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Abstract. Stephen Wolfram's work, and especially his *New Kind of Science*, presents as much a new science as a new natural philosophy - natural computationalism. In the same way as Andrew Hodges, based on Alan Turing's pioneering work on computability and his ideas on morphological computing and artificial intelligence, argues that Turing is best viewed as a natural philosopher we can also assert that Wolfram's work constitutes natural philosophy. It is evident through natural and formal computational phenomena studied in different media, from the book with related materials to programs and demonstrations and computational knowledge engine. Wolfram's theoretical studies and practical computational constructs including Mathematica and Wolfram Alpha reveal a research program reminiscent of Leibniz' *Mathesis universalis*, the project of a universal science supported by a logical calculation framework. Wolfram's new kind of science may be seen in the sense of Newton's *Philosophiæ Naturalis Principia Mathematica* being both natural philosophy and science, not only because of the new methodology of experimental computer science and simulation, or because of particular contributions addressing variety of phenomena, but in the first place as a new unified scientific framework for all of knowledge. It is not only about explaining special patterns seen in nature and models of complex behaviors; it is about the computational nature derived from the first computational principles. Wolfram's as well as Turing's natural philosophy differs from Galileo's view of nature. Computation used in modeling is more than a language. It produces *real time behaviors of physical systems*: computation is *the way nature is*. Cellular automata as explored by Wolfram are a whole fascinating computational universe. Do they exhaust all possible computational behaviors that our physical universe exhibit? If we understand *physical processes as computations* in a more general sense than the computations performed by symbol manipulation done by our current computers, then universal Turing machines and universal cellular automata exhibit only a subset of all possible information-processing behaviors found in nature. Even though mathematically, there is a principle of computational equivalence, in physical nature exists a hierarchy of emergent processes on many levels of organization that exhibits different physical behavior and thus can be said compute with different expressive power. This article argues that, based on the notion of computing nature, where computing stands for all kinds of information processing, the development of natural computationalism have a potential to enrich computational studies in the same way as the explorations in the computational universe hold a promise to provide computational models applicable to the physical universe.

Evolving Ideas of Systèmes du Monde

Cosmogonies as accounts of the origin and the nature of the universe evolve with growth of human knowledge through allegories, myths, models, theories and paradigms. This development goes in parallel with the increase in the size of the known universe – from

immediate surroundings in the age of great myths, to the earth, solar system, Milky Way, to astonishing 500 billion galaxies - according to current state of knowledge. After a long history of mythopoetic and allegoric accounts of the origins and functioning of the universe, Antiquity formulated first natural philosophical and scientific theories. For Pythagoras, numbers were the essence and the principle of the universe, while for Plato geometry was fundamental. Plutarch (*Convivialium disputationum, liber 8,2*) reports: "Plato said God geometrizes continually". This was in modern times re-interpreted by Gauss as "o theos arithmetizei," or "God computes", Svozil (2005). Irrespective of the choice of arithmetic or geometry, the laws of the universe are governed by mathematical principles, even though one is discrete and the other continuous.

Leibniz (1646-1716) with his philosophy of Monadology holds a special place when it comes to the *Systèmes du Monde*. Monads were defined as elementary automata constituting the complex world through communicating networks, Mainzer (1994). In the Section 18 of *Monadology*, Leibniz depicts a monad as follows: "All simple substances or created Monads might be called Entelechies, for they have in them certain perfection (echousi to enteles); and a certain self-sufficiency (autarkeia) which makes them the sources of their internal activities and, so to speak, incorporeal automata." Leibniz had visionary ideas about calculating machines, he introduced binary notation and argued for the essential role of formal languages, Davis (2000). Wiener, in *The Human Use of Human Beings*, describes Leibniz as a forerunner of cybernetics "Leibniz, dominated by ideas of communication, is in more than one way the intellectual ancestor of the ideas of this book for he was also interested in machine computation and automata.", (Wiener 1988, p. 19). According to contemporary informational interpretation of Uchii (2009), Leibniz's monads can be interpreted as information carriers programmed by divine code to change informational contents of their internal states. The divine coding guaranteed correspondence between the activities of monads and the world of phenomena.

