Method for measuring the refractive index and the thickness of transparent plates with a lateral-shear, wavelength-scanning interferometer

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A new method for measuring simultaneously the thickness and the refractive index of a transparent plate is proposed. The method is based on a simple, variable lateral-shear, wavelength-scanning interferometer. To achieve highly accurate measurements of both refractive index n and thickness d we use several means to determine these two quantities. We finely tune a distributed-feedback diode laser light source to introduce a phase shift into the detected signal, whereas we make the sample rotate to produce variable lateral shearing. Phase shifting permits precise determination of the optical thickness, nd, whereas refractive index n is obtained from the retrieved phase of the overall interference signal for all incidence angles. © 2003 Optical Society of America

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1. Introduction

Knowledge of their refractive indices and of their thicknesses is fundamental to the characterization of crystals that are useful in optical devices and substrates for microelectronic applications. A long-standing problem, common to all interferometric methods when they are applied for measuring refractive index, is that interferometers are sensitive to the optical path difference, i.e., to the product nd of refractive index n and thickness d. In fact, to employ interferometric systems for determining the refractive index of a thin plate requires accurate knowledge of the thickness and vice versa. It is highly desirable to have a single method to obtain both thickness and refractive index, and efforts have been made to find valid approaches to measuring n and d simultaneously or separately or at least by using the same apparatus. Various interesting methods of addressing this task have been proposed. The use of low-coherence interferometry in combination with confocal microscopy has been demonstrated to be a valid approach to measuring simultaneously n and d of single plates as well as multiple-layer structures.[1] Furthermore, low-coherence interferometry has also been used in combination with an optical heterodyne microscope, with the group refractive index taken into account.[2–4] Recently, independent measurements of both refractive index and thickness were obtained by a combination of wavelength-scanning interferometry and confocal microscopy.[5] Wedge plates can be characterized by use of the same apparatus but different sources with wavelength-scanning interferometry and spatial fringes detection.[6] Moreover, simultaneous determination of thickness and refractive index of a thin film have also been demonstrated by an interferential spectrogoniometry.[7] However, these methods require complex instrumentation, such as translation stages with high resolution. We propose here a new method for measuring n and d of transparent plates by means of an extremely simple lateral shear interferometer, which is quite similar to that proposed by Wyant for measuring the optical transfer functions of optical systems.[8] We employ the same interferometer, but the object of our investigation is the shear plate itself. A tunable laser source is added to the lateral shear interferometer, transforming it into a lateral-shear, wavelength-scanning interferometer. In this paper we demonstrate the effectiveness of the proposed method in determining n and d for two types of ma-

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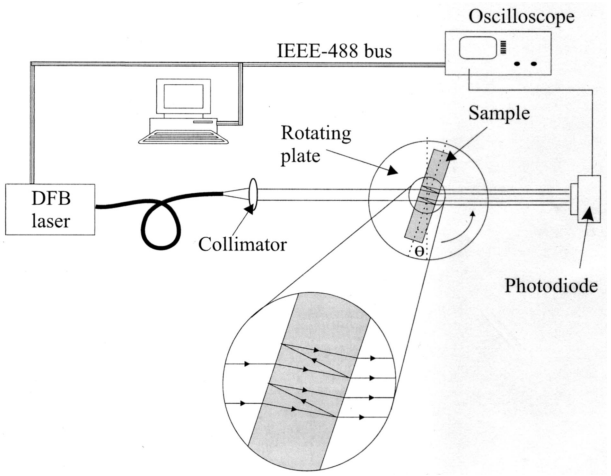

Fig. 1. Experimental setup for determining both refractive indices and thicknesses of transparent plates: DFB, distributed feedback.

terial: crystalline silicon (Si) and a uniaxial crystal of lithium niobate ($LiNbO_3$).

2. Method

Figure 1 presents schematically the optical configuration of the system. A collimated beam from a laser source propagates though the sample under investigation. The radiation passing through the sample experiences an even number of reflections at the sample–air interfaces. The emerging wave fronts are laterally sheared,[8] with a shear that depends on the actual angle of incidence θ. The sheared beams will be mutually coherent and interfere in the area where they are superimposed, producing an interference fringe pattern.

The geometry of the fringe pattern will depend on the phase wave front of the beams. If the laser beam is well collimated, the resultant fringe pattern will be a constant fringe field because the emerging wave fronts have optical path differences that are multiples of $2nd$, depending on the path followed inside the sample. A small detector along the traveling path of the beams and inside the interference area will detect the resultant intensity. If the sample is rotated, the angle of incidence will change and the interference fringe pattern will change its phase because the interfering wave fronts will be have a different optical path difference at each angle of incidence. When the sample is rotated at a constant angular speed, the intensity detected will be a chirped interference signal, as illustrated below.

