Modelling Rerouting Phenomena through Dynamic Traffic Assignment in Rolling Horizon

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Abstract

This article addresses the problem of simulating en-route path choices on transport networks. In particular, by rerouting, we mean changing the currently chosen path, after receiving some information about a traffic event. Indeed, when the forecasted performance pattern of travel times and costs, known or only perceived, changes significantly, drivers may react by shifting their current route to a better one. The representation of such situations is particularly challenging if the information reaches a driver who is already travelling toward the destination. At the state-of-the-practice, most traffic assignment models are not capable of reproducing these phenomena. We will model rerouting in the framework of within-day Dynamic Traffic Assignment (DTA). Two different solutions are presented here, both exploiting the rolling horizon technique. The first solution can be summarized as an alternate sequence of two fixed point problems for each traffic event: a Dynamic User Equilibrium (DUE) with warm start through saved flows, and a Dynamic Network Loading (DNL) for given route choices; this model is called DTA with Rolling Horizon Events (DTA-RHE) and allows setting the information time of each event before or after the event itself. The second one is a simplified version of the first one, under the assumption that all the events are communicated not later than their start time; this model is called DUE with Rolling Horizon Events (DUE-RHE), and is a sequence of Dynamic User Equilibria with warm start. Numerical examples show the results of the proposed models where rerouting phenomenon can be observed.

Keywords: traffic events and information, en-route path choice, Dynamic User Equilibrium, Dynamic Network Loading, Link Transmission Model.
1 Introduction

The main objective behind the proposed models is to reproduce drivers’ behaviour in terms of en-route path choices when facing unexpected events in traffic networks. By unexpected event we mean any relevant traffic information that is not known in advance by at least some percentage of drivers and implies changes in the perception of the supply side, as well as different travel times. This can include: incident, road closure, longer queue, different signal plan, sport event, demonstration, planned road work, etc. In the above cases, the driver behaviour is the following: travel along the usual route, until the information of the event is received; then find a new route subject to known events, as well as their forecasted consequences, and follow it, until the next information is received, or the destination is reached.

Classic paradigms of Dynamic Traffic Assignment (DTA) are incapable of handling such cases. In a destination based dynamic Route Choice Model, the shortest trees are calculated for the given arrival time. This implies that the route is chosen at the origin based on future states of the network, taken from the previous iteration of the supply model – i.e. from previous experiences, in behavioural terms. It implicitly means that the drivers departing from origin know the state of the network up to the time of arrival at the destination. For recurrent traffic congestion this is perfectly consistent with the learning process that defines the routing behaviour on the next day, based on the experience acquired in previous days. Unfortunately such paradigm is wrong for modelling unexpected traffic events, which by definition are not known in advance nor recurrent, but change significantly the performance pattern of travel cost and times for that day.

For these cases, we propose to perform a set of simulations executed in rolling horizon, where the information on unknown events cannot be used by travellers before its communication. Our model is capable of updating route choices based on new information about the network state and apply them to traffic flows which are already on the network.

2 Dynamic Traffic Assignment

This section introduces the mathematical formulation of two versions of Dynamic Traffic Assignment (DTA) as a fixed point problems, namely the Dynamic User Equilibrium (DUE) and the Dynamic Network Loading (DNL), whose framework will be utilized in the reminder of the paper. In this paper, the focus is on the demand side, mostly Route Choice Model (RCM) and Network Flow Propagation (NFP), while the supply side, with the Link Transmission Model (LTM) is treated here as a black-box.

As the analysis is carried out within a dynamic context, all model variables are temporal profiles, here represented as piecewise $C^1$ functions of the time variable $\tau$. Users trips on the road network are modelled through a strongly connected oriented graph $G = (N, A)$, where $N$ is the set of the nodes and $A$ is the set of the arcs.

Notation:

$q_{od}^{\text{dem}}(\tau)$ demand of users from origin $o$ to destination $d$ willing to depart at time $\tau$
The DTA model consists of the following sub-models:

1. **Link Transmission Model (LTM)** – can be any kind of model which takes as an input: splitting rates \( p_a(\tau) \), demand flows departing at origins \( q_o(\tau) \), network characteristics \( \delta_a(\theta) \), and allows to obtain for each arc the performances as a function of time \( \tau \). In our case the General Link Transmission Model [Gen10b] is used to yield inflows \( f_a(\theta) \) and outflows \( e_a(\theta) \); we use here a different symbol for time just to emphasize that the LTM is typically implemented at a much denser time discretization than the rest of the assignment model (\( \theta \) is in the order of seconds, while \( \tau \) is in the order of minutes). This model can be easily substituted by a micro or meso simulation.

2. **First-in-first-out Travel Time (FTT)** – can be seen as complementary model to calculate travel times \( t_a(\tau) \) from the results of the LTM in terms of cumulative inflows and outflows [Gen05].
- Arc Cost Function (ACF) – in general works on each element of the network separately and calculates, starting from $t_a(\tau)$, its generalized cost $c_a(\tau)$ based on network parameters and user preferences. The non separability in time and space of the supply model is in the travel times.

