Passivity and output feedback passification of switched continuous-time systems with a dwell time constraint

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This paper is concerned with the passivity analysis and output feedback passification for a class of switched continuous-time systems. A switching law and control units are designed to guarantee the passivity of closed-loop switched systems. Different from the exist results in state-dependent switching framework, we construct a novel switching law such that the switched system obeys a minimal dwell time between any two consecutive switchings. This avoids the possible arbitrarily fast switching caused by state-dependent switching law. Moreover, the switching law uses only the lower bound of the dwell time and partial measurable states of the closed-loop system, which is applicable in output feedback framework. Using multiple discretized storage functions, the controllers working on different time intervals are obtained, which ensure the closed-loop system is passive under the switching law. Finally, two examples are provided to illustrate the effectiveness of the developed results.

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1. Introduction

Over the past years, passivity and passification problems have attracted great attention in the field of systems and control [1–3]. For a control systems, passivity is an expected behavior, which guarantees the systems cannot generate more energy than that they dissipate [4]. Storage function is an important tool for the study of the passivity as well as the passivity-based control. By constructing different storage functions, the systems can with different performance such as input passivity and output passivity. Moreover, the storage function also serves as a natural candidate for Lyapunov function. This means stability can be satisfied if the passivity problem is solvable [5]. Because the passivity theory is useful for linear or nonlinear systems, the problems for passivity analysis and passification have been thoroughly studied for a variety of control systems including T-S fuzzy systems [6], time-delay systems [7], complex dynamical networks [8], Markovian jump systems [9], to mention a few.

Switched systems, which appear in a variety of practical applications, such as power electronics [10], fight control systems [11], robot manipulators [12], have gained considerable attention [13–17]. The system is a special hybrid system, which consists of a finite number of subsystems and a logical rule called switching law orchestrating the switching behavior between these subsystems [18]. Since switched systems can have better performance under some proper switching law, the design of switching law is widely considered [19–21]. Roughly speaking, the switching types can be classified into time-dependent and state-dependent switching [22]. In time-dependent switching framework, the switching laws satisfying dwell time or average dwell time (ADT) are proposed by using multiple Lyapunov functions (MLFs) [23,24]. Under the switching laws, the switched systems can be stability or have desired performance if the switching between the subsystems is sufficiently slow. On the other hand, different from the time-dependent switching law, state-dependent switching one guarantees the switching events are triggered when the state trajectory hits the switching surfaces. This implies the relationship between some regions consisting of the states and the performance of switched systems.

The issue of the passivity analysis for switched systems has been considered in [25–27]. However, the corresponding issue of passification is still in the early state of development, and only a few results have been reported in the literature. Under the ADT switching, [28,29] focus on the feedback passification of switched systems via state feedback, and the proposed switching law with ADT leads to slow switching between subsystems. However, the switching law allows one stays at arbitrary subsystems all the time, which means the switched system is passive only if all subsystems
of the system are passive beforehand. An improved method is to design the state-dependent switching law [30], which guarantees the switched system is passive without the requirement of passivity of each subsystem. However, it should be pointed out that state-dependent switching cannot guarantee any dwell time for each subsystem. This means it may cause arbitrarily fast switching [31]. In practice, too frequently switching is not allowed because it may cause instability of switched systems even if each subsystem is stable. Furthermore, many real systems need a maximal allowed switching frequency because the components of the systems cannot support too frequent switching. In this case, a dwell time constraint is very necessary for switched systems. Moreover, it has been illustrated in [32], adding the dwell time constraint may achieve better results. Motivated by this method, it is meaningful to design a state-dependent switching law which obeys a dwell time constraint such that the switched system is passive, and with both the advantages of slow switching and state-dependent switching. Meanwhile, the other question should be considered: for output feedback control purpose, when the state of system plant is unavailable, how to design the desired switching law? To the best of the authors’ knowledge, the problems have not yet been fully investigated, and it is our intention to shorten such a gap.

This paper is devote to passive analysis and output feedback passification for switched systems. The main contributions are as follows: (i) An improved state-dependent switching method is presented, where the switching law is not only dependent states but also a minimal dwell time. This avoids possible too frequent switching. (ii) The proposed switching law uses only the partial measurable states of the closed-loop system, which is easy to realize in output feedback framework. (iii) Based on the switching law, time-dependent output feedback controllers are designed to guarantee the passivity of the closed-loop system. The existence of the switching law and controllers are derived in terms of the solution to the linear matrix inequalities.

This paper is organized as follows. Problem formulation and some preliminaries are introduced in Section 2. The problems for passivity analysis and output feedback passification of the switched systems are solved in Section 3. To demonstrate the effectiveness of the proposed method, two examples are given in Section 4. Finally, Section 5 concludes this paper.

Notations: Standard notations are used in this paper. For a matrix $P$, $P^T$ denotes its transpose. $P>0$ and $P<0$ denote positive definite and negative definite, respectively. The Hermitian part of a square matrix $M$ is denoted by $He(M):=M+M^T$. The symbol $*$ within a matrix represents the symmetric entry. 0 and $I$ denote the zero matrix and identity matrix with appropriate dimensions, respectively.

2. Problem formulation and preliminaries

2.1. System description

Consider a continuous-time switched system described by

$$\begin{align*}
\dot{x}(t) &= A_r x(t) + B_r u(t) + B_{or} a(t) \\
y(t) &= C x(t) + D_{or} a(t) \\
z(t) &= E x(t) + D_{or} a(t)
\end{align*}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^p$ is the measured output, $z(t) \in \mathbb{R}^m$ is the regulated output, $a(t) \in \mathbb{R}^r$ is the exogenous input, which belong to $L_2(0, \infty)$. $\sigma: [0, \infty) \rightarrow \mathbb{L} = \{1, \ldots, N\}$ denotes the switching function, which is assumed to be a piecewise constant function, $N$ is the number of the subsystems. The switching instants are expressed by a switching sequence $\{(i_0, t_0), \ldots, (i_h, t_h), \ldots\}$, $i_h \in \mathbb{L}$, $h=0, 1, \ldots$, which means the $i_0$th subsystem is activated when $t \in [t_0, t_{i_0})$. $A_i$, $B_i$, $B_{or}$, $C_i$, $D_{or}$, $E_i$ and $E_{or}$ are real constant matrices of appropriate dimensions.

