Double-fed Induction Machine – Dynamic Modeling
using Winding Function Approach

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Abstract - This paper describes a dynamic model of double-fed induction machine in a natural frame of reference. Winding function approach is used for inductance calculations, taking into account all MMF space harmonics simultaneously, which makes this model suitable for motor (generator) current signature analysis. Presented results from numerical experiments where double-fed induction machine have been analyzed in motoring and generating sub or super synchronous regime, illustrate the power of the model.

I. INTRODUCTION

The principle of operation of double-fed induction machine is known long years ago. However, rather complicated principle of operation was a limiting factor for effective use of this machine until the invention of static converters. Namely, the principle of operation of this machine is based on inserting voltages from the rotor side in order to control the speed of the machine as well as active and reactive power. The inserted voltages should be of variable frequency, i.e., their frequency should be in any time identical with the rotor induced emfs frequency. Beside frequency, care must be taken about adequate phase sequence of inserted voltages. Inserted voltages that are in phase or out of phase regarding to rotor induced emfs, have impact on the rotor speed of the machine. Other phase relationships have, on the first place, impact on the reactive power which the machine takes or gives to the grid, [1].

In the last two decades, double-fed induction machine experiences full affirmation, on a first place, in wind generators. Double-fed induction generator has a unique property that in the case that is connected to the grid, generate electrical energy of grid frequency independently of rotor speed. Other important characteristic of this generator is the possibility of speed regulation using the inserted voltages when this machine could work in generator regime under or above the synchronous speed.

This paper describes the multiple coupled circuit model of double-fed induction machine using winding function for the inductance calculation. By this way, all MMF space harmonics are taken into account simultaneously, what enables analysis of stator and rotor currents spectrum for diagnosis and condition monitoring purposes.

II. MATHEMATICAL MODEL

Double-fed induction machine is described by the following set of matrix equations:

\[ U_s = R_s I_s + \frac{d\psi_s}{dt} \]
\[ U_r = R_r I_r + \frac{d\psi_r}{dt} \]
\[ \psi_s = L_{ss} I_s + L_{sr} I_r \]
\[ \psi_r = L_{rs} I_s + L_{rr} I_r \]

Derivatives of stator and rotor flux linkages are,

\[ \frac{d\psi_s}{dt} = L_{ss} \frac{dI_s}{dt} + L_{sr} \frac{dI_r}{dt} + \frac{dL_{ss}}{d\theta_s} \frac{d\theta_s}{dt} I_s \]
\[ \frac{d\psi_r}{dt} = L_{rs} \frac{dI_s}{dt} + \frac{dL_{ss}}{d\theta_r} \frac{d\theta_r}{dt} I_s + L_{rr} \frac{dI_r}{dt} \]

where

\[ L_{ss} = L'_{ss} \]
\[ \frac{d\theta_s}{dt} = \omega \]

Additional equations describes the mechanical part of the system:

\[ T_m = 0.5 \left( I_s \frac{dL_{ss}}{d\theta_s} I_s + I_r \frac{dL_{ss}}{d\theta_r} I_r \right) \]
\[ \frac{d\theta}{dt} = \frac{1}{J}(T_m - T_f) \tag{10} \]

Above system of equations could be easily solved using some of the numerical techniques for known parameters of the model.

### III. Model Parameters

The key parameters of any induction machine model are winding inductances. In this paper these inductances are calculated using winding function theory, [2-5].

Due to the uniform air gap and common assumption of infinitely permeable iron, self-inductances of stator and rotor windings are independent of rotor position. Consequently, self inductance of rotor winding, say \( a \),

\[ L_a = \frac{\mu_0 r l}{g_0} \int_0^{\pi} n_a(\theta) N_a(\theta) d\theta \tag{11} \]

is simply constant value. In (11) \( r \), \( l \) and \( g_0 \) are mean radius of air gap, axial length of the machine and effective length of air gap, respectively. Under the integral are so called turns and winding function, respectively, which are function of angle measured with datum on rotor for rotor winding, or on stator for stator winding.