- Route Choice Model (RCM) – is destination based and it calculates the arc conditional probabilities $p_{ad}(\tau)$ for given performance pattern of travel times and costs. Dynamic shortest trees are calculated preliminary to obtain trajectories for specific arrival times at the destination. Then a Dial like algorithm can be used to obtain a Logit loading on the efficient arcs, thus passing from deterministic to stochastic model [Bel05, Gen07]. As an alternative, a temporal layer approach can be adopted. The latter allows for a simpler presentation, but its practical implementation with discrete time intervals introduces relevant approximations (systematic flow shifts in time) with respect to the continuous theoretical solution; see [Gen04] for details.

- Network Flow Propagation (NFP) – loads the demand $q_{od}(\tau)$ towards a single destination using the arc probabilities $p_{ad}(\tau)$ from RCM and travel times $t_a(\tau)$ from LTM. When adopting a trajectory approach, the travel times from each node to the destination are actually provided by the RCM for given arrival times. NFP calculates arc inflows $q_{ad}(\tau)$ destination by destination.

- Aggregation (SUM) – simply sums-up the destination specific inflows $q_{ad}(\tau)$ into origin flows $q_o(\tau)$ and arc inflows $q_a(\tau)$. The latter are used to compute the splitting rates $p_a(\tau)$.

- Elastic Demand Model (EDM) – is an optional sub-model that computes the actual travelling flows $q_{ad}^{dem}(\tau)$ based on potential demand and travel costs, through the probability $p_{ad}^{dem}(\tau)$ to make the trip. The latter are the result of a stochastic discrete choice model.

**Figure 1:** Fixed point formulations of Dynamic Traffic Assignment.
The Dynamic User Equilibrium (DUE) can be then formalized as a fixed-point problem in terms of the arc (and origin) flows, as shown in Figure 1:

\[
\text{DUE} = \text{LTM} \rightarrow \text{FTT} \rightarrow \text{ACF} \rightarrow \text{RCM} (\rightarrow \text{EDM}) \rightarrow \text{NFP} \rightarrow \text{SUM} \rightarrow [\text{MSA}] \rightarrow \text{LTM}.
\]

The Dynamic Network Loading (DNL) is a sub-problem of DUE, which consists of seeking, for given route choices, an arc flow pattern consistent with the travel times through the arc performance model. DNL can be seen as a simplified DUE, without route-choice RCM. However, it has still a circular dependency to be solved iteratively to guarantee temporal consistency (not more than few iterations in practice). Arc (and origin) flows can be again considered as pivot variables of this fixed-point problem, as shown in Figure 1:

\[
\text{DNL} = \text{LTM} \rightarrow \text{FTT} \rightarrow \text{ACF} \rightarrow \text{NFP} \rightarrow \text{SUM} \rightarrow [\text{MSA}] \rightarrow \text{LTM}.
\]

Both fixed point problems, DUE and DNL, can be solved through the Method of Successive Averages (MSA). Although this algorithm does not converge very well in practice, it is a very flexible and robust tool; no handy alternative is yet available.

### 3 Rolling Horizon Assignments

A Rolling Horizon Assignment (RHA) can be seen as a sequence of fixed point DTA models. For each new (set of) information regarding traffic events in chronological order of communication, we introduce a new restart.

The first proposed solution is called Dynamic Traffic Assignment with Rolling Horizon Events (DTA-RHE). For \(n\) restarts, \(2(n+1)\) classic DTA runs are needed, with saving flows for the instant \(\tau_e\) of information relative to the \(e\)-th event. At each run different network characteristics need to be considered for LTM (the real travel times) and RCM (the perceived performances) to obtain appropriate results. DTA-RHE is an alternate sequence of two fixed point models for each event: a Warm DUE and a Cold DNL, as defined below.

#### 3.1 Warm DUE

The Warm DUE (scheme of Figure 2) is a Dynamic User Equilibrium with warm start based on saved number of vehicles \(n_{ad}(\tau_e)\) directed to each destination, that runs from \(\tau_e\) (the time when the \(e\)-th event is communicated) to the end of the simulation. It allows to determine the performance pattern in terms of travel cost \(c_{ae}(\tau)\) and times \(t_{ae}(\tau)\) estimated by users to make their routing choices, taking into account the first \(e\) events that have been already communicated.