2.2. Preliminaries

For later development, the following definition and lemmas are introduced.

Definition 1. Switched system (1) with $u(t)=0$ is said to be passive under the switching signal $a(t)$, if there is a scalar $\gamma > 0$ such that

$$\int_0^T a^T(s)z(s)ds \geq -\gamma \int_0^T a^T(s)\sigma(s)\,ds$$

holds for all $T>0$, and all solutions of (1) under the zero initial condition.

Lemma 1. [34] Given a nonsingular matrix $V$ and symmetric matrices $Y$ and $X_i$ for all $i \in \mathbb{L}$ satisfying

$$\begin{bmatrix} Y & I \\ * & X_i \end{bmatrix} > 0$$

it is possible to determine nonsingular matrices $U_i$ and symmetric matrices $\tilde{Y}_i$ and $\tilde{X}_i$ such that

$$\begin{bmatrix} Y & V \\ * & \tilde{Y}_i \end{bmatrix} > 0, \quad \begin{bmatrix} X_i & U_i \\ * & \tilde{X}_i \end{bmatrix} > 0$$

Lemma 2. [33] (Projection Lemma) Given a symmetric matrix $\Phi$ and two matrix $\Gamma$, $\Lambda$, there exists a decision matrix $X$, that satisfies

$$\Phi + \Gamma \Phi \Gamma^T + \Lambda \Phi \Lambda^T < 0$$

if and only if the following projection inequalities with respect to $X$ are satisfied

$$\Gamma^T \Phi \Gamma < 0$$

$$\Lambda^T \Phi \Lambda < 0$$

2.3. Control objective

Main aim in the paper is to design a switching law and output feedback controllers such that the closed-loop switched system is passive and satisfies the following dwell time constraint:

$$t_{i_0+1} - t_{i_0} \geq T_D, \quad h = 0, 1, \ldots$$

where $T_D$ is a given positive scalar, which represents the minimal dwell time of each subsystem.

Remark 1. In [26,30], the problem of feedback passification for switched has been investigated. However, the proposed state-dependent switching law cannot guarantee any dwell time between any two consecutive switchings. This may cause too frequent switching. Therefore, we are interested in designing a switching law subject to a minimal dwell time $T_D$. Based on the viewpoint, $T_D$ can be given according to the allowed maximal switching frequency of control systems in practice.

3. Main results

To guarantee (8), time-dependent output feedback controllers are constructed:

$$\begin{align*}
\dot{x}(t) &= A_r x(t) + B_r y(t) \\
u(t) &= K_s(t) x(t)
\end{align*}$$
where \( \hat{z}(t) \in \mathbb{R}^n \) is the state of the controllers. \( K_i(t), A_{i_{nl}}(t) \) and \( B_{i_{nl}}(t) \), \( i \in L \) are time-varying matrices to be determined.

Combining Eqs. (1) and (9), the augmented system is given as
\[
\begin{align*}
\dot{\xi}(t) &= \xi(t), \quad \dot{\xi}(t) + B_{i_{nl}}(t)\alpha(t) \\
\dot{z}(t) &= \xi(t) + E_{i_{nl}}\alpha(t)
\end{align*}
\]
where
\[
\begin{align*}
\dot{\xi}(t) &= \begin{bmatrix} \dot{\xi}(t) \\ \dot{\xi}(t) \end{bmatrix}, \quad \dot{\xi}(t) = \begin{bmatrix} A_i & B_{i_{nl}}(t) \\ B_{i_{nl}}(t)C_i & A_{i_{nl}}(t) \end{bmatrix}, \quad E_{i_{nl}} = \begin{bmatrix} B_{i_{nl}} \\ B_{i_{nl}}(t)D_{i_{nl}} \end{bmatrix}
\end{align*}
\]

Summarily, the control press is shown in the following block diagram (Fig. 1).

Now, we are ready to present the first result.

### 3.1. Passivity analysis

**Theorem 1.** For a given integer \( H \geq 1 \), scalars \( \gamma > 0 \) and \( \rho_{ij} \geq 0, i \neq j \), \( \forall i, j \in L \), assume switched system (10) switched at time \( t = T_0 \) and \( \sigma(t_0) = i \), then, the adjacent switching instant \( t = t_{h+1} \) is chosen by the following switching law:

For \( \forall t \in [t_h, t_0 + T_D) \), \( \sigma(t) = i \)

For \( \forall t \in [t_h + T_D, \infty) \),
\[
\begin{align*}
\sigma(t) &= i, \text{ if } \xi^T(t)P_{i_{ni}}\xi(t) \leq \xi^T(t)P_{i_{nj}}\xi(t), i \neq j, \quad \forall j \in L \\
\sigma(t_{h+1}) &= \arg \min_{j \in L} \xi^T(t_{h+1})P_{i_{nj}}\xi(t_{h+1}), \quad \text{otherwise}
\end{align*}
\]

Then, the system (10) is passive and satisfies (8) under the switching law (11), if there exist positive matrices \( P_{i_{ni}}, i = 0, \ldots, H - 1, \) such that the following inequalities hold:

\[
\begin{align*}
\text{He}^T(\bar{A}_i(t)P_{i_{ni}} + \Pi_{i_{ni}}) - 2\gamma T_D &< 0 \\
\text{He}^T(\bar{A}_i(t)P_{i_{nj}} + \Pi_{i_{nj}}) - 2\gamma T_D &< 0 \\
\text{He}^T(\bar{A}_i(t)P_{i_{nj}} + \Phi_i) - 2\gamma T_D &< 0
\end{align*}
\]

where
\[
P_{i_{ni,j}} = \frac{P_{i_{ni,j}} - P_{i_{nj}}}{T_D/H}, \quad \Phi_i = \sum_{j \neq i}^N \rho_{ij}(P_{i_{nj}} - P_{i_{nj}})
\]