When there is imperfect collimation, the situation is not much different. In fact, if a small amount of positive or negative (respectively, diverging or converging wave front) defocus is present, the interfering beams will produce, at the exit pupil of the system, some fringes parallel to the rotation axis of the sample. In this case the fringes should be large enough to cover the entire area of the detector to produce the same result as is obtained when there is perfect collimation. So we assume here, without loss of generality, that the laser beam is well collimated.[9]

Considering the direct transmitted beam and the first internally reflected beam, light intensity $I(\theta)$ at the detector, as a function of both the rotation angle and laser wavelength λ, is given by

$$I(\theta) = I_0 + \gamma \cos\left[\frac{4\pi nd}{\lambda}\left(1 - \frac{\sin^2\theta}{n^2}\right)^{1/2}\right], \quad (1)$$

where I_0 is a constant offset that is due to the intensities of two interfering beams and γ is related to the degree of coherence of light. In principle, by applying a phase retrieval technique to the experimental data it is possible from Eq. (1) to determine refractive index n and thickness d of the sample. Unfortunately, the nonlinear least-squares fit of the phase function in Eq. (1) does not in general provide accurate results for both n and d. When the sample thickness is known, the refractive index can be accurately determined, as was recently demonstrated with a technique for phase determination based on a Mach–Zehnder interferometer and a Fourier transformation.[10] Also, a Michelson interferometer was used for measuring the refractive index of a liquid,[11] with good results. In our method we made several simultaneous measurements of refractive index and thickness. The wavelength-scanning interferometry approach permitted us to measure optical thickness nd by the phase shifting method. When the wavelength of the laser diode is changed in steps Δλ, provided that $\Delta\lambda/\lambda \ll 1$, Eq. (1) experiences a phase shift that depends on angle θ, given by

$$\Delta\phi(\theta) = \frac{4\pi nd}{\lambda}\left(\frac{\Delta\lambda}{\lambda}\right)\left(1 - \frac{\sin^2\theta}{n^2}\right)^{1/2}, \quad (2)$$

which, at normal incidence ($\theta = 0$), simply becomes a constant phase shift, i.e., $\Delta\phi \cong 4\pi nd\Delta\lambda/\lambda^2$. Optical thickness nd can be obtained by a sort of calibration of the phase shift by a nonlinear fit by use of an equation with six parameters, A, B_1, B_2, B_3, C, and D, given by

$$I_{\theta=0}(\lambda) \approx A + \sum_{m=1}^{3} B_m \cos\left(m\frac{C}{\lambda} + D\right), \quad (3)$$

where $C = 4\pi nd$. We employed the interference among four beams to take into account multiple reflections that occur for $\theta = 0$ instead of using the simplest Eq. (1) that considers just the interference between two beams. Thus, fitting the experimental data to Eq. (3) allows the product nd to be evaluated independently.

We have described the first step of the procedure employed in the proposed method. Next we can calculate the phase of Eq. (1) from the retrieved phase of the overall interference signal for all available incidence angles. Two approaches can be adopted for analyzing the interferometric signal: use of a phase-shifting algorithm or of a fast Fourier transform (FFT).[10]

Independently of the approach employed (phase shifting algorithm or FFT), the refractive index of the sample is measured from the calculated phase by use of the previously evaluated optical thickness nd. Finally, geometric thickness d is obtained straightforwardly from knowledge of both nd and n. With birefringent materials it is possible to measure both the ordinary n_o and the extraordinary n_e refractive indices by separating the two perpendicular polarization states. In this case there are two equations each for n_o and n_e, namely,[10,12]

$$I_{n_p}(\theta) = I_0 + \gamma \cos\left[\frac{4\pi n_o d}{\lambda}\left(1 - \frac{\sin^2\theta}{n_p^2}\right)^{1/2}\right], \quad p = o, e \tag{4}$$

Note that the two equations for the ordinary and extraordinary refractive indices differ only in the term inside the square root, so calculation times can be speeded up.