Thus \(\delta_e\) does not include the real state of the network, but only the base scenario \(\delta_0\) and the first \(e\) known events – note that an event can be communicated after its start time. This performance pattern should also reflect the idea that user have on the consequences of communicated events. In Warm DUE we assume that each user is capable to forecast not only the direct consequences of events he gets informed of, but also the reaction of the other users. This strong hypothesis can be alleviated by performing a smaller number of DUE iterations, like if the learning process has not been completed. For \(e = 0\) the Warm DUE is the base DUE.
3.2 Cold DNL

The Cold DNL (scheme of Figure 3) is a Dynamic Network Loading that runs from $\tau_e$ to $\tau_{e+1}$. It allows to simulate the network and propagate the flows under real-time modifications of the supply $\delta^{\text{real}}$ with respect to the expected scenario. Modifications of arc flows due to traffic measures are also allowed in the LTM, but they will affect drivers flow pattern only through travel times. The demand side (route choice and elastic trips) is instead consistent with the performance pattern $c_{e}(\tau), t_{e}(\tau)$ computed in the previous Warm DUE. The Cold DNL reproduces the state that actually occurs on the network, rather than a perceived state. Travel choices are stochastic; this allows to retrieve through the RCM and the EDM the same pattern $p_{\text{ad}}(\tau), p_{\text{od}}^{\text{dem}}(\tau)$ resulting from the Warm DUE for given performances $c_{e}(\tau), t_{e}(\tau)$.

The idea underlying the Cold DNL is that until a user does not get a new information at time $\tau_{e+1}$ he will follow the choice pattern based on his current status of knowledge $c_{e}(\tau), t_{e}(\tau)$, although the actual travel times and costs may be different from those expected. In particular, the route choice pattern is given by the arc probabilities $p_{\text{ad}}(\tau)$; that is, a user does not necessarily follow a given path but adapts his route accordingly with the times he reaches the different nodes. Travel times may actually change, given that real events occur in the meanwhile and can change the characteristics of the arcs and the flows on the network. Note that the DNL is different from the LTM since propagating the flows accordingly with $p_{\text{ad}}(\tau)$ under the occurrence of travel times changes does not guarantee that the OD matrix is satisfied.
3.3 DUE with Rolling Horizon Events

If we assume that all events are communicated not later than their start time (perfect traffic information), then there is no need to make a distinction between reality and perception (like in DTA-RHE), while the only difference with a classical DUE is that some users are informed of events after they already began their trip.

For this case we then propose an alternative model, called Dynamic User Equilibrium with Rolling Horizon Events (DUE-RHE). This is a sequence of DUE with warm start in rolling horizon, where each simulation, possibly performed in real time at time $\tau_{e+1}$ as in [Gen11], starts at time $\tau_e$, incorporates (in particular in the LTM) all events that are communicated in the interval $[\tau_e, \tau_{e+1})$, makes a picture of the flows $n_{ad}(\tau_{e+1})$ – i.e. the number of vehicles for each arc distinguished by destination – at time $\tau_{e+1}$, that will be the next re-starting time. The scheme of this model is depicted in Figure 4.

4 Numerical examples

Both models DTA-RHE and DUE-RHE were tested on a toy network using the software Traffic Realtime Equilibrium (TRE), by SISTeMA (www.sistemaitc.com), which includes also procedures for DUE and DNL.

The toy network (see Figure 5) was designed for a single OD pair with two connections: lower - fast and efficient, and upper - alternative used only when lower connection is affected by the event. 'Escape-links' between two routes are inefficient to travel from O to D, they can be useful only to avoid event effects on lower route. The assignment lasts one hour, the event
Figure 4: DUE-RHE – Dynamic User Equilibrium with Rolling Horizon Events.

happens at 40th minute at the lower route and cause speed drop to 5km/h (from 50 km/h). When the event is active, major connection is no longer efficient and the optimal choice is to a) take upper route at the origin, or b) escape lower route at the earliest convenience for users already on the network.

Two simulation results are presented below (figures 6-9): a) for an event communicated with 5 minute delay, and b) for an event known far in advance. Results for three links are presented here: 1) lower link towards the event, 2) ‘escape-link’, 3) initial link of upper route, thick vertical line indicates the time of the event.

If the event is known far in advance, people will reroute even before the event happens (~31st min) and no one uses ‘escape-links’. In this case, the event doesn’t cause any congestion, because by the time of the event the affected link is already empty. When everyone is informed, results of DTA-RHE and DUE-RHE do not vary from DUE.

On the contrary, DTA-RHE simulation of unknown event shows that users use ‘escape-links’ to avoid the effects of the event and reroute only after the event is communicated (45th minute). Flow is propagated accordingly with the base DUE route choice for five minutes after the event happened (during this time it was affecting supply side - LTM, but not demand side - RCM) causing congestion at the event link. At $\tau_e$ the new RCM is calculated and upper route becomes effective. The escape links are now used by users who were at the lower route at the time of event communication. Flows departing from origin after $\tau_e$ use upper route, no one chooses lower route after the event is communicated. This situation cannot be simulated by DUE-RHE or DUE.
Figure 5: The toy network.

Figure 6: Flows against time at three result links for event communicated far in advance.

Figure 7: Flows on the network at the time when event communicated far in advance happens (40th minute).

Figure 8: Flow against time at three links for event communicated with 5 minute delay.

Figure 9: Flows on the network shortly after unknown event is communicated (47th minute).

References

The short list of references presented below is only aimed at showing the research stream followed by the authors. A proper literature review goes beyond the scope of the extended abstract.


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