**Proof.** Noting that (8), we first divide the time interval \( [t_h, t_{h+1}) \) into two segments:
\[
[t_h, t_{h+1}) = [t_h, t_{h} + T_D) \cup [t_h + T_D, t_{h+1})
\]

For \( i = 0, \ldots, H - 1 \), denoting \( T_i = T_D/H \) and \( T_{h+i} = [t_h + T_i, t_{h+i} + T_i] \), where \( T_H = T_D/H \), we obtain \( T_0 = 0 \) and \( T_H + T_D = 0 \), which follows
\[
[t_h, t_{h+1}) = \bigcup_{i=0}^{H-1} [t_h + iT_D, t_{h+i}]
\]

Based on (16), we have \( \{t_h, t_{h+1}\} = \bigcup \{t_h + T_i, t_{h+i} + T_i\} \bigcup \{t_h + T_H, t_{h+1} + T_D\} \). Thus, \( \{t_h, t_{h+1}\} \) is divided into \( H + 1 \) segments. Next, for system (10), we choose the following multiple discretized storage functions matching the \( H + 1 \) time intervals:
\[
\begin{align*}
V(t) &= V_{\sigma(\xi(t))}(\xi(t)) = \xi^T(t)P_{\sigma(\xi(t))}(t)\xi(t) & (17)
\end{align*}
\]

where
\[
P_{i}(t) = \begin{cases} 
P_{i_{nl}}(\eta), & t \in [t_h + T_i, t_{h+i}], \\
P_{i_{H+1}}, & t \in [t_h + T_H, t_{h+i} + T_D]
\end{cases} \quad i = 0, 1, \ldots, H - 1
\]

for \( \sigma(\xi(t)) = i \), \( P_{i_{nl}}(\eta) = P_{i_{nl}} + (P_{i_{nl+1}} - P_{i_{nl}})\eta, \eta = (t - t_h - T_D)/T_H \) and \( P_{i_{H+1}} > 0 \). Then, based on (18), we can see that matrix function \( P_i(t) \) linearly changes from \( P_i(t) \) to \( P_{i_{H+1}} \).

To satisfy passivity property (2), we introduce a prescribed positive scalar \( \gamma \) and denote
\[
J = -2\gamma T_D^2(t)/T_D - 2\gamma T_D^2(t)/T_D^2(0)
\]

For \( \forall t \in [t_h + T_i, t_{h+i} + T_D) \), we have
\[
\begin{align*}
\dot{V}(t) + J(t) &= V_{\sigma(\xi(t))}(\xi(t)) + J(t) \\
&= \xi^T(t)P_{i_{nl}}(\eta)\xi(t) + 2\gamma T_D^2(t)/T_D^2(0) \xi^T(t)P_{i_{nl}}(\eta)\xi(t) + J(t) \\
&= \xi^T(t)\left[ (P_{i_{nl}} + (P_{i_{nl+1}} - P_{i_{nl}})\eta)\xi(t) + 2\gamma T_D^2(t)/T_D^2(0) \xi^T(t)(1 - \eta)P_{i_{nl}} + \eta P_{i_{nl+1}}\xi(t) \right. \\
&+ J(t) = \xi^T(t)\left( 1 - \eta \right)P_{i_{nl}}\xi(t) + 2\gamma T_D^2(t)/T_D^2(0) \xi^T(t)(1 - \eta)P_{i_{nl}} + \eta P_{i_{nl+1}}\xi(t) + J(t) \\
&= \left. \xi^T(t)\left( 1 - \eta \right)P_{i_{nl}}\xi(t) + 2\gamma T_D^2(t)/T_D^2(0) \xi^T(t)(1 - \eta)P_{i_{nl}} + \eta P_{i_{nl+1}}\xi(t) + J(t) \right)
\end{align*}
\]

Clearly, based on (20), we know that (12) and (13) guarantee
\[
\dot{V}(t) + J(t) < 0
\]

for \( \forall t \in [t_h + T_i, t_{h+i} + T_D), i = 0, 1, \ldots, H - 1 \).

Next, for \( \forall t \in [t_h + T_D, t_{h+1}), \) using (18), we have
\[
\begin{align*}
\dot{V}(t) &= 2\gamma T_D^2(t)/T_D^2(0) \xi^T(t)P_{i_{H+1}}\xi(t) + J(t) \\
&= \xi^T(t)\left[ P_{i_{H+1}}(\xi(t) + \xi(t)) + 2\gamma T_D^2(t)/T_D^2(0) \xi^T(t)P_{i_{H+1}}(\xi(t) + \xi(t)) \right. \\
&+ \xi^T(t)\left. P_{i_{H+1}}(\xi(t) + \xi(t)) + 2\gamma T_D^2(t)/T_D^2(0) \xi^T(t)P_{i_{H+1}}(\xi(t) + \xi(t)) \right]
\end{align*}
\]

Under the switching law (11), we get
\[
\sum_{j=1}^N\rho_{ij}\xi^T(t)P_{i_{nl}}\xi(t) = 0, \quad \forall i, j \in L, \quad i \neq j
\]
This guarantees
\[ V(t) + J(t) \leq V(t) + J(t) + \sum_{i=1}^{N} \rho_i \dot{x}^T(t)(P_{i,H}-P_{i,0})\dot{x}(t) \]  
for \( \forall t \in [t_n + T_d, t_{n+1}) \). Then, it is easy to see switching law (11) and condition (14) guarantee
\[ V(t) + J(t) < 0 \]  
for \( \forall t \in [t_n + T_d, t_{n+1}) \). Further, we have
\[ \int_0^{t_{n+1}} \dot{x}^T(s)\dot{x}(s)ds + \sum_{k=n}^{n+1} \int_{t_k}^{t_{k+1}} \dot{x}^T(s)\dot{x}(s)ds < 0 \]  
Then, for any \( T > 0 \), we know that
\[ V(T) < V(0) + \int_0^{T} \dot{x}^T(s)\dot{x}(s) + 2\gamma \dot{x}^T(s)\omega(s)ds \]
Based on the zero initial condition and \( V(T) \geq 0 \), we have \( \int_0^{T} \dot{x}^T(s)\dot{x}(s)ds \geq -\gamma \int_0^{T} \dot{x}^T(s)\omega(s)ds \), which completes this proof. \( \square \)

**Remark 2.** In Theorem 1, the proposed switching law (11) guarantees the minimal dwell time \( T_d \). On the other hand, when \( t \in [t_n + T_d, t_{n+1}) \), the switching law is converted into the min-switching strategy, which guarantees the storage function is non-increasing at the switching instants \( t_{n+1} \).