Système du Monde of the Clockwork (mechanistic) universe is an example of a flawlessly lawful scientifically-based universe, in the form of a perfect machine, governed by the laws of physics. Laplace (1749-1827) believed that a Supreme Intelligence, based on the laws of nature and on knowledge of the positions and velocities of all particles in the universe at any moment could infer the state of the universe at any future or past time according to the laws of mechanics discovered by Newton (1642-1727). Even though the universe-automaton is a physical system, Galileo (1564-1642) in his book *The Assayer - Il Saggiatore*, points to vital connection between physics and mathematics, claiming that the way to understand nature is through mathematics.

The mechanistic world is based on the following principles, Dodig Crnkovic and Müller (2011):

- (M1) The ontologically fundamental entities of the physical reality are physical structures (space-time & matter-energy) and change of physical structures (dynamics).
- (M2) All the properties of any complex physical system can be derived from the properties of its components.
- (M3) Change of physical structures is governed by laws.
- (M4) The observer is outside of the system observed.

Mechanistic models assume that the system is closed, isolated from the environment, and laws of conservation (energy, mass, momentum, etc.) thus hold. Environment, if modeled at all, is treated as a perturbation for the steady state of the system.

The limits of a mechanistic universe and determinism were uncovered by the increasing use of computers as tools of exploration, especially in the biological world. What begins to emerge nowadays is a fundamentally new paradigm of not only sciences but even a more general paradigm of the universe, comparable in its radically novel approach with its historical predecessors the Mytho-poetical Universe, the Universe of Ideal Mathematical Principles and the Mechanistic Universe. This new paradigm is dubbed Info-Computational Universe; for the details, see Dodig –Crnkovic (2006).

Our current understanding of the fundamentality of *information* and *computation* for the structure and dynamics of the natural world, has led to an articulation of the universe as a computer, a network of computational processes on informational structures.

The Computing Universe – Naturalist Computationalism

“Will we find the whole of physics? I don’t know for sure. But I think at this point it’s sort of almost embarrassing not to at least try.” Wolfram talk from the 2010 TED Conference

The idea of computing nature (natural computationalism, pancomputationalism) is old, and in a general sense can be traced back to Leibniz. Among the first contemporary researchers sharing computational view of nature are Konrad Zuse, Edward Fredkin, Tommaso Toffoli and Stephen Wolfram, together with Jürgen Schmidhuber, Seth Lloyd, Charles Seife, and Gregory Chaitin.

Konrad Zuse was the first to suggest in 1967 that the physical behaviour of the entire universe is being computed on a basic level, possibly on cellular automata, by the universe itself, which he referred to as "Rechnender Raum" or Computing Space, Zuse (1969; 1970). "The idea that space might be defined by some sort of causal network of discrete elementary quantum events arose in various forms in work by Carl von Weizsäcker (ur-theory), John Wheeler (pregeometry), David Finkelstein (spacetime code), David Bohm (topochronology) and Roger Penrose (spin networks). General arguments for discrete space were also sometimes made--notably by Edward Fredkin, Marvin Minsky and to some extent Richard Feynman--on the basis of analogies to computers and in particular the idea that a given region of space should contain only a finite amount of information.", Wolfram (2002, p. 1026). Zuse had the idea of "going beyond quantum mechanics in discretizing physics, a vision he shared with the late Einstein and many researchers, among others Fredkin, Toffoli, Margolus, and Wolfram.", Svozil (2005).

Wolfram (2002), based on extensive studies of cellular automata, advocates for a pancomputationalist view as a new dynamic kind of reductionism in which the complexity of behaviors and structures found in nature are derived (generated) from a few basic structures and processes:

“I strongly suspect that the vast majority of physical laws discovered so far are not truly fundamental, but are instead merely emergent features of the large-scale behavior of some ultimate underlying rule. And what this means is that any simplicity observed in known physical laws may have little connection with simplicity in the underlying rule. So perhaps in the end there is the least to explain if I am correct that the universe just follows a single, simple, underlying rule.” (Wolfram, 2002, p. 471)

Wolfram and Fredkin (1990), in the similar vein as Zuse, assume that the universe is, on a fundamental level, a discrete system. Following the principle that “the ultimate model of physics is to be as simple as possible” Wolfram (2002, p. 475) expects the features of the universe to emerge “purely from properties of space”. This presupposes that space is the independent first principle. It is however also possible that space-time and matter-energy emerge at once; that there is no space without matter-energy. But in this context, this is a detail. The most important is the expressive power, productivity and internal coherence of models, and models can differ.