Mutual inductance between two stator or two rotor windings are also rotor position independent and could be calculated on same manner. For \( A \) and \( B \) stator windings,

\[ L_{AB} = \frac{\mu_0 r l}{g_0} \int_0^{\pi} n_A(\theta) N_B(\theta) d\theta \tag{12} \]

However, mutual inductance between stator and rotor windings (phases) are rotor position dependent. They could be calculated on same manner as above, but for different rotor positions,

\[ L_{aA}(\theta_r) = \frac{\mu_0 r l}{g_0} \int_0^{\pi} n_a(\theta) N_A(\theta_r) d\theta \tag{13} \]

Mutual inductance between stator phase \( A \) and rotor phase \( a \), as a function of rotor position, as well as derivative of this inductance is given in Fig 1.

![Fig. 1. Mutual inductance between stator and rotor phase winding in function of rotor position](image1)

Obviously, shape of inductance curve differs from harmonic function. This is due to the accounting of all MMF space harmonics from stator as well as from rotor side. By other words, exact position of stator and rotor windings, number of turns in individual coils as well as manner of coil interconnection are taken into account using winding and turn functions of phase windings.

### IV. Results of Simulation

For purpose of illustration of described model, numerical experiment on three phase machine, which parameters are given in Appendix, is conducted.

#### A. Motoring regime

As a first step, ordinary induction motor was analyzed. Fig. 2. shows the speed, electromagnetic torque and rotor phase current during run-up transient regime. At moment \( t=4s \), motor was loaded with \( T_l=50Nm \). Rotor slip in steady state: \( s=3.18\% \).

![Fig. 2. Start-up of induction motor and loading with \( T_l=50Nm \) at \( t=4s \). Upside down: rotor speed, electromagnetic torque, and rotor phase current](image2)

Stator phase current spectrum in steady state is presented on Fig.3. The most significant higher frequency components are two components on frequencies 243 and 337Hz. These components are due to the rotor MMF space harmonics. Namely, rotor MMF waves are given by,

\[ F_s(\theta_r) = F_{s_{max}} \cos(2\pi f_r - s\alpha t) \tag{14} \]
where $\nu = 6g+1$, $g=0, \pm 1, \pm 2, \ldots$. For uniform air gap, magnetic flux density waves, from rotor side, could be described by the similar expression,

$$B_s(\theta) = B_{s,\text{max}} \cos(\nu\theta - \omega_0 t)$$  \hspace{1cm} (15)$$

All of these harmonics induce currents in stator winding. Among them, the most significant are 5th, 7th and so called principal slot harmonics of order $R/p+1$ and $R/p-1$. In stator reference frame, using following transformation,

$$\theta = \theta_0 + \frac{1-s}{p} \omega_0 t$$  \hspace{1cm} (16)$$

flux density waves are,

$$B_s(\theta) = B_{s,\text{max}} \cos(\nu\theta - (\nu(1-s) - s)\omega_0 t).$$  \hspace{1cm} (17)$$

Therefore, in stator winding could be expected currents of following frequencies:

$$f_s = [\nu(1-s) - s]f_1$$  \hspace{1cm} (18)$$

The most significant rotor space harmonics, 5th, 7th gives

$$f_s = \frac{5 \pi}{\nu} \rightarrow 243 \text{ [Hz]}$$
$$f_s = \frac{7 \pi}{\nu} \rightarrow 337 \text{ [Hz]}$$

Principal rotor slot harmonics, $\nu = -17$ and $\nu = 19$, also induces high frequency stator current components, however, of much smaller magnitudes.

It should be noted that, opposite to the wound rotor machine, cage rotor, beside fundamental MMF harmonic, produce only principal slot harmonics, [4]. So, stator currents of cage induction machine have no frequency components in this, lower part of spectrum (around 300Hz for 50Hz machines or around 360Hz in 60Hz machines).

Double-fed induction motor has following, unique property. It can work in motoring regime even at super synchronous speed. It could be realized by inserting the proper frequency and phase sequence rotor voltages. Such working regime is illustrated on Fig.4, where at $t=6s$, in rotor circuit was inserted three phase inverse sequence voltages whose amplitude was $8\sqrt{2}$V and frequency $f_2=0.03 \cdot f_1$. Power balance in this operating regime is given in Table I.