**Remark 3.** Based on slow switching method, passivity of switched systems is guaranteed under the switching law satisfying average dwell time (ADT) [28]. Since each subsystem is allowed to be activated all the time under the switching law, they must be passive beforehand. However, using switching law (11), the passivity of each subsystem need to be guaranteed only in some regions \( \Omega(t) = \{ \dot{x}(t) : \sum_{i=1}^{N} \rho_i \dot{x}^T(t)(P_{i,0}-P_{i,H})\dot{x}(t) \geq 0, \forall i \in I, i \neq j \} \), not everywhere. This means the overall switched system can be passive without the requirement of passivity of each subsystem. Thus, the proposed switched law relaxes the performance requirements of subsystems.

**Remark 4.** In real world applications, Theorem 1 is trivial since the state \( \dot{x}(t) \) of \( \dot{z}(t) \) is usually unavailable. When the state \( \dot{x}(t) \) of system (1) is fully unmeasurable, it is impossible to design switching law (11). To make our result applicable, we solve switching law using only the partial measurable state \( \dot{x}(t) \) of closed-loop system will be designed in the following theorem. This guarantees the effectiveness of the proposed switching and control strategies in output feedback framework. In addition, the results can be regarded as an extension for existing ones in state feedback passification framework [28,29].

### 3.2 Feedback passification via output feedback

**Theorem 2.** For a given integer \( H \geq 1 \), scalars \( \gamma > 0 \) and \( \rho_{ij} = 0, i \neq j \), \( \forall j \in I \), assume switched system (10) switched at \( t = t_n \) and \( \sigma(t_n) = i \), then, the adjacent switching instant \( t = t_{n+1} \) is chosen by the following switching law:
For \( \forall t \in [t_n, t_{n} + T_d) \), \( \sigma(t) = i \)
For \( \forall t \in [t_{n} + T_d, t_{n+1}) \), \( \sigma(t) = j \), if \( \dot{x}^T(t)P_{3,j}0\dot{x}(t) \leq \dot{x}^T(t)P_{3,i}0\dot{x}(t), i \neq j \), \( \forall j \in I \)
(28)
\[ \sigma(t_{n+1}) = \arg \min_{j \in I} \dot{x}^T(t_{n+1})P_{3,j}0\dot{x}(t_{n+1}), \text{ otherwise} \]
where

\[ P_{3,i}0 = V^T(X - Y)^{-T}P_{3,i}0(X - Y)^{-1}V \]  

\[ \text{Then, the system (10) is passive and satisfies (8) under switching law (28), if there exist positive matrices } P_{i,j}, P_{i,H}, P_{i,0}, P_{j,H}, P_{j,0}, \text{ matrices } X, Y, \text{ and matrices } P_2, P_{2l,q}, \text{ and matrices } P_1, P_{1l,q}, \text{ for all } i = 1, \ldots, H - 1, m = 0, \ldots, H, q = 1, \ldots, H - 1 \text{ such that the following inequalities hold:} \]

\[ -\text{He}(Y)P_{i,l} - Y + \Psi_{3,l} \leq 0 \]
\[ -\text{He}(Y)P_{i,l+1} - Y + \Psi_{3,l+1} \leq 0 \]
\[ -\text{He}(Y)P_{i,l} - Y + \Psi_{3,l} \leq 0 \]

\[ \text{where} \]
\[ P_{i,j} = \begin{bmatrix} P_{j,i} & P_{j,j} & P_{j,q} \\ * & P_{j,0} + P_{i,j} & P_{i,j} \\ * & * & P_{i,j} \end{bmatrix} \]
\[ \Psi_{3,l} = \begin{bmatrix} \Psi_{3,l} \\ * & * & * \end{bmatrix} \]

\[ \Sigma = \begin{bmatrix} 0 & 0 \\ 0 & \sum_{j \neq l}^{N} \rho_{ij}(P_{3,j}0 - P_{3,l}0) \end{bmatrix} \]

\[ \Psi_{2l} = H \begin{bmatrix} P_{2l,q} - \Sigma_{2l} \end{bmatrix} + \text{He}(\Phi_i) \]