3. Description of the Measuring Setup of the Lateral-Shear Wavelength-Scanning Interferometer

On the basis of the explanation above, we describe the experimental configuration adopted in our proposed measurement method. The laser source consists of a distributed-feedback (DFB) diode laser (Notel LC155GC-20A) with a peak wavelength near 1535 nm and a time-averaged spectral linewidth at −27 dB of 0.4 nm. We achieved linear wavelength tuning by varying the laser temperature. The linear relationship between wavelength emission and temperature was previously calibrated by a fiber optical spectrum analyzer (ANDO AQ-6315B) with a wavelength resolution of 0.02 nm. A fiber optic collimator was mounted upon the fiber pigtail end of the DFB laser. An InGaAs photodiode (Thorlab DET410) with an 0.8-mm^2 sensitive area and a 800–1800-nm spectral range was used as a detector along the path of the emerging beams.

The sample was mounted vertically upon a rotating dc motor, and a stable dc voltage applied to the motor guaranteed a constant rotation speed. The interferometric signal was digitized on an oscilloscope (Tektronix TDS540) with a sampling frequency of 2.5×10^6 points/s. Another InGaAs photodiode was positioned in the setup to detect the signal reflected by the sample. This signal was employed to feed the trigger to the oscilloscope. In this way it was possible to average the detected signal over 50 periods of rotation, reducing the noise. Moreover, the trigger signal was fundamental for independent measurement of the period of rotation. Laser temperature setting and interferometric signal recording were totally automated by means of an IEEE 488 bus control and a personal computer. In this way, interferometric signals were obtained as a function of rotation angle θ for each laser wavelength λ. In our experiments the laser wavelength variation was in the range [1535.28–1536.04] nm, with 150 steps of $\Delta\lambda = 0.005$ nm.

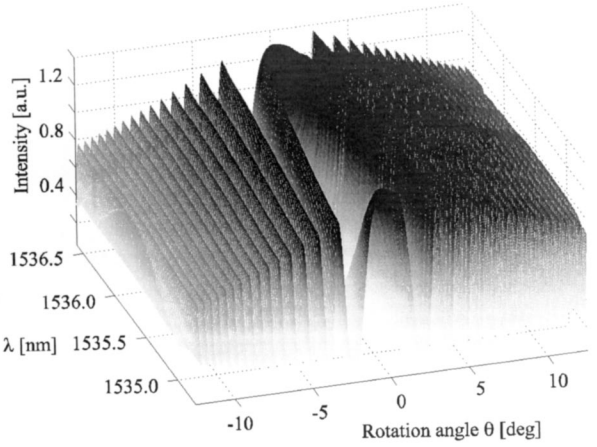

Fig. 2. Experimental interferometric signal versus wavelength λ and rotation angle θ for a Si sample.

Furthermore, we measured the ordinary n_o and the extraordinary n_e refractive indices of a $LiNbO_3$ crystal, too. For this purpose, a Glan–Thompson polarizer was inserted into the optical setup between the collimator and the crystal sample to permit selection of the polarization state of the incident light[9] by use of Eq. (4).

4. Experimental Results

We tested both a ⟨100⟩-type silicon sample and a uniaxial $LiNbO_3$ crystal z-cut sample. The silicon sample had a nominal refractive index of 3.47 and a thickness of 483 ± 5 μm; the $LiNbO_3$ sample had nominal ordinary and extraordinary refractive indices of 2.20 and 2.13, respectively, with a thickness of 500 ± 50 μm. The thickness and refractive-index values are commercial specifications for the samples used and are only approximate values. In Fig. 2 the overall interferometric signal for the Si sample phase shifted by means of DFB laser wavelength scanning, is shown as a function of incidence angles. For each wavelength the signal is composed of 5000 sampled points, for an angular range of −25° to +25°. The sampling rate that we adopted corresponds to an angular resolution of approximately 50–5000 = 0.01° per point. The rotation period for the Si sample is 14.60 ms. According to the approach described above, a nonlinear fit of the signal at $\theta = 0$ has to been made by use of relation (3) to evaluate the product nd. Figure 3 shows both the experimental data and the nonlinear fitting curve at normal incidence from which nd was estimated. By using the measured value of nd, however, one can obtain the refractive index by retrieving the phase of the overall interference signal. Figures 4(a) and 4(b), respectively, show the experimental signal and the corresponding cosine of the retrieved phase determined by the FFT method at a fixed wavelength. A nonlinear least-squares fitting procedure[13] applied to the data shown in Fig. 4(b) gives refractive index n for each wavelength. The average of all evaluated indices provides the determination of the refractive index.