Moreover, even though discrete models possess many attractive features, physics regularly uses both. (Lesne, 2007) argues for the necessity of continuum in physical modeling of the world. Here is the summary:

“This paper presents a sample of the deep and multiple interplay between discrete and continuous behaviours and the corresponding modellings in physics. The aim of this overview is to show that discrete and continuous features coexist in any natural phenomenon, depending on the scales of observation. Accordingly, different models, either discrete or continuous in time, space, phase space or conjugate space can be considered.” (Lesne, 2007, p.185)

However the computing universe (natural computationalism) does not critically depend on the discreteness of the models of the physical world. There are digital as well as analog, discrete and continuous-state models as well as computers. On a quantum-mechanical level, the universe performs, on characteristically dual wave-particle objects, both continuous and discrete computation, Lloyd (2006).

Turing and the Computing Nature

Not only Leibniz can be seen as a predecessor of natural computationalism, Turing can be added to the list as well, based on his conviction that machines (can be made that) can think and on his work on unorganized machines (neural networks) and morphogenesis.

Turing is well known in the first place for his contributions to the theory of computation, computer science, (Turing machine model) Turing (1936; 1950), and artificial intelligence (Turing test), but for his biographer Hodges, Turing is ultimately a natural philosopher:

“He thought and lived a generation ahead of his time, and yet the features of his thought that burst the boundaries of the 1940s are better described by the antique words: natural philosophy.” (Hodges, 1997, p.3)

It is important to notice that Turing's natural philosophy goes further than Galileo's view about the language of nature:

"Philosophy [i.e. physics] is written in this grand book — I mean the universe — which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth." (Galileo, 1623, p.237)

Computing differs from mathematics in that computers not only calculate numbers, but more importantly produce real time behaviors. Turing studied a variety of natural phenomena and proposed their computational modeling. He made a pioneering contribution in the elucidation of connections between computation and intelligence and his work on morphogenesis provides evidence for natural philosophers' approach.

Turing's paper on morphogenesis proposed a chemical model as the basis of the development of biological patterns, Turing (1952). He did not originally claim that the physical system producing patterns actually performs computation through morphogenesis. Nevertheless, from the perspective of contemporary natural computationalism and particularly info-computationalism we can argue that morphogenesis is a process of morphological computing, Dodig-Crnkovic (2012).

Physical process, though not computational in the traditional sense, presents natural (unconventional), physical, morphological computation. An essential element in this process is the interplay between the informational structure and the computational process – information self-structuring.

The process of computation implements physical laws which act on informational structures. Through the process of computation, structures change their forms, as argued in Dodig-Crnkovic (2011). All computation on some level of abstraction can be viewed as morphological computation – a form-changing/form-generating process on informational structures, Dodig-Crnkovic (2012).

Generation of Form by Morphogenetic and Morphological Computing

*"With this background it becomes understandable that we need no **intelligent design** of complex structures, but only very simple rules for local elements that generate global structures during their evolution."* (Mainzer and Chua, 2011, p.9)

Generation of form can be studied by cellular automata based on rules defining updated configurations of a grid of cells, and equivalent rules for other simple programs, but it can also be studied in physical systems undergoing morphogenesis or metamorphoses. In such systems

underlying physical laws express themselves as a computation causing changes of existing forms. This process has been studied in robotics and nano-systems Mac Lennan (2010), and recently even on the macroscopic scales in materials and architecture as material computation, computing matter and material computation, Menges (2012), but it deserves more attention as a basic phenomenon of form generation in physical matter, especially intricate in living systems.