Moreover, \( A_{i}(t), B_{i}(t) \) and \( K_{i}(t) \) can be constructed by

\[ A_{i}(t) = \begin{cases} V^{-1}(N_{i,l} - YA_{i} - YX^{-1}W_{1,l}) \\ -T_{i,l}C_{i}(X - Y)^{-1}V, \quad t \in [t_{n} + T_{i}, t_{n} + T_{i+1}) \end{cases} \]
\[ B_{i}(t) = \begin{cases} V^{-1}T_{i,l}, \quad t \in [t_{n} + T_{i}, t_{n} + T_{i+1}) \end{cases} \]
\[ K_{i}(t) = \begin{cases} (B_{i}^{T}B_{i})^{-1}B_{i}^{T}W_{1}^T(X - Y)^{-1}V, \quad t \in [t_{n} + T_{i}, t_{n} + T_{i+1}) \end{cases} \]
Proof 2. For \(\forall \tau \in [t_0 + T_1, t_0 + T_{l+1}]\), \(l = 0, 1, \ldots, H - 1\), we have \(\sigma(t) = i\). Based on Theorem 1, (12) and (13) are equivalent to the following inequalities:
\[
\begin{bmatrix}
\bar{\mathbf{A}}_i(t) & I \\
\mathbf{B}_{ii}(t) & 0
\end{bmatrix}
\begin{bmatrix}
\Sigma_{ii,i} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{A}_i(t) & \mathbf{E}_{ii}(t) \\
\mathbf{B}_{ii}(t) & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
I & I
\end{bmatrix}
\begin{bmatrix}
\mathbf{E}_{ii} & 0 \\
I & I
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_i & \mathbf{E}_{ii} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{T} & 0 \\
0 & I
\end{bmatrix}
< 0
\]
where
\[
\Sigma_{ii,i} = \begin{bmatrix} P_{i,i} \quad \Delta_{i,i} \\ \Delta_{i,i}^T \quad -\mathbf{E}_i^T \end{bmatrix}, \quad \Sigma_{ii,i+1} = \begin{bmatrix} P_{i,i+1} \quad \Delta_{i,i} \\ \Delta_{i,i}^T \quad -\mathbf{E}_i^T \end{bmatrix}, \quad \Pi_1 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}, \quad \Pi_3 = \begin{bmatrix} 0 & -2\gamma L \end{bmatrix}
\]
Denoting
\[
F_{ii} = \begin{bmatrix} \bar{\mathbf{A}}_i(t) & \mathbf{B}_{ii}(t) \\ I & 0 \\ 0 & I \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix}, \quad F_3 = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}
\]
We can see that (37) can be rewritten as
\[
F_{ii}^T F_2 \Sigma_{ii,i} F_3 + F_3 \Sigma_{ii,i+1} F_2 F_{ii} < 0
\]
where
\[
\Pi_1 = \begin{bmatrix} 0 & -\mathbf{E}_i^T \\ -\mathbf{E}_i^T & -\mathbf{E}_i^T - 2\gamma L \end{bmatrix}
\]
Further, letting \(F_{ii} = F_2 = \mathbf{A}_i(t) I 0 0\), and noting (5) in Lemma 2, we know that (39) is satisfied if the following conditions hold for given matrices \(F_{ii}^1\) and \(\mathbf{X}\), where
\[
\mathbf{Y} = [\mathbf{A}_i(t) \mathbf{A}_i(t) \mathbf{A}_i(t) \mathbf{A}_i(t)]
\]
Based on \(F_{ii}^1 F_{ii}^1 = \begin{bmatrix} -I & 0 \\ 0 & 0 \end{bmatrix}^T\), we construct \(F_{ii}^1\) and \(\mathbf{X}\) as follows.
\[
F_{ii} = \begin{bmatrix} -I \\ \mathbf{B}_{ii}(t) \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} \Lambda \quad \Lambda \end{bmatrix}
\]
where \(\Lambda\) is a symmetric matrix. Then, it follows from (40) that
\[
\begin{bmatrix}
-\text{He}(\Lambda) & \mathbf{P}_{i,i} + \Lambda^T \mathbf{A}_i(t) - \mathbf{A}_i^T \mathbf{B}_{ii}(t) \\
\mathbf{E}_{ii} & 0
\end{bmatrix}
\begin{bmatrix}
\Sigma_{ii,i} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{A}_i(t) & \mathbf{E}_{ii}(t) \\
\mathbf{B}_{ii}(t) & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
I & I
\end{bmatrix}
\begin{bmatrix}
\mathbf{E}_{ii} & 0 \\
I & I
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_i & \mathbf{E}_{ii} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{T} & 0 \\
0 & I
\end{bmatrix}
< 0
\]
where
\[
\Sigma_{ii,i} = \Delta_{i,i} + \text{He}(\mathbf{A}_i(t)\Lambda)
\]
It follows from (31) that
\[
\mathbf{Y} > 0
\]
where \(\mathbf{Y}\) is defined in Theorem 2, which implies \(\mathbf{X}\) and \(\mathbf{Y} - \mathbf{X}\) are nonsingular. Denoting \(\mathbf{X}^{-1} = \mathbf{X}\), we have
\[
\begin{bmatrix}
\mathbf{Y} & I \\
I & \mathbf{X}\end{bmatrix} > 0
\]
Using Lemma 1, the following matrices are constructed:
\[
\Lambda = \begin{bmatrix} \mathbf{Y} & \mathbf{V} \\
\mathbf{V}^T & 0 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} \mathbf{X} & \mathbf{U} \end{bmatrix}
\]
where \(\Lambda^{-1} = \mathbf{V}\), then we have
\[
\mathbf{U} = (I - \mathbf{XY})\mathbf{Y}^{-1}
\]
Denoting \(\mathbf{X} = \mathbf{V}\), we then can obtain \(\Delta_1\) and \(\Delta_2\) are full rank of column. Next, left-multiplying \(\mathbf{Y} \mathbf{Y}^{-1}\) and right-multiplying \(\mathbf{Y} \mathbf{Y}^{-1}\) to (42) and noting that \(\Delta_2^2 \mathbf{Y} - \Delta_2 \mathbf{Y} \mathbf{Y}^{-1} \Delta_2 = \mathbf{Y}\), we can obtain
\[
\begin{bmatrix}
-\text{He}(\mathbf{Y}) & \mathbf{P}_{i,1} + \mathbf{A}_i(t) - \mathbf{Y} \\
\mathbf{B}_{ii}(t) & -\mathbf{E}_i^T - \mathbf{E}_i^T - 2\gamma L
\end{bmatrix} < 0
\]
where
\[
\mathbf{Z}_{ii} = \text{He}(\mathbf{A}_i(t)) + \mathbf{Y} \mathbf{Y}^{-1} \mathbf{E}_i, \quad \mathbf{F}_{i,m} = \mathbf{Y} \mathbf{Y}^{-1} \mathbf{P}_{i,m}, \quad m = 0, \ldots, H
\]
Setting
\[
\mathbf{P}_{i,0} = \begin{bmatrix} \mathbf{P}_{i,0} & \mathbf{P}_{i,1} \\
\mathbf{P}_{i,0}^T & \mathbf{P}_{i,0} \end{bmatrix}, \quad \mathbf{P}_{i,H} = \begin{bmatrix} \mathbf{P}_{i,H} & \mathbf{P}_{i,H} \\
\mathbf{P}_{i,H}^T & \mathbf{P}_{i,H} \end{bmatrix}, \quad \mathbf{P}_{i,i} = \begin{bmatrix} \mathbf{P}_{i,i} & \mathbf{P}_{i,i} \\
\mathbf{P}_{i,i}^T & \mathbf{P}_{i,i} \end{bmatrix}
\]
we obtain (29) and (30). Thus, for \(\forall \tau \in [t_0 + T_1, t_0 + T_{l+1}]\), we conclude that (31) guarantees (37). Further, following the similar constructions, we know that (32) is sufficient for (38). These guarantee (12) and (13).
Next, for \(\forall \tau \in [t_0 + T_D, t_{h+1}]\), we know (14) is equivalent to
\[
\begin{bmatrix}
\mathbf{A}_i(t) & I \\
\mathbf{B}_{ii}(t) & 0
\end{bmatrix}
\begin{bmatrix}
\Sigma_{ii,i} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{A}_i(t) & \mathbf{E}_{ii}(t) \\
\mathbf{B}_{ii}(t) & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 \\
I & I
\end{bmatrix}
\begin{bmatrix}
\mathbf{E}_{ii} & 0 \\
I & I
\end{bmatrix}
\begin{bmatrix}
\mathbf{F}_i & \mathbf{E}_{ii} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{T} & 0 \\
0 & I
\end{bmatrix}
< 0
\]
where
\[
\Sigma_{ii,i} = \begin{bmatrix} 0 & \mathbf{P}_{i,H} \\ \mathbf{P}_{i,H}^T & 0 \end{bmatrix}
\]
Noting that the structure of \(\mathbf{P}_{i,0}\) and \(\mathbf{P}_{i,H}\) in (49), we have
\[
\arg\min_{i \in L} \mathbf{P}_{i,0} - \mathbf{P}_{i,H} \mathbf{B}(t) \mathbf{P}_{i,0} - \mathbf{P}_{i,H} \mathbf{B}(t) = \arg\min_{i \in L} \mathbf{P}_{i,0} - \mathbf{P}_{i,H} \mathbf{B}(t) \mathbf{P}_{i,0} - \mathbf{P}_{i,H} \mathbf{B}(t)
\]
This means the switching law (28) is equivalent to (11), which guarantees
\[ \sum_{j \neq i, j \in L} \rho_{j} x(t) \geq 0 \]
Clearly, (52) is equivalent to (23). Then, based on similar approach as before, we know that (33) guarantees (14). Therefore, the passive is satisfied by switching law (28). This completes this proof.