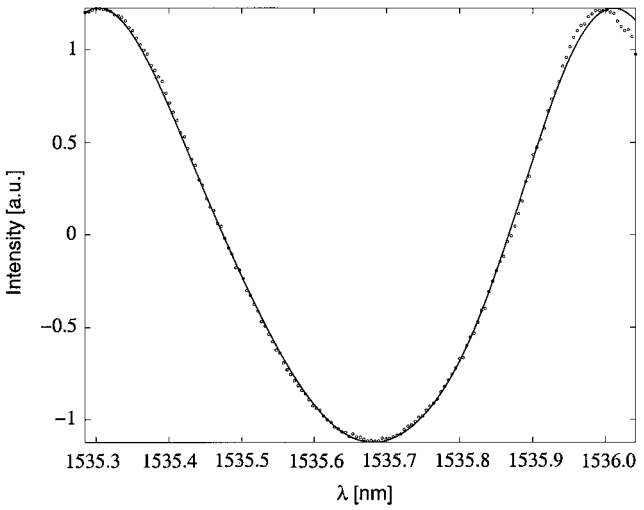

Fig. 3. Experimental (circles) and fitted (solid curve) intensity curves for normal incidence ($\theta = 0$) for a Si sample.

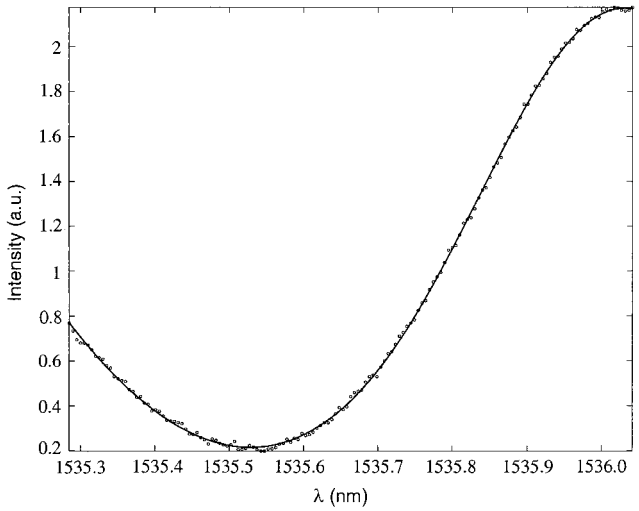

Fig. 5. Experimental (circles) and fitted (solid curve) intensity curves for normal incidence ($\theta = 0$) for a $LiNbO_3$ sample.

Measured values obtained for the Si sample are $n = 3.4755 \pm 0.0003$ and $d = 483.93 \pm 0.04$ μm.

Figure 5 shows the wavelength-scanning data and the fitting curve at normal incidence for the $LiNbO_3$ crystal z-cut plate. At normal incidence, the optic axis of the crystal plate lies along the direction of the incidence beam. Depending on the direction of polarization, the correct expression for Eq. (4) can be used. In fact, the refracted waves experience different indices of refraction, depending on their polarization, i.e., ordinary index n_o for s polarization and extraordinary index n_e for p polarization. Applying the method described above to a signal composed of 5000 sampled points and relative to a rotation period of 22.0 ms, we obtain $n_o = 2.2004 \pm 0.0001$, $n_e = 2.1360 \pm 0.0002$, and $d = 500.0 \pm 0.3$ μm.

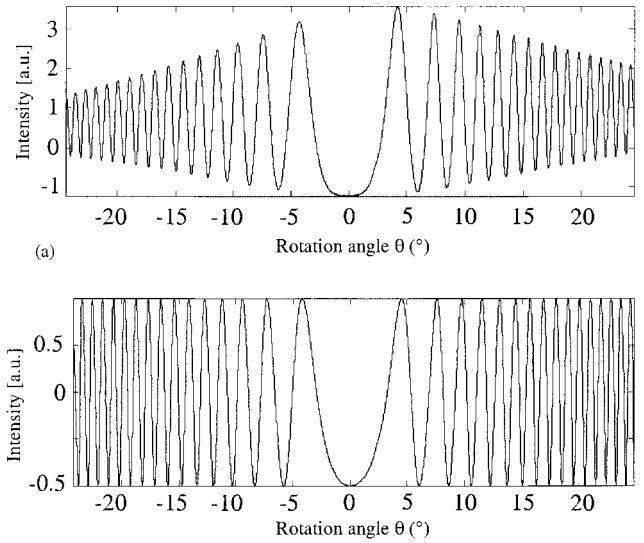

Fig. 4. (a) Experimental signal and (b) corresponding cosine of the retrieved phase at a fixed wavelength (1535.68 nm).