Despite the mathematical principle of computational equivalence¹, in physical nature there is a hierarchy of emergent processes on many levels of organization exhibiting different physical behaviors. Based on the notion of computing nature, where computing stands for all kinds of information processing, the development of natural computationalism enriches our understanding of computation by computational studies of physical systems, in the similar way as the explorations in the computational universe provide new models applicable to the physical universe. The process goes in both directions – from the physical to the models and the other way round, Rozenberg and Kari (2008).

Criticisms of the Computational Views of the Universe

In his article on Physical Computation for The Stanford Encyclopedia of Philosophy, Gualtiero Piccinini (2010) presents several critical arguments against Pancomputationalism (Naturalist computationalism). The unlimited Pancomputationalism, the most radical version of Pancomputationalism, according to Piccinini asserts that “every physical system performs every computation—or at least, every sufficiently complex system implements a large number of non-equivalent computations”. I argue these to be two substantially different claims. The first one, that every system executes every computation, has little support in physics and other natural sciences. Different sorts of systems perform different sorts of dynamical behaviors. The second claim, that a sufficiently complex systems implement a large number of different computations, is in accordance with natural sciences and essentially different from the claim that every system performs every computation, Dodig-Crnkovic in Dodig-Crnkovic and Müller (2011).

As for the sources of Naturalist computationalism, Piccinini identifies several:

One source is “a matter of relatively free interpretation” which computation a system performs. This may well be true of human computational devices like fingers, pebbles, abacuses, and computers even though interpretations once chosen are kept constant (thus no longer free), in order to allow social communication of results.

Another source of Pancomputationalism is the causal structure of the physical world. That claim goes one step further than the first one, actually searching for the basis of “free

¹ “Almost all processes that are not obviously simple can be viewed as computations of equivalent sophistication.” (Wolfram 2002, pp.5 and 716-717). “Almost any dynamical system that doesn't lead to random or transparently fixed or oscillatory behavior, is likely to be a universal computer.” (Goertzel, Dynamical Psychology, 2002)

interpretation". We can freely chose systems used for calculation/computation, but the computational operations performed are predictable because of the laws of physics which guarantee that physical objects behave in the same way and according to physical laws so that we can predict and use their behaviour for computation.

Info-computationalism is in the Piccinini scheme based on the third source:

"A third alleged source of pancomputationalism is that every physical state carries information, in combination with an information-based semantics plus a liberal version of the semantic view of computation. According to the semantic view of computation, computation is the manipulation of representations. According to information-based semantics, a representation is anything that carries information. Assuming that every physical state carries information, it follows that every physical system performs the computations constituted by the manipulation of its information-carrying states (cf. Shagrir 2006). Both information-based semantics and the assumption that every physical state carries information (in the relevant sense) remain controversial."

The use of the word "manipulation" seems to suggest a conscious intervention, while computation in general, as understood within the framework of Computing Nature/Natural Computationalism/Pancomputationalism, is a natural dynamical process that drives (through the physical interaction mechanisms) changes in informational structures. Notwithstanding Piccinini's skepticism, there are well established theories in computer science which do exactly the job of connecting computational processes and informational structures as suggested by info-computationalism, Dodig-Crnkovic (2011).

Recently, Piccinini made a substantial move in the direction of Natural Computationalism by advocating, what he calls *the modest view of the physical Church-Turing thesis* Piccinini (2011). Here his claim in short is that not all of physical computation is Turing computable. This view agrees with our best knowledge about Natural Computation today and it also brings us closer back to the Turing's work concerning unorganized machines with oracles (advice, learning).

Yet another interesting source of criticism towards Natural Computationalism and in particular Info-Computationalism is expressed by Vincent Müller in Dodig-Crnkovic and Müller (2011):

"There might be a set of computing procedures that is larger than the one defined by Church-Turing – and there is certainly a mathematical set of computable functions larger than that computable by Turing machine (e.g. that computable by Turing's idea of his machine plus "oracle"). (...) My understanding of 'computer', as suggested by [Turing, 1936], is that such machines characteristically go beyond mere calculators (like those already invented by Leibniz and Pascal) in that they are universal; they can, in principle, compute any algorithm, because they are programmable – in this sense, Zuse's Z3 was the first computer (1941). If this feature of universality is a criterion for being a computer, then analog machines do not qualify because they can only be programmed in a very limited sense. (...) First, how can you guarantee that the

notion of 'computing' you are using here is in any sense unified, i.e. one notion?" (Dodig-Crnkovic and Müller, 2011, p.162.)