**Remark 5.** The switching law (28) in Theorem 2 is dependent only on \( T_{0} \) and the states of controllers (9). When the state vector of system mode (1) is not accessible, the proposed switching strategy is easy to design and realize.

**Remark 6.** To satisfy (8), \( H + 1 \) controllers working on the corresponding time intervals are designed for subsystem \( i \). Roughly speaking, a larger \( H \) presents more degree of freedom of the controller design during the time interval \([t_{h}, t_{k+1}]\), which leads to less conservatism.

**Remark 7.** In the absence of exogenous input \( o(t) \), the following corollary can be easily obtained, which presents the novel sufficient conditions guaranteeing the asymptotic stability of the switching system (10) with the dwell time constraint (8) via output feedback.

**Corollary 1.** For given scalars \( \gamma > 0 \), \( \rho_{j} \geq 0 \), \( i \neq j \), \( j \in L \), assume switched system (10) switched at \( t = t_{h} \) and \( \sigma(t_{h}) = i \), then, the adjacent switching instant \( t = t_{k+1} \) is chosen by the switching law (28). Under the switching law, the system (10) is asymptotically stable and satisfies (8), if there exist positive matrices \( \overline{P}_{1,i}, \overline{P}_{1,H}, \overline{P}_{2,i,H}, \overline{P}_{3,i,H}, \overline{P}_{1,i}, \overline{P}_{1,i}, \overline{P}_{i,j}, \overline{X}_{i}, \overline{Y}_{i}, \) and matrices \( \overline{P}_{2}, \overline{P}_{2,i}, \overline{P}_{3,i}, N_{i,m}, W_{i,m}, l = 1, \ldots, H - 1, m = 0, \ldots, H, \)

\[
\begin{align*}
\begin{array}{c}
\begin{bmatrix}
-He(\overline{Y}) & \overline{P}_{1,i} - \overline{Y} + \Psi_{i,1}i \\
* & \Psi_{2i,i} 
\end{bmatrix} < 0 \\
\begin{bmatrix}
-He(\overline{Y}) & \overline{P}_{1,i} - \overline{Y} + \Psi_{i,1}i \\
* & \Psi_{2i,i} 
\end{bmatrix} < 0 \\
-He(\overline{Y}) & \overline{P}_{1,i} - \overline{Y} + \Psi_{i,1}i \\
\begin{bmatrix}
* & \Psi_{2i,i} 
\end{bmatrix} < 0 \\
\end{array}
\end{align*}
\]

where \( \overline{Y}, \overline{Psi}_{i}, \overline{Psi}_{2i} \) denote the matrix, \( \overline{Y}, \overline{Psi}_{i}, \overline{Psi}_{2i}, \) are defined in Theorems 1 and 2. Moreover, \( A_{ci}(t), B_{ci}(t) \) and \( K_{ci}(t) \) can be constructed by (34)–(36), respectively.