5. Discussion

In this section, first we analyze the sensitivity of the method. Next, we discuss briefly the possible causes of errors, and finally we evaluate limitations of the method in measuring the refractive index.

Reported experimental results show an uncertainty of less than 10^{-4} in determination of the refractive index, and it is easy to illustrate that the proposed method allows this level of accuracy. In fact, the refractive index is obtained, once the product nd has been evaluated, by processing of the fringe signal like that shown in Fig. 4(a). The interferometric signal was processed by the FFT method.[14]

It is well known that the FFT method has accuracy and resolution typically limited to $\lambda/20$ and $\delta\varphi = 2\pi/20 \approx 0.3$ rad in terms of wavelength and phase, respectively.[12] However, from expression (3) it is easy to estimate the sensitivity of the method by considering the phase change that corresponds to a refractive-index variation of 0.0001. In fact, from expression (3), it is possible to obtain the phase change $\delta\varphi(\theta)$ in the interferometric signal corresponding to a refractive change δn by evaluating $\delta\varphi = (\delta\varphi/\delta n)\delta n$. In Fig. 6, the phase change $\delta\varphi(\theta)$ for $\delta n = 0.0001$ is plotted for the Si plate. It is clear from the figure that the phase variation is even greater than 0.3 [$\delta\varphi(\theta) > 0.3$ rad], meaning that the proposed method has enough sensitivity to permit such small refractive-index variations to be measured.

Provided that the rotation speed is kept constant, the accuracy of the method is essentially limited by two factors. The first is the accuracy of measurement of the rotation period, which allows the angle of incidence to be measured, and the second factor is the sampling rate adopted for acquisition of the interference signal at the photodiode. The sampling rate at the oscilloscope determines the accuracy in measurement of the angle of the interferometric signal. Systematic speed variations or accidental external

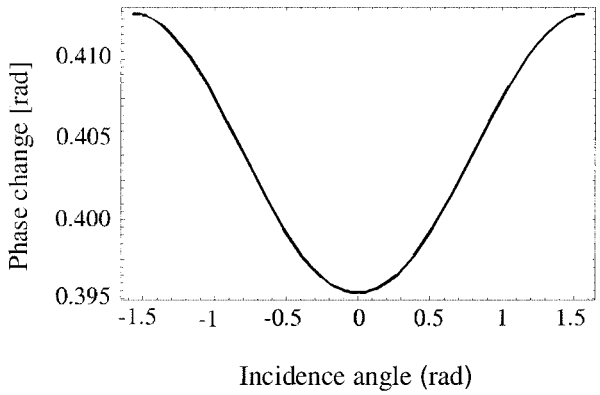

Fig. 6. Calculated sensitivity of phase change $\delta\varphi(\theta) = (\partial\varphi/\partial n)\delta n$ of the interferometric signal corresponding to the refractive change $\delta n = 0.0001$ for a Si sample.

effects that perturb the rotation of the sample are quite easily detectable by observation of the signal of Fig. 2 from another prospective. Figures 7(a) and 7(b) show density plots of two discarded interferometric signals, for the Si and the $LiNbO_3$ samples, respectively; variations and fluctuations of the rotating speed owing to the presence of external effects are clearly visible.

To determine nd by nonlinear fit with the model given by expression (3), the phase shift needs to satisfy the condition

$$4\pi n d\Delta\lambda_{\mathrm{Max}}/\lambda^2 \geq \pi.$$

This requirement poses a limitation on the minimum thickness of the plate to be characterized as well as on wavelength-scanning range $\Delta\lambda_{\mathrm{Max}}$. Thickness d has to be greater than

$$d_{\mathrm{min}} = \lambda^2/(4n\Delta\lambda_{\mathrm{Max}}).$$

In our case $\Delta\lambda_{\mathrm{Max}} = 0.76$ nm, and the corresponding minimum thickness is $d_{\mathrm{min}} = 220$ μm. Of course, a larger scanning range permits testing of thinner samples, but the contribution of the dispersion effect should be evaluated. In our case, the index change that is due to dispersion can safely be assumed to be negligible. For Si we have $dn/d\lambda = -7.8 \times 10^{-5}$ nm^{-1}, whereas for $LiNbO_3$ $dn_o/d\lambda = -3.4 \times 10^{-5}$ nm^{-1} and $dn_e/d\lambda = -2.9 \times 10^{-5}$ nm^{-1}.