![Fig.4. Start-up of induction motor, loading with $T_L=50Nm$ at $t=4s$ and turnover to super synchronous motoring regime at $t=6s$. Upside down: rotor speed, electromagnetic torque, and rotor phase current.](image)

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>POWER BALANCE IN SUB AND SUPER SYNCHRONOUS MOTOING REGIME</th>
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<tbody>
<tr>
<td>$s=3.18% \ (4s&lt; t &lt; 6s)$</td>
<td>$s=-2.95% \ (t &gt; 6s)$</td>
</tr>
<tr>
<td>$P_1 \ [W]$</td>
<td>8450</td>
</tr>
<tr>
<td>$P_2 \ [W]$</td>
<td>0</td>
</tr>
<tr>
<td>$P_{\text{Cu}} \ [W]$</td>
<td>570</td>
</tr>
<tr>
<td>$P_{\text{Cu}} \ [W]$</td>
<td>258</td>
</tr>
<tr>
<td>$P_{\text{mech}} \ [W]$</td>
<td>7602</td>
</tr>
</tbody>
</table>

In described super synchronous regime of operation, induction motor observed from the stator side, work with capacitive power factor, Fig.5.

![Fig.5. Stator phase voltage and current, during transient process at $t=6s$.](image)

**B. Generating regime**

Single fed induction machine works in generator regime above synchronous speed. On the other side, double-fed induction generator could push electrical energy into the grid.
even at sub synchronous speed. It could be realized by inserting the proper frequency and phase sequence rotor voltages. Such working regime is illustrated on Fig.6. After start-up of the machine as a motor, at \( t=4s \) machine is loaded with \( T_L=-50\text{Nm} \), i.e. it is made transition to the "normal" generator regime. At \( t=6s \), in rotor circuit was inserted three phase inverse sequence voltages whose amplitude was \( 8\sqrt{2}\text{V} \) and frequency \( f_2=0.03f_1 \). Power balance for this operating regime is given in Table II.

![Fig.6. Start-up of induction motor, loading with \( T_L=-50\text{Nm} \) at \( t=4s \) and turnover to sub synchronous generating regime at \( t=6s \). Upside down: rotor speed, electromagnetic torque, and rotor phase current](image)

### Table II

<table>
<thead>
<tr>
<th>( )</th>
<th>Power Balance in Super and Sub Synchronous Generating Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 ) [W]</td>
<td>( s=-2.62% ) (4s&lt; ( t ) &lt;6s)</td>
</tr>
<tr>
<td>( P_2 ) [W]</td>
<td>-7283</td>
</tr>
<tr>
<td>( P_{el} ) [W]</td>
<td>520</td>
</tr>
<tr>
<td>( P_{mech} ) [W]</td>
<td>215</td>
</tr>
<tr>
<td>( P_{inc} ) [W]</td>
<td>-8060</td>
</tr>
</tbody>
</table>

### Conclusion

A new approach to double-fed induction machine modeling, based on winding function approach, has been introduced in this paper. The model is based directly on the geometry of the induction machine and physical layout of all windings. Several operating regimes of the double-fed induction machine have been simulated to demonstrate its versatility.

### Appendix

Three phase machine parameters:

- \( P_{in}=415\text{V}, f=50\text{Hz} \), stator winding connection \( \Delta \), rotor winding connection \( Y \)
- \( r=0.08\text{m}, l=0.11\text{m}, g_0=0.8\text{mm}, N_s=28 \) turns per coil, \( N_r=28 \) turns per coil
- number of stator slots \( S=48 \), number of rotor slots \( R=36 \), \( p=2 \), \( R_r=1.75\Omega \), \( R_s=0.1\Omega \), \( J=0.0754\text{kgm}^2 \), rotor slot skewing \( \gamma=2\pi/48 \)

- Stator winding scheme:
  \( A-1-16'-2-15'-3-14'-4-13'-25-40'-26-39'-27-38'-28-37'-X \)

- Rotor winding scheme:
  \( a-1-10'-2-11'-3-12'-21-12'-20-11'-19-19'-18-28'-20-29'-21-30'-3-30'-2-29'-1-18'-x \)

### References