**Proof 3.** When \( o(t) = 0 \), the storage function \( V(t) \) satisfying (17) is natural converted into the multiple discretized Lyapunov functions. Noting that (18), (49) and following the same lines as before, it is easy to see that \( P(t) \) is piecewise linear, which guarantees overall Lyapunov function \( V(t) \) is continuous at interval instants \( t_{h} + T_{i} \). Moreover, conditions (53)–(55) guarantee \( V(t) \) is decreasing during the time interval \([t_{h} + T_{i}, t_{h} + T_{i+1}]\). Noting that switching law (28) guarantees the \( V(t) \) is non-increasing at the switching instants \( t_{i+1} \), we know that \( V(t) \) is decreasing, which means the asymptotic stability of system (10) with \( o(t) = 0 \) is satisfied.

**Remark 8.** When the states of system (1) are unavailable, the existing approaches in [31,32] are invalid for stabilizing the switched system via state feedback because the state-dependent switching law and controllers cannot be obtained. Compared with these results, Corollary 1 presents a novel design method to stabilize the switched system via output feedback, and the proposed switching law (28) depending only on \( x(t) \) is applicable in this case.

**Fig. 2.** State response of open-loop subsystem 1 of system (1).

**Remark 9.** For a switched system with \( N \) subsystems, when \( N \) is very large, if the proposed LMIs (31)–(33) hold, the passivity of the switched system is still guaranteed. In Example 2 of this paper, the effectiveness of the proposed switching and control strategies for a switched system with four unstable subsystems is tested, which illustrates our results are valid for complex switched systems.

**4. Examples**

In this section, two examples are presented to demonstrate the effectiveness of the proposed method.

**Example 1.** Consider a switched system with two unstable subsystems, where the subsystem matrices are obtained from the model of HiMAT vehicle in [35]:

\[
A_{1} = \begin{bmatrix}
-1.87 & 0.98 \\
12.6 & -2.63 \\
17.1 & -1.85 
\end{bmatrix}, \quad A_{2} = \begin{bmatrix}
-1.35 & 0.98 \\
1.6 & -2.36 \\
1.2 & -1.85 
\end{bmatrix}
\]

where the state \( x(t) = [x_{1}(t) x_{2}(t)]^{T} \) of system (1) denote the angle of the attack and pitch rate, respectively. Further, we suppose other matrices to be

\[
B_{1} = \begin{bmatrix}
0.9 \\
1.81 \\
0.34 \\
0.25 \\
0.15 \\
0.15 
\end{bmatrix}, \quad B_{2} = \begin{bmatrix}
1.93 \\
0.85 \\
0.41 \\
0.26 \\
0.26 \\
0.26 
\end{bmatrix}
\]

\[
E_{1} = \begin{bmatrix}
0.32 \\
0.17 \\
0.17 \\
0.15 \\
0.15 \\
0.15 
\end{bmatrix}, \quad E_{2} = \begin{bmatrix}
0.9 \\
0.85 \\
0.41 \\
0.26 \\
0.26 \\
0.26 
\end{bmatrix}
\]

Figs. 2 and 3 show the states of the open-loop subsystems of switched system, which means the system is non-passive.

Next, to show the effectiveness of the proposed switching and control strategies in output feedback framework, the following assumption is introduced.

**Assumption 1.** The states of switched system (1) are fully unmeasurable.

Then, based on Theorem 2, we first fix \( H = 1 \), this means there are two controllers for each subsystem over the time interval \([t_{h}, t_{h+1}]\). Setting \( \gamma = 1.1, T_{D} = 10 \) and using (29)–(36), we can obtain the gain matrices of the output feedback controllers:
Furthermore, the switching logical matrices are

\[
\begin{bmatrix}
A_{31,0} & B_{31,0} \\
K_{1,0}
\end{bmatrix} = \begin{bmatrix}
13.8410 & -268.1546 & -0.3003 \\
5.3465 & -82.4361 & -0.6083 \\
-18.1446 & 311.7692 & \\
\end{bmatrix},
\begin{bmatrix}
A_{32,0} & B_{32,0} \\
K_{2,0}
\end{bmatrix} = \begin{bmatrix}
22.8360 & -318.5632 & -0.7189 \\
5.4071 & -61.0343 & -0.7968 \\
-14.1969 & 190.6076 & \\
\end{bmatrix},
\begin{bmatrix}
A_{31,1} & B_{31,1} \\
K_{1,1}
\end{bmatrix} = \begin{bmatrix}
11.3023 & -250.8007 & -0.2914 \\
4.6299 & -77.7191 & -0.5793 \\
-16.0429 & 307.3480 & \\
\end{bmatrix},
\begin{bmatrix}
A_{32,1} & B_{32,1} \\
K_{2,1}
\end{bmatrix} = \begin{bmatrix}
21.6556 & -314.4431 & -0.6710 \\
5.0660 & -59.2199 & -0.7551 \\
-14.1949 & 197.6148 & \\
\end{bmatrix}
\]

Then, the switching signal is generated by switching law (28) depending only on \(\hat{x}(t)\) and \(T_0\). Under the switching law, to check the convergence of \(x(t)\), the state response of system (1) and control input are shown in Figs. 4 and 5, which mean the proposed switching and control laws stabilize the system.

Finally, to show the passivity of the closed-loop system, we denote the following functions:

\[
\alpha(t) = \omega^T(t)\dot{z}(t) + \gamma \omega^T(t)\omega(t)
\]

(56)

\[
\beta(t) = \int_0^t [\omega^T(s)\dot{z}(s) + \gamma \omega^T(s)\omega(s)]ds
\]

(57)

where \(\omega(t) = 0.15e^{-0.1t}\cos(15t)\), then \(\alpha(t)\) and \(\beta(t)\) are shown in Figs. 6 and 7, respectively. Clearly, \(\beta(t) > 0.02 > 0\). Thus, our method guarantees the close-loop system is passive.