So in what way is physical computation/natural computation important? One of the central questions within computing, cognitive science, AI and other related fields is about computational modeling (and simulating) of intelligent behaviour. What can be computed and how? It has become obvious that we must have richer models of computation, beyond Turing machines, if we are to efficiently model and simulate biological systems. What exactly can we learn from nature and especially from intelligent organisms?

It has taken a more than sixty years from the first proposal of Turing test he called the "Imitation Game", described in Turing (1950) p. 442, to the recent (2011) IBM's Watson machine winning Jeopardy by purely computational means. That is just the beginning of what Turing believed one day will be possible – a construction of computational machines capable of generally intelligent behavior as well as the accurate computational modeling of natural world.

Computation vs. Universal Computation

Computation is a process that a physical system undergoes when processing information (computing). Computation as a phenomenon is studied within several research fields: theory of computation, including computability theory, physics, biology, logic, and so on. It is worth noticing that the German, French and Italian languages use the respective terms "Informatik", "Informatique" and "Informatica" (Informatics in English) to denote Computing, indicating close relationships between computation and information. In Dodig-Crnkovic (2006) it is argued that information constitute the structure, the fabric of the universe, while computation is synonymous with physical process that, implementing physical laws, incessantly changes informational structures.

The ability of a computer to perform universal computation (i.e. to process not only input data but also the code describing any other computing machine) is considered central. Here is the explanation given by Svozil (2005): *"The notion of universal computation is robust in the sense that any universal computer can emulate any other universal computer (regardless of efficiency and overhead), so that it does not really matter which one is actually implemented. (...) So, when it comes to their generic properties, it is not really important whether automaton universes are modeled to be Cellular Automata, Turing Machines, colliding billiard balls [8], or biological substrates."* Fredkin (1991) notices: *"This is also the disadvantage. It is hard to think about the properties of the members of a class when each member can do everything. The field of Computer Science has very few examples of useful or meaningful analytic solutions as to what some digital system will or won't do. On the contrary, there is a celebrated proof that, in general, there are no analytical shortcuts that can tell the future state of some general computation any quicker than doing the computation step by step (this is the so called "halting problem" for Turing Machines, Turing 1936). **There are normally no solutions in closed form. There is not yet any good hierarchy of concepts that express complex behavior in terms of simpler behavior, as is done in physics.**"* (Emphasis added)

This is the core of the problem: There is no hierarchy. In physics there is natural encapsulation, so in principle separation between different levels of organization. A meta-language is information compression of the level below. However, discrete automata are all on the same organizational level, even though they show temporal development. That is why a universal automaton, which as an input takes arbitrary machine and executes its algorithm cannot do any better than the machine itself, as it operates on the same information. The way to make it possible for a universal machine to be more powerful is to first separate levels of abstraction between metalevel (universal) and object level (particular algorithm). That is what is done in physics “for free” by self-organization based on natural laws on different organization levels (spatial scales).

In general it is not necessary for computation belonging to different classes of processes to be universal. Physical processes in quantum mechanics are different from processes in the classical clockwork universe and it is not a big problem if they are modeled differently, by different classes of computers. That is what present day physics does – it produces different theoretical frameworks for different levels of organization – from quarks to galaxies. We have different frameworks executed on the same sort of computer. In the future we can have the same framework executed on different sorts of computers.

Questions Beyond Present Computational Experiments

Even within the world of cellular automata, there are number of interesting questions for future investigations. Mainzer and Chua (2011) propose:

“Going beyond the numerical experiments of Steven Wolfram, it is argued that cellular automata must be considered complex dynamical systems in their own right, requiring appropriate analytical models in order to find precise answers and predictions in the universe of cellular automata. Indeed, eventually we have to ask whether cellular automata can be considered models of the real world and, conversely, whether there are limits to our modern approach of attributing the world, and the universe for that matter, essentially a digital reality.”