Because accurate determination of normal incidence $\theta = 0$ is critical for precise measurement, we adopted an optical alignment for setting the zero angle of the plate. We obtained the alignment by superimposing the spot of the beam reflected by the crystal surface upon that of the incident beam on target. Furthermore, we checked the consistency of the normal incidence by determining the angular po-

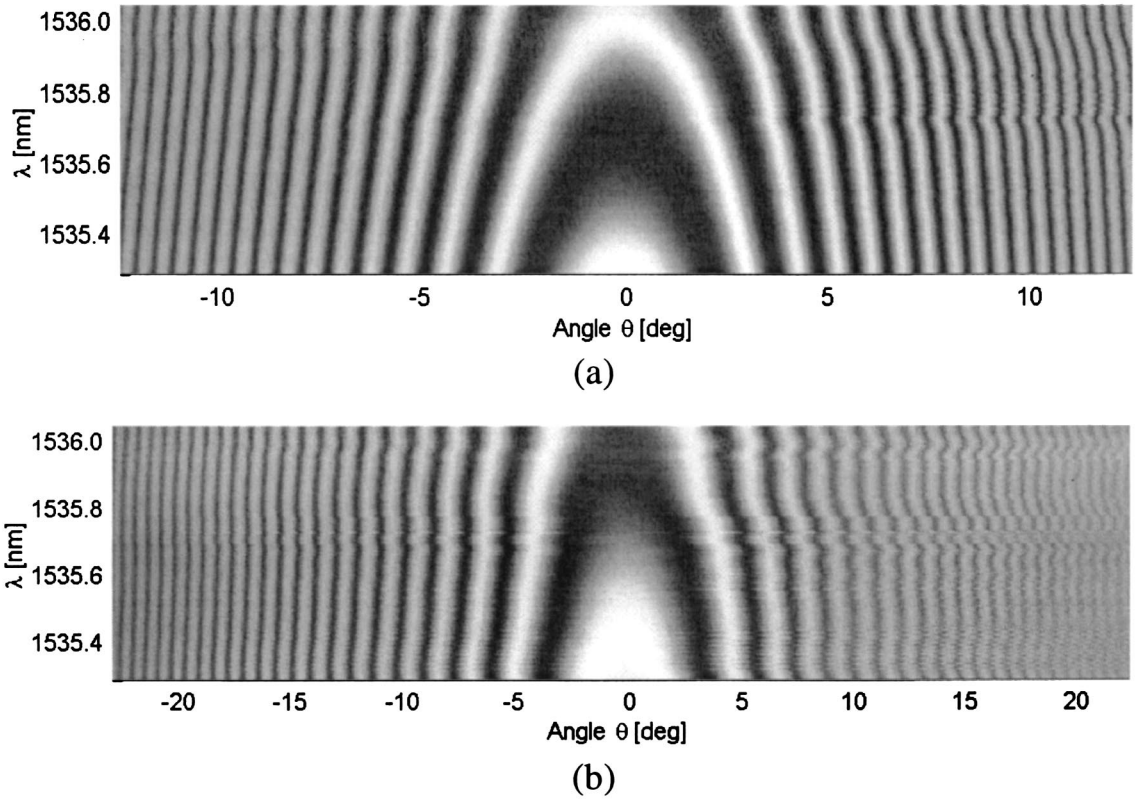

Fig. 7. Density plot of experimental interferometric signals as function of wavelength λ and rotation angle θ for (a) a Si sample and (b) a $LiNbO_3$ sample. These signals were discarded because external effects perturbed the rotation period, as is clearly illustrated by local discontinuities in the pattern.

sition for $\theta = 0$ from the symmetry of the signal of the wrapped phase shown in Fig. 4(b).

Finally, we could make a rough estimation of the time for measurement by taking into account the rotation period ($\sim$20 ms), the number of averages (50), and the rate of change of the laser wavelength ($<$0.1 s). They led us to estimate a total measurement time of $\sim$165 s. However, improvements of the experimental setup to reduce the number of averages could result in shorter measurement times.

6. Conclusion

In conclusion, we have described a new method for measuring both refractive indices and thicknesses of plates without the need to know accurately either of the two quantities. This constitutes a significant improvement on other interferometric techniques in which the main limitation is the limited accuracy in knowledge of d. With respect to other proposed methods for simultaneous measurement of n and d, our experimental setup is simple, as it requires a minimum of optical alignment and does not require high-precision motorized translation stages, as in confocal microscopy and low coherence methods. However, a lateral-shear, wavelength-scanning interferometer can be used for measuring only single-layer structures at present.

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