The following example will shown the effectiveness of the proposed results for a more complex switched system, where there are many subsystems for the system.
Example 2. Consider a switched system with four unstable subsystems:

\[
A_1 = \begin{bmatrix} -1.6 & 0.8 \\ 10 & -2.57 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.82 \\ 0.81 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.57 \end{bmatrix}, \quad E_\omega = \begin{bmatrix} 0.42 \end{bmatrix}
\]

\[
C_1 = \begin{bmatrix} 0.95 & 0.87 \end{bmatrix}, \quad D_{1,1} = 0.45, \quad E_2 = \begin{bmatrix} 0.53 \\ 0.71 \end{bmatrix}, \quad E_\omega = 0.74 \]

\[
A_2 = \begin{bmatrix} -1.44 & 0.9 \\ 15.72 & -1.21 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.93 \\ 0.32 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.71 \end{bmatrix}, \quad E_\omega = 0.66 \]

\[
C_2 = \begin{bmatrix} 1.02 & 0.84 \end{bmatrix}, \quad D_{2,1} = 0.69, \quad E_3 = \begin{bmatrix} 0.35 \\ 0.54 \end{bmatrix}, \quad E_\omega = 0.61 \]

\[
A_3 = \begin{bmatrix} -1.21 & 0.54 \\ 8.17 & -1.98 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.69 \\ 0.51 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0.41 \end{bmatrix}, \quad E_\omega = 0.69 \]

\[
C_3 = \begin{bmatrix} 0.63 & 0.19 \end{bmatrix}, \quad D_{3,1} = 0.33, \quad E_3 = \begin{bmatrix} 0.28 \\ 0.56 \end{bmatrix}, \quad E_\omega = 0.38 \]

\[
A_4 = \begin{bmatrix} -1.18 & 0.49 \\ 8 & -1.82 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0.51 \\ 0.43 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 0.69 \end{bmatrix}, \quad E_\omega = 0.42 \]

\[
C_4 = \begin{bmatrix} 0.35 & 0.19 \end{bmatrix}, \quad D_{4,1} = 0.61, \quad E_4 = \begin{bmatrix} 0.27 \\ 0.36 \end{bmatrix}, \quad E_\omega = 0.99 \]

Based on Theorem 2, we obtain the following output feedback controllers:

\[
\begin{bmatrix} A_{c1, 0} & B_{c1, 0} \\
K_{1, 0} & \end{bmatrix} = \begin{bmatrix} 32.7527 & -381.0965 \\ 6.5510 & -66.5835 \end{bmatrix}, \quad \begin{bmatrix} 6.1023 \\ -1.8233 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{c2, 0} & B_{c2, 0} \\
K_{2, 0} & \end{bmatrix} = \begin{bmatrix} 37.0637 & -437.6976 \\ 6.1329 & -60.0086 \end{bmatrix}, \quad \begin{bmatrix} -23.4071 \\ -1.9110 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{c3, 0} & B_{c3, 0} \\
K_{3, 0} & \end{bmatrix} = \begin{bmatrix} 42.1995 & -498.6655 \\ 6.6538 & -66.2165 \end{bmatrix}, \quad \begin{bmatrix} -20.8423 \\ -2.4317 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{c4, 0} & B_{c4, 0} \\
K_{4, 0} & \end{bmatrix} = \begin{bmatrix} 49.6997 & -549.2849 \\ 8.9210 & -75.9809 \end{bmatrix}, \quad \begin{bmatrix} -107.1416 \\ -0.9452 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{c1, 1} & B_{c1, 1} \\
K_{1, 1} & \end{bmatrix} = \begin{bmatrix} 32.1720 & -391.6736 \\ 6.3864 & -67.6162 \end{bmatrix}, \quad \begin{bmatrix} -22.0749 \\ -2.0134 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{c2, 1} & B_{c2, 1} \\
K_{2, 1} & \end{bmatrix} = \begin{bmatrix} 37.0637 & -437.6976 \\ 6.1329 & -60.0086 \end{bmatrix}, \quad \begin{bmatrix} -23.4071 \\ -1.9110 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{c3, 1} & B_{c3, 1} \\
K_{3, 1} & \end{bmatrix} = \begin{bmatrix} 42.1995 & -498.6655 \\ 6.6538 & -66.2165 \end{bmatrix}, \quad \begin{bmatrix} -20.8423 \\ -2.4317 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{c4, 1} & B_{c4, 1} \\
K_{4, 1} & \end{bmatrix} = \begin{bmatrix} 49.6997 & -549.2849 \\ 8.9210 & -75.9809 \end{bmatrix}, \quad \begin{bmatrix} -107.1416 \\ -0.9452 \end{bmatrix}
\]

and the switching matrices are

\[
P_{31, 0} = \begin{bmatrix} 13.9676 & -0.5871 \\ -0.5871 & 13.2938 \end{bmatrix}, \quad P_{32, 1} = \begin{bmatrix} 13.9661 & -0.9397 \\ -0.9397 & 11.9889 \end{bmatrix}
\]

\[
P_{33, 0} = \begin{bmatrix} 41.5465 & -2.9595 \\ -2.9595 & 36.9493 \end{bmatrix}, \quad P_{33, 1} = \begin{bmatrix} 40.1356 & -3.2674 \\ -3.2674 & 34.4963 \end{bmatrix}
\]

\[
P_{34, 0} = \begin{bmatrix} 37.0527 & -2.2120 \\ -2.2120 & 38.8318 \end{bmatrix}, \quad P_{34, 1} = \begin{bmatrix} 34.6280 & -2.2185 \\ -2.2185 & 35.9901 \end{bmatrix}
\]

Then, the state response of system (1) and $\beta(t)$ are shown in Figs. 8 and 9. Based on the figures and noting that $\beta(t) > 0.05 > 0$, we know the passivity of the switched system is guaranteed.
5. Conclusion

In this paper, we investigated the passivity and feedback passification of switched continuous-time systems satisfying a dwell time constraint. By multiple discretized storage functions, a state-dependent switching law with minimal dwell time is designed such that the switched system is passive via output feedback. Moreover, the switching law uses only the measurable states of controllers, which are easy to apply in practice. Thus, our approach is effective and supposed to be of further improve the previous results.

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