Instead of exploring cellular patterns from a phenomenological point of view, Mainzer & Chua (2011) apply analytical methodology like the one used in mathematical physics.

I would add some of questions that came to my mind when reading Wolfram’s book. Here are some of them.

Cellular automata get updated synchronously. How about diachronic processes? If they are modeling physical world as we know it, it should be possible to model an event originated in the past (like a photon created in the Big Bang) to interact with the informational structure in the contemporary universe (trigger a detector today). How about non-local systems?

Cellular automata and simple programs have demonstrated surprisingly rich expressive power in modeling self-organization and emergent properties in systems consisting of similar units but

there are phenomena in nature that seem to be radically different: How about interactions in totally heterogeneous systems? Can they be reduced to the properties of the underlying grid?

How about evolution? How could evolution and development be implemented in the world of cellular automata? Deacon (2011) for example proposes constructive mechanisms to explain (the unavoidable) evolution from thermodynamic to anticipatory (teleological) systems that are in agreement with natural computation (physical computation).

It is possible that computation on a mathematical level is "all or nothing" (computational equivalence), but if we want to ascribe computational characteristics to the physical world and explain its full complexity, we must admit that there are hierarchical structures in physical systems that have complex systemic properties. How could the architecture be build up out of form-generating algorithms? In nature there is a hierarchical succession of levels of organization and every higher level can be described by meta-language with respect to previous level. How about second order algorithms, or algorithms changing algorithms?

Could that be that the underlying cellular automaton of the universe expands producing expanding universe which we observe? What would that mean for the properties of the automaton?

Conclusion. The Dream of Leibniz Coming True?

"Could it be that someplace out there in the computational universe we might find our physical universe? Will we find the whole of physics? ... I think computation is destined to be the defining idea of our future." Stephen Wolfram, TED talk, filmed Feb. 2010

Wolfram's New Kind of Science is one of his closely interconnected projects that can be understood in relation to the Leibniz's quest for automation of reason in a universal science, Mathesis universalis (1695). Leibniz's characteristica universalis was envisaged as algebra expressing conceptual thought by a formal system based on the rules for symbolic manipulation of calculus ratiocinator. There are two opposed interpretations of Leibniz's calculus ratiocinator: the first is analytic view relating calculus to software and "algebra of logic", and the second, synthetic view, found in cybernetics, understands calculus ratiocinator as referring to a "calculating machine". This duality may be seen as reflecting the dichotomy between mathematical and physical view of computation.

The development of formal systems, Hilbert's program and the development of programmable computational machinery all contributed to the gradual realization of the formalization project of Leibniz. However, at the same time the development of human knowledge run into increasing fragmentation and specialization which has reached alarming proportions. So, for example, at present no individual can have general knowledge of physics broad enough to cover all its different fields – from string theory to astrophysics.

Wolfram's project, contrary to the general division into disparate knowledge compartments, runs towards common synthetic framework using tools of formal reasoning and Mathematica as calculus ratiocinator, achieving a wide-ranging synthesis of knowledge. Adding Wolfram Alpha's capability to accumulate and compute general knowledge, this project bears a resemblance to the ambitions of *Mathesis universalis*, and brings renewed renaissance optimism about the human capability to know the world based on natural laws, with computation as an organizing principle of all knowledge.

