. BS = B-spline registration. Dem
= Demons. Mor = Morphons. (a) SSD between fixed and
deformed images for the virtual cube. (b) Norm of differ-
ence (ND) in pixels between deformation fields for the vir-
tual cube. (c) ND between registration and simulation of the
virtual cube. (d) SSD between fixed and deformed images for the
phantom. (e) ND between deformation fields for the
phantom. (f) Distance (in pixels) between markers in the
fixed and deformed images of the phantom.](image)
4. DISCUSSION

Experiment showed that the elastic behavior of deformation field computed by registration could be improved, and that different methods gave much closer results with the proposed filter for both applications. In particular, the cross-effects that occur within highly incompressible materials are not taken into account with traditional regularization schemes, whereas incompressibility is well simulated with this filter (as illustrated in figure 4). This requires the knowledge of Poisson’s ratio for each material contained in the image. This is an advantage when the real physical deformation has to be estimated using prior knowledge.

By avoiding using a mesh, the method needs relatively low memory for a good spatial resolution of the field. On the other hand, this method may require a large number of iterations before convergence. Therefore, when used with multiresolution registration processes, it is suggested to apply it at the end of each resolution step.

Another advantage of the method is the good compromise between regularization and matching. Indeed, as the global matching of two images mainly depends on the matching of their gradients, changing the field in homogeneous regions does not introduce significant errors in the matching. Therefore, the anisotropic behavior of the filter allows us to regularize the field only in regions with little information, while keeping the matching of high information voxels.

Fig. 4. Comparison of deformation field computed by the morphons registration without (left) and with (right) elastic regularization. Notice that both fields lead to almost the same deformed image, in spite of the fact that their behavior is different inside the volume.

5. CONCLUSION

We developed a method to improve physical behavior of any dense field image registration. The method was tested on a virtual and a real elastic cube for 3 different registration methods. The bias between methods was reduced as each solution converged towards a more realistic deformation, without increasing the final metric. Linear elasticity is not the only model which can be incorporated in this method, other behaviors could be simulated in the same way.

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7. REFERENCES