References

- Davis, M., 2000. *The Universal Computer: The Road from Leibniz to Turing*. WW Norton.
- Deacon, T., 2011. *Incomplete Nature: How Mind Emerged from Matter*. New York: W.W. Norton & Company.
- Dodig-Crnkovic, G., 2006. *Investigations into Information Semantics and Ethics of Computing*, Mälardalen University Press.
- Dodig-Crnkovic G., and V. Müller, 2011. A Dialogue Concerning Two World Systems: Info-Computational vs. Mechanistic. In: *INFORMATION AND COMPUTATION*, World Scientific Publishing Co. Series in Information Studies. Editors: G Dodig-Crnkovic and M Burgin.
- Dodig-Crnkovic, G., 2011. Info-Computational Philosophy of Nature: An Informational Universe with Computational Dynamics, In: *From First to Third via Cybersemiotics*, the Festschrift for Prof. Søren Brier; Thellefsen, T., Sørensen, B. and Cobley, P., Eds.; CBS Copenhagen, Denmark, p.97.
- Dodig-Crnkovic, G., 2012. Info-computationalism and Morphological Computing of Informational Structure, in *Integral Biomathics, PART II: Mathematics and Computation*; Simeonov, P., Smith, L. and Ehresmann, A. (Eds.). Springer Series on Computational Intelligence and Complexity.
- Fredkin, E., 1991. Finite Nature, *Proceedings of the XXVIth Rencontre de Moriond*, pp. 283-297.
- Fredkin, E., 1990. Digital Mechanics: An Information Process Based on Reversible Universal Cellular Automata, *Physica D*, 45. pp.254–270.
- Galileo, G., 1623. *Il Saggiatore* (in Italian); *The Assayer*, English trans. Stillman Drake (1957), *Discoveries and Opinions of Galileo* pp.237-238.
- Hodges, A., 1997. *Turing. A Natural philosopher*. Phenix. London
- Lesne, A., 2007. The discrete versus continuous controversy in physics. *Mathematical Structures in Computer Science*, 17, pp.185-223.
- Lloyd, S., 2006. *Programming the Universe: A Quantum Computer Scientist Takes on the Cosmos*. New York: Knopf.
- MacLennan, B.J., 2010. Models and Mechanisms for Artificial Morphogenesis, *Natural Computing*, Springer series, *Proceedings in Information and Communications Technology (PICT) 2*, ed. by F. Peper, H. Umeo, N. Matsui, and T. Isokawa, Tokyo: Springer, , pp. 23–33.
- Mainzer, K. and Chua, L.O., 2011. *The Universe as Automaton: From Simplicity and Symmetry to Complexity*, Springer-Verlag Berlin and Heidelberg GmbH & Co.
- Menges, A., ed., 2012. *Material Computation – Higher Integration in Morphogenetic Design, Architectural Design*, Vol. 82 No. 2, Wiley Academy, London.

- Piccinini, G., 2010. Computation in physical systems. The Stanford Encyclopedia of Philosophy, Fall 2010 ed.; Stanford University. Available online: <<http://plato.stanford.edu/>>
- Piccinini, G., 2011. The Physical Church-Turing Thesis: Modest or Bold? *British Journal for the Philosophy of Science* 62 (4). pp.733-769.
- Rozenberg, G. and Kari, L., 2008. The many facets of natural computing, *Communications of the ACM*, 51. pp.72–83.
- Svozil, K., 2005. Computational universes, *Chaos, Solitons & Fractals*, Volume 25, Issue 4, pp.845-859.
- Turing, A.M., 1952. The Chemical Basis of Morphogenesis. *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, Vol. 237, No. 641. (Aug. 14, 1952), pp. 37-72.
- Turing, A.M., 1950. Computing Machinery and Intelligence, *Mind*, LIX(236). pp.433–460.
- Turing, A.M., 1936. "On Computable Numbers with an Application to the Entscheidungsproblem", *Proceedings London Math. Soc.*, series 2, 43, pp.544-546.
- Uchii, S., 2009. *An Informational Interpretation of monadology*. Preprint <<http://philsci-archive.pitt.edu/4635/>>
- Wiener, N., 1988. *The Human Use of Human Beings: Cybernetics and Society*. Da Capo Press.
- Wolfram, S., 2002. *A New Kind of Science*, Champaign, IL: Wolfram Media.
- Zuse, K., 1970. *Calculating space*. MIT technical translation AZT-70-164-GEMIT, MIT (Proj. MAC). Cambridge, MA.
- Zuse, K., 1969. *Rechnender Raum*, Friedrich Vieweg & Sohn, Braunschweig.